Homework #2

16.2 The allowed mode frequencies are given by $\nu_m = mc/(2nL)$, so the allowed mode wavelengths are $\lambda_m = c/\nu_m = (2nL)/m$. The mode number is then

$$m = \frac{2nL}{\lambda} = \frac{2(3.6)(8 \times 10^{-4})}{850 \times 10^{-9}} = 6770$$

The frequency spacing between adjacent modes is

$$\delta\nu = \frac{c}{2nL} = \frac{3 \times 10^8}{2(3.6)(8 \times 10^{-4})} = 5.2 \times 10^{10}\ \mathrm{s}^{-1} = 52\ \mathrm{GHz}$$

and the corresponding wavelength spacing is

$$\delta\lambda = \frac{\lambda^2}{c}\,\delta\nu = \frac{(850 \times 10^{-9})^2}{3 \times 10^8}\,(5.2 \times 10^{10}) = 1.25 \times 10^{-10}\ \mathrm{m} = 0.125\ \mathrm{nm}$$

16.4 The optical intensity decays in time according to

$$I(t) = I(0)e^{-t/\tau_c}$$

where the cavity lifetime τ_c is given approximately by

$$\tau_c \simeq \frac{2L}{c(1-R^2)}$$

Setting $I(t) = 0.2I(0)$ at time t = 806 ns, we have $0.2 = \exp(-t/\tau_c)$, which gives

$$\tau_c = t/\ln 5 = (806 \times 10^{-9})/\ln 5 = 5 \times 10^{-7}\ \mathrm{s}$$

The mirror reflectivity is then found from

$$1 - R^2 = \frac{2L}{c\tau_c} = \frac{2(0.45)}{(3 \times 10^8)(5 \times 10^{-7})} = 6 \times 10^{-3}$$

Solving for R gives $R^2 = 1 - 6 \times 10^{-3}$, or $R = 0.997$.

16.7 (a) The frequency spacing between adjacent modes is

$$\delta\nu = \frac{c}{2nL} = \frac{3 \times 10^8}{2(3.5)(8 \times 10^{-4})} = 5.36 \times 10^{10}\ \mathrm{s}^{-1}$$

and the corresponding wavelength spacing is

$$\delta\lambda = \frac{\lambda^2}{c}\,\delta\nu = \frac{(830 \times 10^{-9})^2}{3 \times 10^8}\,(5.36 \times 10^{10}) = 1.23 \times 10^{-10}\ \mathrm{m} = 123\ \mathrm{pm}$$

The frequency width of the modes is

$$\Delta\nu_{1/2} = \frac{(3 \times 10^8)\ln(1/0.309)^2}{4\pi(3.5)(8 \times 10^{-4})} = 2.0 \times 10^{10}\ \mathrm{Hz}$$

where the facet reflectities were obtained from

$$R_1 = R_2 = \left(\frac{3.5 - 1}{3.5 + 1}\right)^2 = 0.309$$

Putting this in terms of wavelength gives

$$\Delta\lambda = \frac{\lambda^2}{c}\,\Delta\nu = \frac{(830 \times 10^{-9})^2}{3 \times 10^8}\,(2 \times 10^{10}) = 4.6 \times 10^{-11}\ \mathrm{m} = 46\ \mathrm{pm}$$

(b) Using the results of problem 16.6, we have

$$Q = \frac{4\pi nL}{\lambda \ln(1/R_1R_2)} = \frac{4\pi(3.5)(8 \times 10^{-4})}{(830 \times 10^{-9})\ln(1/0.309^2)} = 1.8 \times 10^4$$

$$\mathcal{F} = \frac{2\pi}{\ln(1/R_1R_2)} = \frac{2\pi}{\ln(1/0.309^2)} = 2.7$$

16.8 (a) The mode spacing is the same as before, $\delta\lambda = 123$ pm, since the cavity length is the same.

