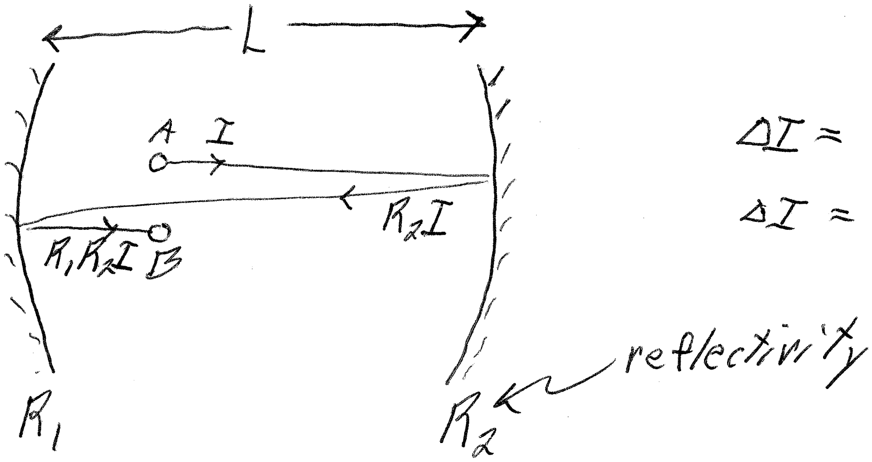
Photon Lifetime (or cavity lifetime)



$$\Delta I = R_1 R_2 I - I$$

$$\Delta I = I(R_1 R_2 - 1)$$

Time interval for change ΔI is $\Delta t_{RT} = \frac{2L}{c}$

so

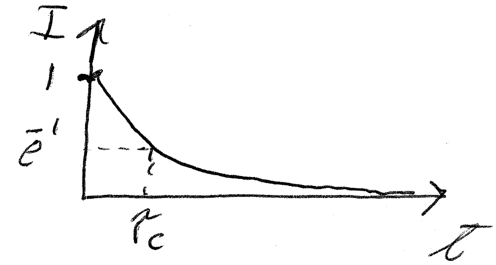
$$\frac{\Delta I}{\Delta t} = -\frac{(1 - R_1 R_2)}{2L/c} I$$

Define $\tau_c \equiv \frac{2L}{c(1 - R_1 R_2)} = \frac{\Delta t_{RT}}{1 - R_1 R_2}$ cavity lifetime

If $\tau_c \gg \Delta t_{RT}$, then

$$\frac{dI}{dt} \simeq -\frac{1}{\tau_c} I$$

Solution $I(t) = I_0 e^{-t/\tau_c}$



Cavity lifetime is time for intensity to decay by factor e^{-1}

For $\tau_c \sim \Delta t_{RT}$ do not have exponential decay. But can still define τ_c as time for intensity to drop by factor e^{-1}

$p \equiv$ # round trips in time τ_c

$$\frac{I}{I_0} = (R_1 R_2)^p = e^{-1}$$

$$p \ln(R_1 R_2) = -1$$

$$p \ln\left(\frac{1}{R_1 R_2}\right) = 1$$

$$p = \frac{1}{\ln\left(\frac{1}{R_1 R_2}\right)}$$

$$p = \frac{\tau_c}{\Delta t_{RT}}$$

$$p = \frac{\tau_c}{2L/c}$$

So,
$$\frac{\tau_c}{2L/c} = \frac{1}{\ln(1/R_1 R_2)}$$

or
$$\boxed{\tau_c = \frac{2L/c}{\ln(1/R_1 R_2)}}$$

replace $c \to c/n$ in medium

Exercise: use $\ln(1+x) \approx x$ for $x \ll 1$ to show this agrees with previous result for $R_1 R_2 \approx 1$

Mode frequency width

$$\Delta\omega \, \tau_c \sim 1$$

$$\Delta\omega \sim (1 - R_1 R_2) \frac{c}{2L} \qquad \text{for } R_1 R_2 \approx 1$$

