Homework #5

21.3 (a) The frequency spacing is

$$\delta\nu = \frac{c}{2nL} = \frac{3 \times 10^8}{2(1.8)(3 \times 10^{-3})} = 2.78 \times 10^{10} \text{ Hz} = 27.8 \text{ GHz}$$

(b) Taking $\alpha \approx 0$, the threshold gain coefficient is

$$\gamma_{th} = \alpha + \frac{1}{2L} \ln\left(\frac{1}{R_1 R_2}\right) = \frac{1}{2(3 \times 10^{-3})} \ln\left(\frac{1}{[0.995][0.98]}\right) = 4.2 \text{ m}^{-1}$$

(c) Using the lineshape function given in Eq.(18-49), the variation of gain coefficient with frequency is

$$\gamma(\nu) = \gamma_{max} \frac{1}{1 + \left(\frac{\nu - \nu_0}{\Delta\nu/2}\right)^2}$$

where $\Delta\nu = 5000$ GHz is the full width at half maximum. The mode nearest the one with maximum gain has a frequency difference $\nu - \nu_0 = \delta\nu = 27.8$ GHz, and a gain coefficient

$$\gamma = \frac{\gamma_{max}}{1 + \left(\frac{27.8}{2500}\right)^2} = \frac{\gamma_{max}}{1.00012}$$

The gain of adjacent modes therefore only differs by about 0.012%. Because of fluctuations in pumping level and other environmental perturbations, it is likely in this case that the laser will lase on several modes simultaneously, or at least hop from one mode to another.

21.4 (a) The frequency spacing is

$$\delta\nu = \frac{c}{2nL} = \frac{3 \times 10^8}{2(1.1)} = 1.364 \times 10^8 \text{ Hz} = 136.4 \text{ MHz}$$

(b) When $P_{in} = 2P_{th}$, $\gamma_{max} = 2\gamma_{th}$, and when the gain is half its maximum value, $\gamma = \gamma_{th}$. Therefore, all the modes within the full width at half maximum (FWHM) can potentially lase. The number of modes within the FWHM is

$$N = \frac{\Delta\nu}{\delta\nu} = \frac{3.5 \text{ GHz}}{0.1364 \text{ GHz}} = 25.7$$

This should be rounded up to the nearest whole number (because, for example, if N were just slightly larger than 1, then 2 modes could potentially fall within the FWHM). The number of possible lasing modes here is therefore 26.

(c) We need to find the range of frequencies for which lasing can occur. The requirement is that $\gamma > \gamma_{th}$, and we can use Eq.(18-51) to describe the variation of gain with frequency,

$$\gamma(\nu) = \gamma_{max} e^{-b(\nu-\nu_0)^2}$$

where b is given by Eq.(18-52). For pumping at 5 times threshold, we have $\gamma_{max} = 5\gamma_{th}$. The frequency range on one side of the gain maximum is found from

$$\gamma(\nu) = \gamma_{th} = \frac{1}{5}\gamma_{max} = \gamma_{max} e^{-b(\nu-\nu_0)^2}$$

Solving for $(\nu - \nu_0)$ gives

$$\frac{1}{5} = e^{-b(\nu-\nu_0)^2}$$

$$b(\nu - \nu_0)^2 = \ln 5$$

$$\nu - \nu_0 = \sqrt{\frac{\ln 5}{b}} = \sqrt{\frac{\ln 5}{4 \ln 2}}\, \Delta\nu = 0.762\, \Delta\nu$$

Since the full frequency range in which lasing can occur is $2(\nu - \nu_0)$, the number of lasing modes is

$$N = \frac{2(0.762)(3.5 \text{ GHz})}{0.1364 \text{ GHz}} = 39.1$$

which rounds up to a maximum of 40 modes that can potentially lase.

21.7 (a) The frequency spacing of the modes is

$$\delta\nu = \frac{c}{2nL} = \frac{3 \times 10^8}{2(0.25)} = 6 \times 10^8 \text{ Hz} = 0.6 \text{ GHz}$$

Taking the linewidth from Table 23-2 to be $\Delta\nu = 1.5$ GHz, the number of lasing modes is $N = 1.5/0.6 = 2.5$, which rounds up to a maximum of 3 lasing modes.

(b) The mode frequencies are given by

$$\nu_m = m\frac{c}{2L}$$

As L increases, ν_m decreases, and eventually the next highest mode $m + 1$ will move the the frequency previously occupied by mode m. This will occur when the length has changed from L to L', such that

$$m\frac{c}{2L} = (m+1)\frac{c}{2L'}$$

Solving for the new length L', we have

$$\frac{L'}{L} = \frac{m+1}{m} = 1 + \frac{1}{m} = 1 + \frac{c}{2L\nu} = 1 + \frac{\lambda}{2L}$$

Therefore,

$$L' = L + \frac{\lambda}{2}$$

and the change in length is $\Delta L = \lambda/2 = (632.8 \text{ nm})/2 = 316.4$ nm. The fractional change in length is

$$\Delta L/L = \frac{316.4 \times 10^{-9}}{25 \times 10^{-2}} = 1.26 \times 10^{-6}$$

which is a change of 1.26 ppm (parts per million).

22.1 (a) The peak power is (pulse energy)/(pulse duration) = (0.1 J)/(5 $\times 10^{-9}$s) = 2 $\times 10^{7}$ W.

