Homework #4

19.4 Using $G = 200$ and $L = 15$ cm, Eq.(19-15) gives

$$\gamma_0 = \frac{1}{L}\ln G = \frac{1}{15 \text{ cm}}\ln(200) = 0.353 \text{ cm}^{-1}$$

The gain in dB is $10\log_{10}(200) = 23$ dB, and the dB gain per unit length is therefore (23 dB)/(15 cm) = 1.53 dB/cm.

19.5 (a) Here $I \gg I_s$, so

$$G \simeq 1 + \frac{I_s}{I_1}\gamma_0 L = 1 + \frac{(2\times 10^4)(5.3)}{4\times 10^5} = 1.26$$

where the result $\gamma_0 L = \ln(200) = 5.3$ has been used from problem 19.4.

(b) Now $I_1 = I_s$, so Eq.(19-27) becomes

$$\ln G + G - 1 = \gamma_0 L = 5.3$$

$$\ln G + G = 6.3$$

This implicit equation for G can be solved to give $G = 4.74$, which agrees with the value obtained from Fig. 19-9.

19.6 The core area is $A_c = \pi(25\times 10^{-4}\text{ cm})^2 = 1.964\times 10^{-5}\text{ cm}^2$. We need to find the increase in signal power, which is

$$\Delta P = P_2 - P_1 = A_c(I_2 - I_1) = A_c I_1(G-1)$$

where $G = I_2/I_1$ is the gain calculated in problem 19.5. For $I_1 = 4\times 10^5$ W/cm^2, we then have

$$\Delta P = \left(1.964\times 10^{-5}\text{ cm}^2\right)\left(4\times 10^5 \text{ W/cm}^2\right)(1.26-1) = 2.05 \text{ W}$$

while for $I_1 = 2\times 10^4$ W/cm^2 , we then have

$$\Delta P = \left(1.964\times 10^{-5}\text{ cm}^2\right)\left(2\times 10^4 \text{ W/cm}^2\right)(4.74-1) = 1.47 \text{ W}$$

Note that as P_1 increases by a factor of 20, the added signal power changes only by a factor of 1.4, due to saturation of the gain.

20.1 The output mirror reflectivity is 20%, so

$$\gamma_{th} = \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right) = \frac{1}{2(1.5)}\ln\left(\frac{1}{0.2}\right) = 0.536\ \mathrm{m}^{-1}$$

20.3 Using the expansions $(1-x)^{-1} \approx 1+x$, and $\ln(1+x) \approx x$ for $x \ll 1$, and taking $\alpha = 0$, we have

$$\frac{1}{\tau_c} = \frac{c}{2L}\ln\left(\frac{1}{1-x}\right) \simeq \frac{c}{2L}x = \frac{c}{2L}(1 - R_1R_2)$$

where $x \equiv 1 - R_1R_2$. Taking the reciprical of this we obtain Eq.(16- 13).

20.4 The threshold gain coefficient is

$$\gamma_{th} = \frac{1}{c\tau_c} = \alpha + \frac{1}{2L}\ln\left(\frac{1}{R_1R_2}\right) = 0.8 + \frac{1}{2(0.075)}\ln\left(\frac{1}{0.95}\right) = 1.142\ \mathrm{m}^{-1}$$

and the average pump photon energy is

$$h\nu_p = \frac{hc}{\lambda_p} = \frac{(6.63\times10^{-34})(3\times10^{8})}{500\times10^{-9}} = 3.98\times10^{-19}\ \mathrm{J}$$

Combining Eqs.(20-18) and (20-24), the pump threshold is then

$$P_{th} = \frac{V\,h\nu_p\,\gamma_{th}}{\sigma\tau_2} = \frac{(2.3\times10^{-5})(7.5\times10^{-2})(3.98\times10^{-19})(1.142)}{(1.8\times10^{-23})(480\times10^{-6})} = 90.7\ \mathrm{W}$$

The required absorbed pump energy is therefore 90.7 W. If 5% of the electrical power is converted into optical excitation, then $P_{abs} = 0.05P_{elec}$, and $P_{elec} = P_{abs}/0.05 = 90.7/0.05 = 1.81$ kW.