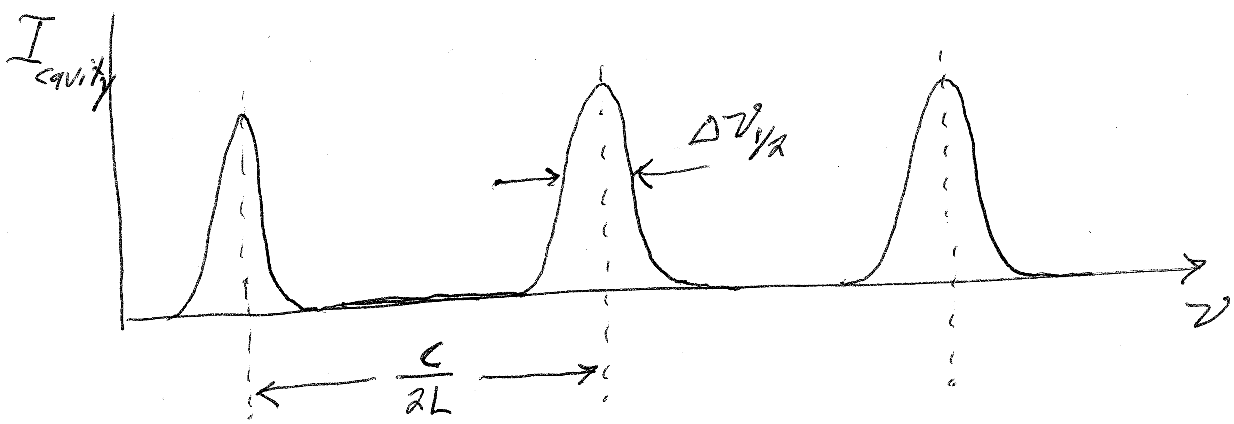
$$\Delta\nu \sim \frac{1}{2\pi}(1 - R_1 R_2)\frac{c}{2L}$$

$$\Delta\nu_{1/2} = \frac{1}{2\pi}(1 - R_1 R_2)\frac{c}{2L} \qquad \text{exact in limit } R_1 R_2 \to 1$$

$\Delta\nu_{1/2} \equiv$ full width at half maximum (FWHM)

For arbitrary $R_1 R_2$,

$$\Delta\nu_{1/2} = \frac{1}{2\pi} \ln(1/R_1 R_2) \frac{c}{2L}$$



- $\boxed{\delta\nu = \frac{c}{2L}}$ free spectral range
- modes are "quantum states" of EM field
- for small $\Delta\nu_{1/2}$ (coherent light) need high $R_1 R_2$
- for high R, use multi-layer dielectric film

Quality Factor

$$Q \equiv \frac{\omega_0}{\Delta\omega_{1/2}} \qquad \text{where} \quad \omega_0 = \text{center of resonance}$$

$$Q = \frac{m\, {}^{c}/_{2L} \cdot 2\pi}{(1 - R_1 R_2)\, {}^{c}/_{2L}} = \frac{m \cdot 2\pi}{1 - R_1 R_2} \qquad (R_1 R_2 \approx 1)$$

* higher Q means "sharper" resonance
* larger mode number m gives higher Q

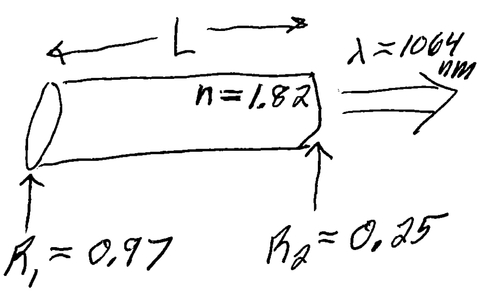
For arbitrary $R_1 R_2$,

$$Q = \frac{m \cdot 2\pi}{\ln(1/R_1 R_2)}$$

Example:

Nd:YAG rod with dielectric mirrors coated on ends. $L = 7$ cm

Find Q.



Solution: $\nu_m = m \dfrac{c}{2nL} = \dfrac{c}{\lambda}$

$$m = \frac{2nL}{\lambda} = \frac{2(1.82)(0.07)}{1.064 \cdot 10^{-6}} = 2.40 \cdot 10^{5}$$

$$Q = \frac{(2.4 \cdot 10^{5}) \cdot 2\pi}{\ln\left(\frac{1}{[.97][.25]}\right)} = \boxed{1.06 \cdot 10^{6}}$$

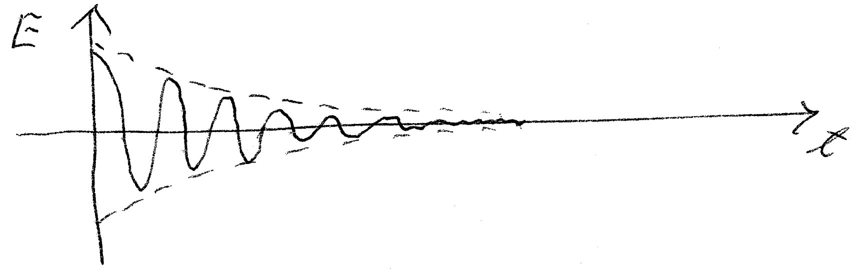
Using approximate expression,

$$Q \approx \frac{m \cdot 2\pi}{1 - R_1 R_2} = 1.98 \cdot 10^{6} \qquad \text{too high}$$

