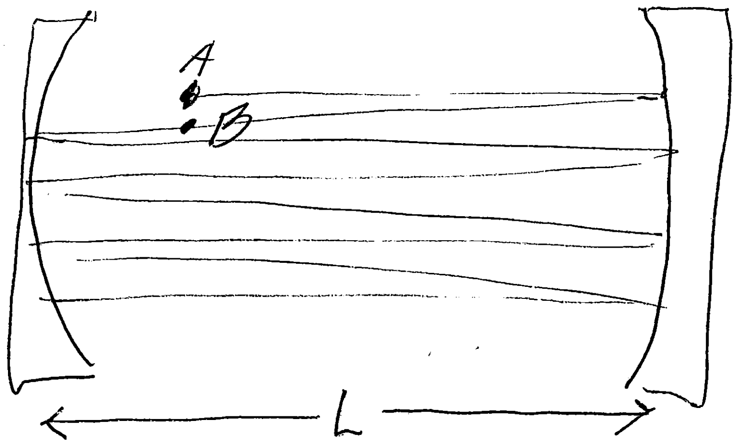
Optical Resonators



Phase at A and B must be the same for standing wave resonance

$$E(x,t) = E_0 \cos(kx - \omega t)$$

In one round trip,

$$\Delta\phi = k\,\Delta x \approx k\,2L = m\,2\pi$$

$$\frac{2\pi}{\lambda} \cdot 2L = m\,2\pi$$

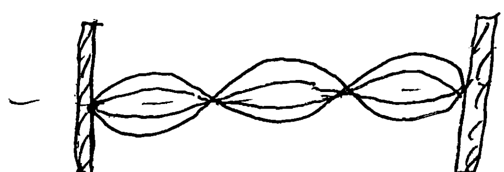
$$\boxed{2L = m\lambda}$$

$L \sim 1\,m$

$\nu \sim 10^{15}\,Hz$

$m = \frac{2L\nu}{c} \sim \frac{2 \cdot 10^{15}}{3 \cdot 10^{8}}$

$m \sim 10^{7}$



$m = 3$

$$\nu_m = \frac{c}{\lambda} = m\,\frac{c}{2L}$$

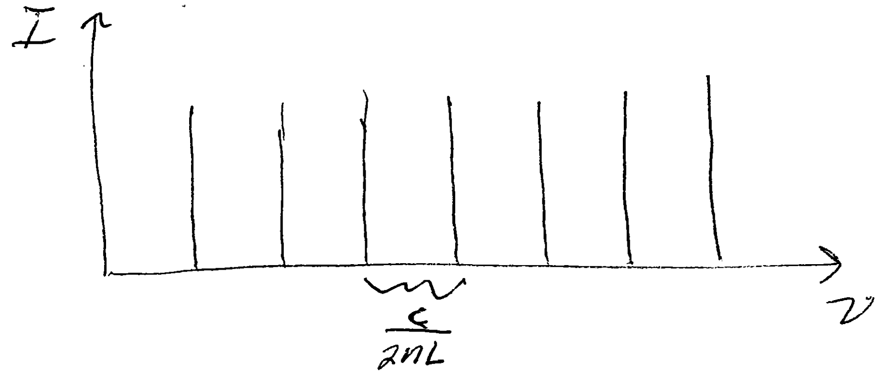
In medium with index n,

$$\boxed{\nu_m = m\,\frac{c}{2nL}}$$

$\frac{2Ln}{c} = \Delta t_{RT}$ round-trip time

so $\nu_m = m\left(\frac{1}{\Delta t_{RT}}\right)$ resonance (like pushing swing)

Mode Spectrum



Example: calculate mode spacing for HeNe laser with L = 15 cm, both in frequency and wavelength

Gas laser: $n \approx 1$

$$\Delta\nu = \frac{c}{2L} = \frac{3\cdot10^{8}\ m/s}{2(0.15\ m)} = 1\cdot10^{9}\ Hz$$

To find $\Delta\lambda$, use $\nu = \frac{c}{\lambda}$

For small $\Delta\nu$, $\Delta\nu \cong \left|\frac{d\nu}{d\lambda}\right|\cdot\Delta\lambda = \frac{c}{\lambda^2}\Delta\lambda$

$$\therefore\ \Delta\lambda = \frac{\lambda^2}{c}\Delta\nu = \frac{(633\cdot10^{-9})^2}{3\cdot10^{8}}(1\cdot10^{9})$$

$$\Delta\lambda = 1.33\cdot10^{-12}\ m$$

$\nu = \frac{3\cdot10^{8}}{6.33\cdot10^{-7}}\ Hz$

$\approx 4.74\cdot10^{14}$

$\frac{\Delta\lambda}{\lambda} \approx -\frac{\Delta\nu}{\nu}$

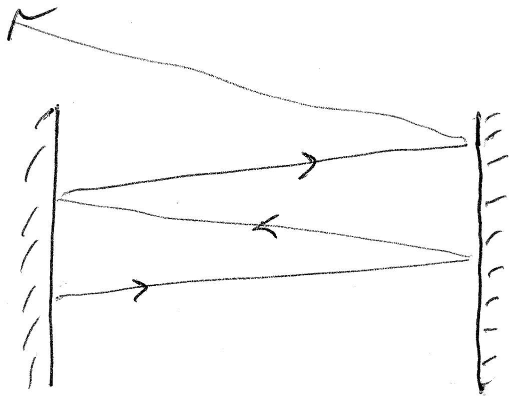
Mode density

linear mode density $= \frac{\#\ \text{modes}}{(\text{length})(\text{freq. interval})} = \frac{2nL}{Lc} = \frac{2n}{c}$ 1-D independent of ν

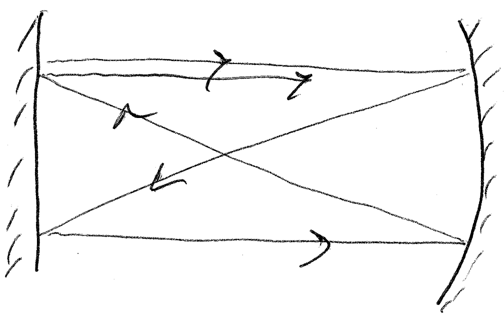
volume mode density $\rho_\nu = \frac{8\pi n^{3}\nu^{2}}{c^{3}}$ for 3-D cavity

see text for derivation

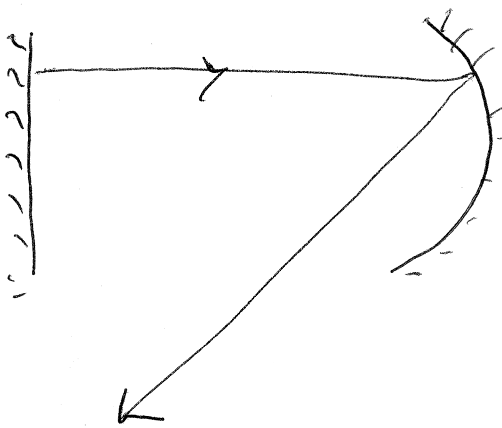
Stability of Modes between Mirrors



unstable



Maybe
Stable



Unstable

Unstable resonators not good for continuous wave (cw) lasers but may be useful for pulsed lasers.