20.7 (a) Since $N_2 \gg N_1$, the gain coefficient is

$$\gamma = N_2\sigma_{em} - N_1\sigma_{abs} \simeq N_2\sigma_{em}$$

$$\gamma = \left(1 \times 10^{19}\ \mathrm{cm}^{-3}\right)\left(6 \times 10^{-21}\ \mathrm{cm}^2\right) = 0.06\ \mathrm{cm}^{-1} = 6\ \mathrm{m}^{-1}$$

If the gain coefficient is constant along the length of the fiber, the total gain is

$$G = e^{\gamma L} = e^{(6)(1)} = 403$$

This corresponds to a dB gain of $10\log_{10}(403) = 26.1$ dB

(b) Neglecting the fiber attenuation, we have

$$\gamma_{th} = \frac{1}{2L}\ln\left(\frac{1}{R_1 R_2}\right) = 6\ \mathrm{m}^{-1}$$

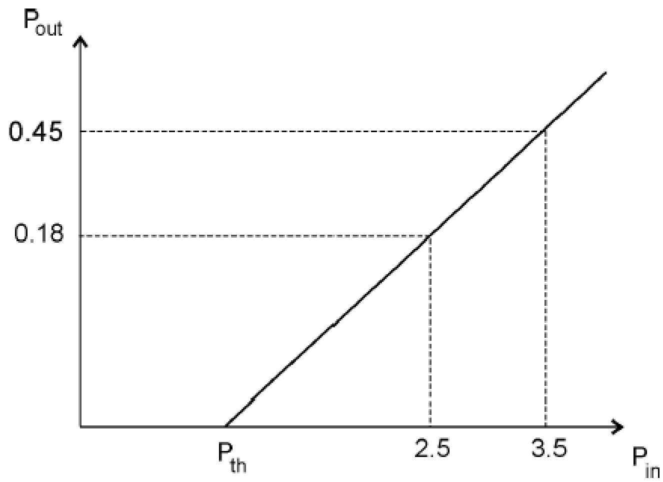
Setting $R_1 = R_2 = R$ and solving for R gives

$$\frac{1}{R^2} = e^{2L\gamma_{th}} = e^{2(1)(6)} = 1.627 \times 10^5$$

$$R = 2.5 \times 10^{-3}$$

(c) The Fresnel reflection of $R \approx 0.04$ for light propagating between glass and air is much larger than that required for lasing, so lasing will readily occur in this situation, even without any external end mirrors. To prevent lasing from occuring, the ends of the fiber must be modified to prevent reflections. This can be done by cleaving the fiber ends at an angle, by depositing a thin anti-reflection film on the end, or by simply inserting the fiber end into an index-matching fluid.

20.8 (a) The slope efficiency is the slope of the P_{out} vs. P_{in} graph, as shown in the figure below. Using the given information we have



$$\eta_s = \frac{\Delta P_{out}}{\Delta P_{in}} = \frac{0.45 - 0.18}{3.5 - 2.5} = 0.27$$

(b) Writing the slope efficiency in terms of the pump threshold,

$$\eta_s = \frac{0.45 - 0}{3.5 - P_{th}}$$

which can be solved for P_{th} to give

$$3.5 - P_{th} = \frac{0.45}{\eta_s} = 1.667$$

$$P_{th} = 1.83 \text{ W}$$

(c) The output power is

$$P_{out} = \eta_s(P_{in} - P_{th}) = 0.27(3 - 1.83) = 0.315 \text{ W}$$

(d) Eq.(20-29) can be written

$$\eta_s \simeq \frac{T}{\delta + T}\,\frac{h\nu}{h\nu_p} = \frac{\lambda_p/\lambda}{1 + \delta/T}$$

Solving this for δ gives

$$1 + \delta/T = \frac{1}{\eta_s}\,\frac{\lambda_p}{\lambda} = \frac{1}{0.27}\left(\frac{810}{1060}\right) = 2.83$$

$$\delta/T = 1.83$$

$$\delta = (1.83)(0.02) = 0.0366$$

(e) Generalizing Eq.(20-28) to include a refractive index n,

$$\frac{2nL}{c\tau_c} \simeq \delta + T$$

$$\tau_c \simeq \frac{2nL}{c(\delta + T)} = \frac{2(1.5)(0.06)}{(3 \times 10^8)(0.0366 + 0.02)} = 1.06 \times 10^{-8} \text{ s} = 10.6 \text{ ns}$$