(b) The number of photons per pulse is (energy per pulse)/(energy per photon), or

$$N = \frac{E_{pulse}}{h\nu} = \frac{E_{pulse}\lambda}{hc} = \frac{(0.1)(1.064 \times 10^{-6})}{(6.63 \times 10^{-34})(3 \times 10^{8})} = 5.35 \times 10^{17}$$

(c) The light intensity inside the laser cavity is related to the output power by Eq.(20-21), $P_{out} = (1/2)IAT$. Solving for I gives

$$I = \frac{2P_{out}}{AT} = \frac{2(2 \times 10^{7})}{\pi(2 \times 10^{-3})^2(0.7)} = 4.55 \times 10^{12}\ \mathrm{W/m^2}$$

The stimulated emission rate (induced rate) is then

$$W^{ind} = \frac{I\sigma}{h\nu} = \frac{I\sigma\lambda}{hc} = \frac{(4.55 \times 10^{12})(28 \times 10^{-24})(1.064 \times 10^{-6})}{(6.63 \times 10^{-34})(3 \times 10^{8})} = 6.8 \times 10^{8}\ \mathrm{s^{-1}}$$

The spontaneous rate is

$$W^{spont} = \frac{1}{\tau} = \frac{1}{230 \times 10^{-6}} = 4.35 \times 10^{3}\ \mathrm{s^{-1}}$$

It can be seen that for the Q-switched pulse, the decay of the excited state is dominated by stimulated emission.

22.3 (a) The cavity lifetime in a cavity filled with index n is

$$\frac{1}{\tau_c} = \frac{c\alpha}{n} + \frac{c}{2nL}\ln\left(\frac{1}{R_1R_2}\right)$$

Using Eq.(5-4) to relate the dB loss to a fractional loss gives $\alpha = 2$ dB/km $= 4.6 \times 10^{-4}$ m^{-1}. The cavity lifetime can then be evaluated as

$$\frac{1}{\tau_c} = \frac{3\times 10^8}{1.5}\left[4.6\times 10^{-4} + \frac{1}{2(5)}\ln\left(\frac{1}{[0.98][0.95]}\right)\right]$$

$$\frac{1}{\tau_c} = 1.522\times 10^6 \text{ s}^{-1}$$

$$\tau_c = 657 \text{ ns}$$

(b) Repeating the above calculation for $L = 1.5$ m, we obtain $\tau_c = 206$ ns.

(c) The cavity lifetime can be further shortened by decreasing the reflectivities of one or both of the mirrors.

22.4 The optimum repetition rate is the reciprical of the upper state lifetime, which for Er is 10 ms. The optimum rate is therefore $1/10^{-2}$ s = 100 pulses/s. The average output power is the repetition rate times the energy per pulse, or (100 pulses/s)(5×10^{-3} J/pulse) = 0.5 W

22.6 (a) We first find the frequency spacing between the cavity modes,

$$\delta\nu = \frac{c}{2L} = \frac{3 \times 10^8}{2(0.9)} = 1.667 \times 10^8 \text{ Hz}$$

and then the minimum and maximum frequencies of the lasing range,

$$\nu_1 = \frac{3 \times 10^8}{870 \times 10^{-9}} = 3.448 \times 10^{14} \text{ Hz}$$

$$\nu_2 = \frac{3 \times 10^8}{720 \times 10^{-9}} = 4.167 \times 10^{14} \text{ Hz}$$

The frequency range over which lasing occurs is $\Delta\nu = \nu_2 - \nu_1 = 7.19 \times 10^{13}$ Hz, and the number of lasing modes is therefore

$$N = \frac{\Delta\nu}{\delta\nu} = \frac{7.19 \times 10^{13}}{1.667 \times 10^8} = 4.31 \times 10^5$$

(b) The pulse width is the reciprical of the lasing frequency width, or

$$\Delta t_p = \frac{1}{\Delta\nu} = \frac{1}{7.19 \times 10^{13}} = 1.39 \times 10^{-14} \text{ s} = 13.9 \text{ fs}$$

(c) The pulse repetition time is $T = 1/\delta\nu = 1/(1.667 \times 10^8) = 6 \times 10^{-9}$ s.

(d) The energy and peak power of each pulse are

$$P_{peak} = N\langle P\rangle = (4.31 \times 10^5)(3) = 1.293 \times 10^6 \text{ W}$$

$$\text{pulse energy} = P_{peak}\Delta t_p = (1.293 \times 10^6)(1.39 \times 10^{-14}) = 1.8 \times 10^{-8} \text{ J}$$

22.11 (a) The time between pulses is

$$T = \frac{2nL}{c} = \frac{2(1.5)(200)}{3 \times 10^8} = 2 \times 10^{-6} \text{ s}$$

(b) The number of oscillating modes is

$$N = \frac{T}{\Delta t_p} = \frac{2 \times 10^{-6}}{1.3 \times 10^{-12}} = 1.54 \times 10^6$$

(c) The frequency range for the modes that are lasing is

$$\Delta\nu = \frac{1}{\Delta t_p} = \frac{1}{1.3 \times 10^{-12}} = 7.69 \times 10^{11} \text{ Hz}$$

and the corresponding wavelength range is

$$\Delta\lambda = \frac{\lambda^2}{c}\Delta\nu = \frac{(1.55 \times 10^{-6})^2}{3 \times 10^8}(7.69 \times 10^{11}) = 6.16 \times 10^{-9} = 6.16 \text{ nm}$$

(d) The peak power is (energy per pulse)/(duration of pulse) or $P_{peak} = (16 \times 10^{-9})/(1.3 \times 10^{-12}) = 1.23 \times 10^4$ W, or 12.3 kW.

(e) The average power is

$$\langle P \rangle = \frac{1}{N} P_{peak} = \frac{1.23 \times 10^4}{1.54 \times 10^6} = 8 \times 10^{-3} \text{ W} = 8 \text{ mW}$$

Alternatively, we can evaluate $\langle P \rangle$ by

$$\langle P \rangle = \frac{\text{energy per pulse}}{\text{time between pulses}} = \frac{16 \times 10^{-9}}{2 \times 10^{-6}} = 8 \times 10^{-3} \text{ W}$$