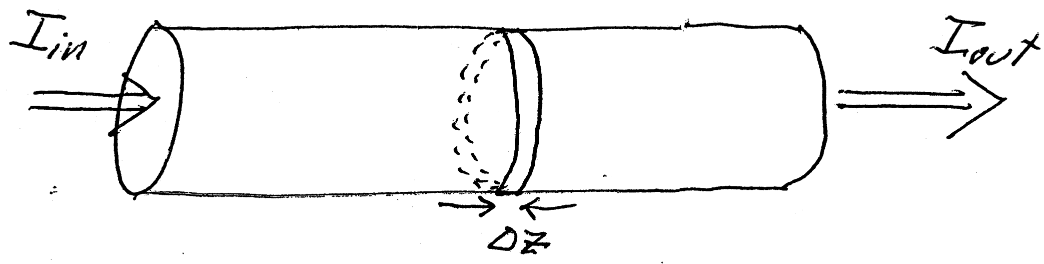
Optical Amplifiers



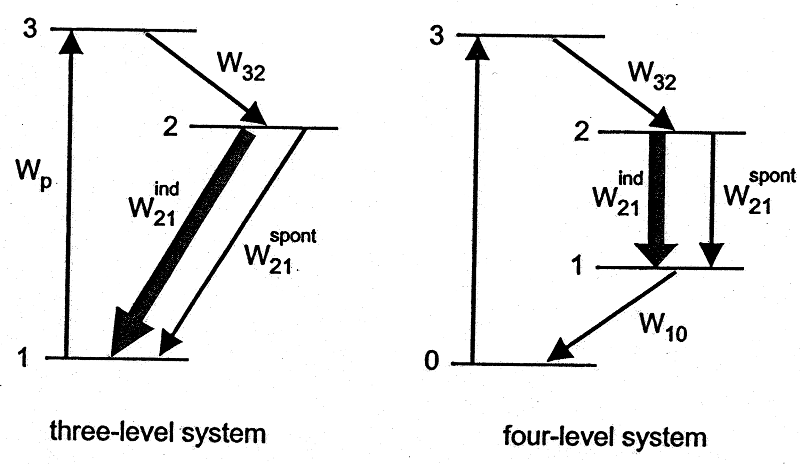
$$\frac{\Delta I}{I} = \gamma \, \Delta z$$

γ may not be constant (saturation)

$$\gamma = (N_2 - N_1)\,\sigma$$

Use rate equations to determine N_1 & N_2

Two types of laser systems:



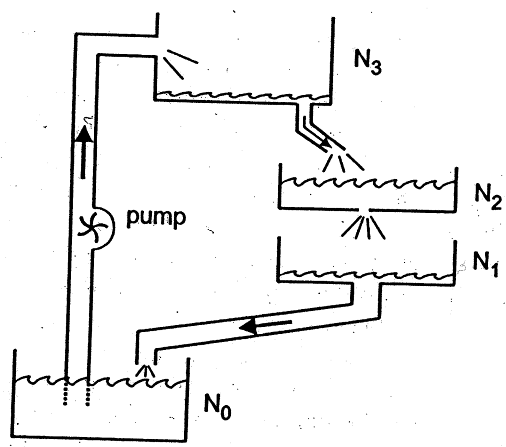
example	example
ruby	Nd: YAG

3-level system: lower laser level is ground state

For positive gain, need $N_2 > N_1$ (population inversion)

So 3-level needs at least half of atoms in excited state $\Longrightarrow$ difficult to achieve

Flow of excitation energy similar to water flow:



larger holes $\Longrightarrow$ faster decay

$W_{32} = \dfrac{1}{\tau_{32}}$ etc.

Assume $\begin{cases} \tau_{32} \ll \tau_{21} \\ \tau_{10} \ll \tau_{21} \end{cases}$

so level 2 is "bottleneck"

$N_3 \ll N_2$

$N_1 \ll N_2$

$N_0 + N_2 \simeq N$ ← $\dfrac{\text{total \# atoms}}{\text{Vol}}$

Rate equation becomes

$$\frac{dN_2}{dt} = N_0 W_p - N_2 W_{21}^{ind} - N_2 \frac{1}{\tau_2}$$

Also assume for now $N_2 \ll N_0$, so $N_0 \simeq N$ is constant

Total decay from level 2

$$W_2 = \frac{1}{\tau_2} \simeq \frac{1}{\tau_{21}} + \frac{1}{\tau_{20}}$$

Assume for now little depletion of ground state:

$$N_2 \ll N_{tot}$$

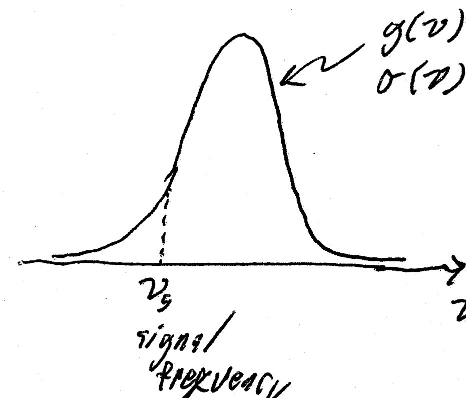
Re-write rate eq. in terms of light intensity