Need $R_1 R_2 > 0.8$ for 10% accuracy

Interpretation of Q:

$$Q = \frac{\omega_0}{\Delta\omega_{1/2}} = \omega_0 \tau_c = 2\pi \cdot (\#\text{oscillations in } \tau_c)$$



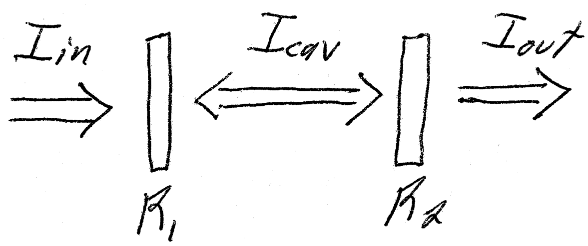
Finesse

$$\mathcal{F} \equiv \frac{c/2L}{\Delta\nu_{1/2}} \qquad \frac{\text{mode spacing}}{\text{mode width}}$$

$$\mathcal{F} = \frac{c/2L}{\frac{1}{2\pi}(1-R_1R_2)\, c/2L} = \frac{2\pi}{1-R_1R_2} \qquad (\text{large } R_1R_2)$$

Note that $\mathcal{F}$ independent of m or L

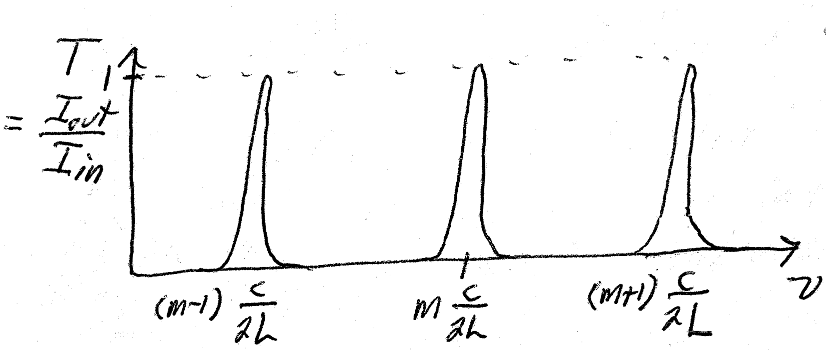
Fabry-Perot Interferometer



$I_{out} = T_2 I_{cav} = (1-R_2) I_{cav}$

But at resonance, $I_{out} = I_{in}$

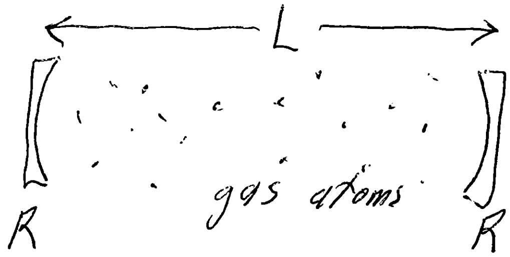
so $I_{cav} = \dfrac{I_{in}}{1-R_2}$



* Have $I_{cav} \gg I_{in}$ at resonance

* Can tune "filter" by varying L (or index of medium between)

Cavity Ring-down Spectroscopy



$\alpha \equiv \dfrac{\text{fraction of light absorbed}}{\text{unit length}}$

Assume $R \simeq 1$

$\alpha L \ll 1$

In one round trip $\Delta t_{RT} = \dfrac{2L}{c}$

$\alpha 2L$ = fraction absorbed in RT

$$\Delta I = (1-\alpha 2L) R^2 I - I$$
$$= -I\left[(1-R^2) + 2\alpha L R^2\right]$$
$$\simeq -I\left[(1-R^2) + 2\alpha L\right]$$

$(1-2\alpha L)$ = fraction not absorbed in RT

$$\frac{dI}{dt} = -\frac{I\left[(1-R^2) + 2\alpha L\right]}{2L/c} = -\frac{I}{\tau_c}$$

where now

$$\frac{1}{\tau_c} = \frac{c}{2L}\left[(1-R^2) + 2\alpha L\right]$$

$$\boxed{\frac{1}{\tau_c} = \frac{1}{\tau_{c0}} + c\alpha}$$

if $\alpha(\lambda)$ then $\tau_c(\lambda)$