$$I(z) = I(0)\, e^{\gamma_0 z}$$

$$G \equiv \frac{I_2}{I_1} = e^{\gamma_0 L} \qquad \text{small sig gain}$$

Often characterize gain by dB:

$$\text{dB gain} = 10 \log_{10} G$$

$$\text{dB gain} = 10\, \gamma_0 L \log_{10} e$$

$$= 4.34\, \gamma_0 L$$

dB	G
0	1
10	10
20	100
30	1000

Note: dB applies to absorption also

$$\text{dB loss} = 4.34\, \alpha L$$

$$\text{dB net gain} = 4.34\, (\gamma_0 - \alpha) L$$

Example:

Fiber amp has a net gain of 25 dB in 8 m. If fiber loss is .02 m^{-1}, find γ_0

$$\gamma_0 - \alpha = \frac{25}{8 \cdot 4.34} = .72\ m^{-1}$$

$$\gamma_0 = .74\ m^{-1}$$

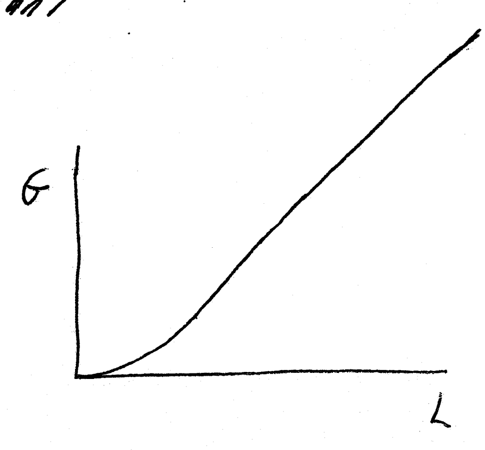
(2) Large Signal limit $\quad I \gg I_s$

$$\frac{dI}{dz} \simeq \gamma_0 I_s = \text{constant}$$

$$\Delta I \simeq \gamma_0 I_s\, \Delta z$$

$$I_2 - I_1 = \gamma_0 I_s L$$

$$G - 1 = \frac{I_s}{I_1} \gamma_0 L$$



Compare power in & out:

$$\left.\begin{aligned} P_{in}^{sig} &= I_1 A \\ P_{out}^{sig} &\approx I_2 A \end{aligned}\right\} \quad \Delta P^{sig} = (I_2 - I_1) A$$

$$= \gamma_0 I_s L A$$

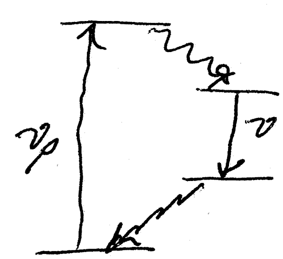
$$= \mathcal{R} h\nu \, LA$$

Pump power absorbed is

$$P_{abs}^{pump} = \mathcal{R}(LA)\, h\nu_p$$

∴ efficiency

$$\eta \equiv \frac{\Delta P^{sig}}{P_{abs}^{pump}} = \frac{h\nu}{h\nu_p}$$



This limit on efficiency referred to as "quantum defect". Applies to laser also.

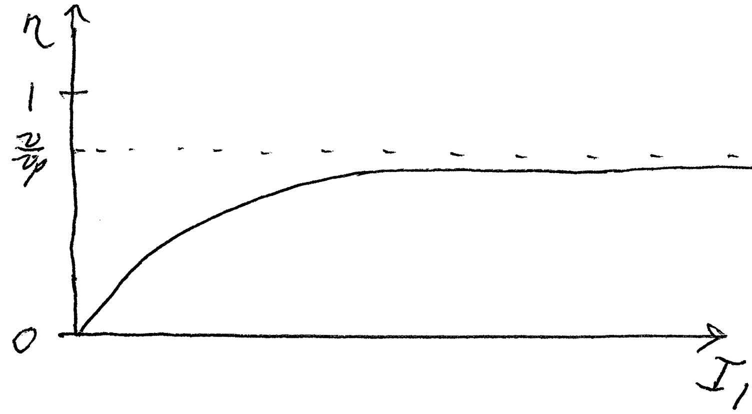
Efficiency much smaller in small-signal limit:

$$\eta_{small\,I} \simeq \frac{(G-1) I_1 A}{\mathcal{R} LA\, h\nu_p} = \frac{(e^{\gamma_0 L} - 1) I_1}{\mathcal{R} L\, h\nu_p}$$