$$W_{21}^{ind} = B_{21} \rho_\nu g(\nu)$$

$$= \left(\frac{A_{21}}{\frac{8\pi\nu^2}{c^3} h\nu}\right)\left(\frac{I}{c}\right) g(\nu)$$

$$= \left(\frac{A_{21} \lambda^2 g(\nu)}{8\pi}\right) \frac{I}{h\nu}$$

$$\boxed{W_{21}^{ind} = \frac{I\,\sigma(\nu)}{h\nu}}$$



Define pumping rate

$$\mathcal{R} \equiv W_p N_0 \qquad \left[\frac{\text{atoms pumped}}{\text{Vol} \cdot \text{time}}\right]$$

Rate eq. becomes

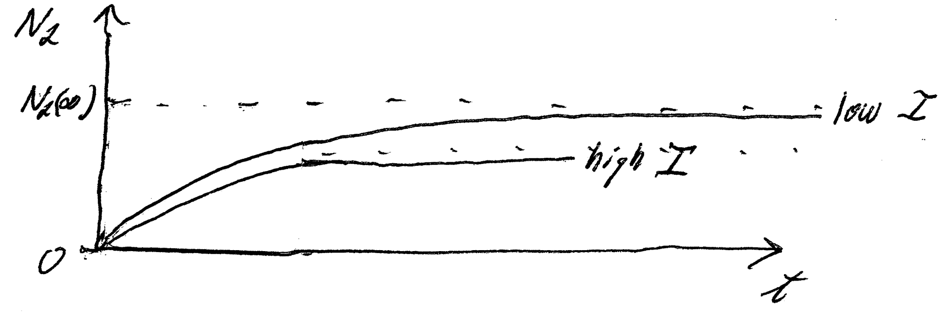
$$\frac{dN_2}{dt} = \mathcal{R} - N_2\left(\frac{I\sigma}{h\nu} + \frac{1}{\tau_2}\right)$$

or $$\frac{dN_2}{dt} = \mathcal{R} - \frac{N_2}{\tau_2'}$$

$$\frac{1}{\tau_2'} \equiv \frac{1}{\tau_2} + \frac{I\sigma}{h\nu}$$

This diff eq similar to that for charging cap:

$$N_2(t) = N_2(\infty)\left[1 - e^{-t/\tau_2'}\right]$$



Solve for $N_2(\infty)$ by setting $\frac{dN_2}{dt} = 0$

$$0 = R - \frac{N_2}{\tau_2'}$$

$$N_2(\infty) = R\tau_2'$$

$$= \frac{R}{\frac{1}{\tau_2} + \frac{I\sigma}{h\nu}} = \frac{R\tau_2}{1 + \frac{I\sigma}{h\nu}\tau_2}$$

Note: as I increases, N_2 rises more quickly (τ_2' decreases), but to a smaller final value.

Steady-state value $N_2(\infty)$ important, since it limits gain:

$$\frac{1}{I}\frac{dI}{dz} \equiv \gamma(\nu) = A_{21}\frac{\lambda^2}{8\pi}g(\nu)\,N_2 = \sigma(\nu)\,N_2$$

$$\gamma(\nu) = \frac{R\,\sigma(\nu)\,\tau_2}{1 + \frac{I\sigma\tau_2}{h\nu}}$$

Define "saturation intensity" where gain drops by factor of 2:

$$\frac{I_s \sigma \tau_2}{h\nu} = 1$$

$$\boxed{I_s = \frac{h\nu}{\sigma \tau_2}}$$

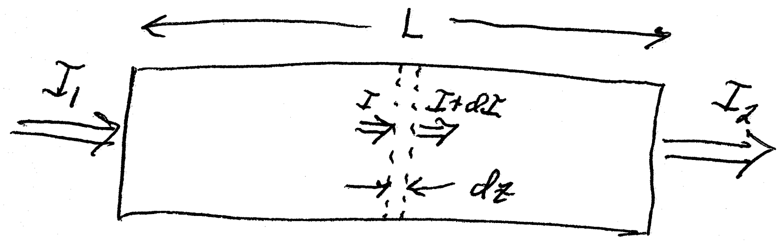
Then can write

$$\gamma = \frac{R \sigma \tau_2}{1 + I/I_s} = \frac{\gamma_0}{1 + I/I_s}$$

where $\gamma_0 \cong R \sigma \tau_2$

Note: long τ_2 gives higher small signal gain but gain saturation occurs at lower I

Total Gain of Optical Amplifier



$$\frac{1}{I}\frac{dI}{dz} = \frac{\gamma_0}{1 + \frac{I}{I_s}}$$

Two limits:

(1) Small Signal $I \ll I_s$

$$\int \frac{dI}{I} \simeq \int \gamma_0 \, dz$$

$$\ln I \Big|_{I_1}^{I_2} = \gamma_0 z \Big|_0^L$$