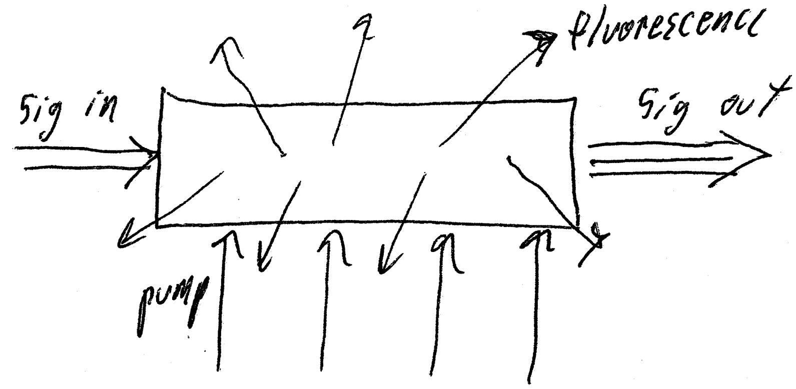
For short length $\gamma_0 L \ll 1$

$$\eta_{small\,I} \simeq \frac{\gamma_0 L\, I_1}{\mathcal{R} L\, h\nu_p} \approx \frac{(\mathcal{R}\sigma\tau_2) I_1}{\mathcal{R}\, h\nu_p} = \frac{I_1 \sigma \tau_2}{h\nu_p}$$

$$\eta_{small\,I} \simeq \frac{I_1}{I_s} \frac{h\nu}{h\nu_p} \ll 1 \qquad \text{since } I_1 \ll I_s$$



Where does energy go when $\eta < 1$?



Amplifier gain: general case:

$$\frac{1}{I}\frac{dI}{dz} = \frac{\gamma_0}{1+\frac{I}{I_s}}$$

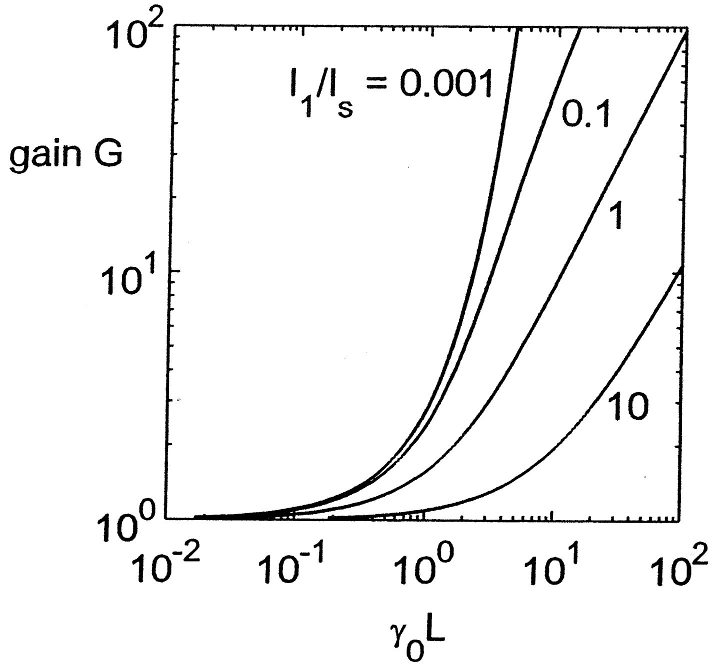
$$\int_{I_1}^{I_2}\left(\frac{1}{I}+\frac{1}{I_s}\right)dI = \int_0^L \gamma_0 \, dz$$

$$\ln\left(\frac{I_2}{I_1}\right) + \frac{I_2 - I_1}{I_s} = \gamma_0 L$$

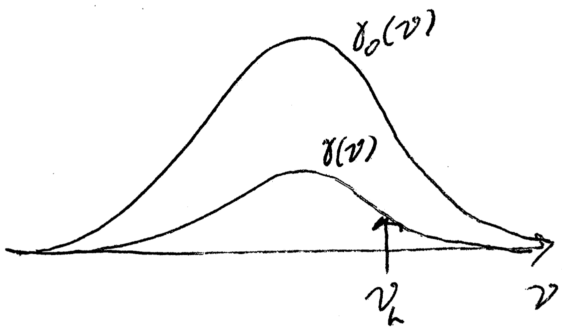
$$\ln G + \frac{I_1(G-1)}{I_s} = \gamma_0 L$$

Implicit eq. for G.

Can plot I_2 vs I_1 by treating G as variable, & solve for I_1 in above. Then $I_2 = G I_1$

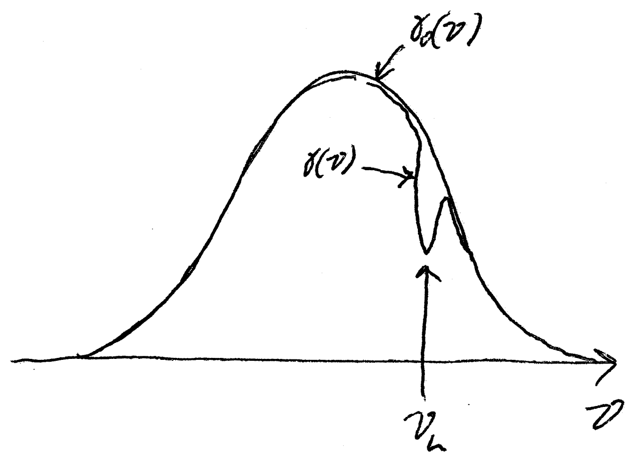


Inhomogeneous Saturation



homogeneous

$$\gamma(\nu, I) = \frac{\gamma_0(\nu)}{1 + I/I_s}$$



inhomogeneous
"hole burning"

$$\gamma(\nu, I) \simeq \frac{\gamma_0(\nu)}{\sqrt{1 + I/I_s}}$$