(b) The frequency width of the modes is

$$\Delta\nu_{1/2} = \frac{(3 \times 10^8)\ln(1/[0.98][0.95])}{4\pi(3.5)(8 \times 10^{-4})} = 6.1 \times 10^8 \text{ Hz} = 610 \text{ MHz}$$

Putting this in terms of wavelength gives

$$\Delta\lambda_{1/2} = \frac{\lambda^2}{c}\Delta\nu = \frac{(830 \times 10^{-9})^2}{3 \times 10^8}(6.1 \times 10^8) = 1.4 \times 10^{-12} \text{ m} = 1.4 \text{ pm}$$

(c) The finesse is

$$\mathcal{F} = \frac{2\pi}{\ln(1/R_1R_2)} = \frac{2\pi}{\ln(1/[0.95][0.98])} = 88$$

or alternatively,

$$\mathcal{F} = \frac{\delta\nu}{\Delta\nu_{1/2}} = \frac{53.6 \text{ GHz}}{0.61 \text{ GHz}} = 88$$

16.9 (a) We can neglect the effect of the semiconductor's refractive index since the length of the semiconductor is very small compared with the mirror separation. The effective cavity length is therefore ≈ 5 cm, and the mode spacing is

$$\delta\nu = \frac{c}{2L} = \frac{3 \times 10^8}{2(0.05)} = 3 \times 10^9 \text{ Hz} = 3 \text{ GHz}$$

(b) The fraction of light transmitted through each face of the semiconductor crystal is $T = 1 - 0.309 = 0.691$, and there are four such transmissions in each round-trip. This loss can be accounted for by replacing R_1R_2 by

$$R_1R_2 = (0.691)^4(0.95)(0.98) = 0.212$$

where the last two factors are the mirror reflectivities. This gives

$$\Delta\nu_{1/2} = \frac{c\ln(1/R_1R_2)}{4\pi nL} = \frac{(3 \times 10^8)\ln(1/0.212)}{4\pi(0.05)} = 7.4 \times 10^8 \text{ Hz} = 740 \text{ MHz}$$

for the mode width.

(c) The finesse is

$$\mathcal{F} = \frac{2\pi}{\ln(1/R_1R_2)} = \frac{2\pi}{\ln(1/0.212)} = 4.05$$

or alternatively,

$$\mathcal{F} = \frac{\delta\nu}{\Delta\nu_{1/2}} = \frac{3 \text{ GHz}}{0.74 \text{ GHz}} = 4.05$$

17.4 Using Eqs.(17-2) and (17-4),

$$\frac{z}{z_0} = \sqrt{\left(\frac{w}{w_0}\right)^2 - 1} = \sqrt{\left(\frac{293}{176}\right)^2 - 1} = 1.331$$

The Rayleigh range is then

$$z_0 = \frac{z}{1.331} = \frac{0.20}{1.331} = 0.150 \text{ m}$$

The wavelength is found from

$$\lambda = \frac{\pi w_0^2}{z_0} = \frac{\pi(176 \times 10^{-6})^2}{0.150} = 6.48 \times 10^{-7} \text{ m} = 648 \text{ nm}$$

17.6 (a) The confocal cavity has $r = L$, so the spot size on the mirrors is given by

$$w^2 = \frac{2\lambda L}{2\pi} = \frac{\lambda L}{\pi}$$

Solving for the cavity length gives

$$L = \frac{\pi w^2}{\lambda} = \frac{\pi(3 \times 10^{-4})^2}{900 \times 10^{-9}} = 0.314 \text{ m}$$

(b) For the confocal cavity, $w_0 = w/\sqrt{2} = 0.3/\sqrt{2} = 0.212$ mm

(c) The mode spacing (including both longitudinal and transverse modes) is $c/(4L) = (3 \times 10^8/(4[0.314]) = 2.39 \times 10^8$ Hz $= 239$ MHz

17.9 Using Eq.(17-22) we have

$$M^2 = \frac{\theta_{eff}\pi w_{0,eff}}{\lambda} = \frac{(1.4 \times 10^{-3})\pi(3 \times 10^{-3})}{1.5 \times 10^{-6}} = 8.8$$

17.11 (a)The spot size at the focus is

$$w_{02} \simeq \frac{\lambda f}{\pi a} = \frac{(1.064 \times 10^{-6})(25 \times 10^{-3})}{\pi(1.5 \times 10^{-3})} = 5.64 \times 10^{-6} \text{ m}$$

(b)Assuming the lens has a sufficiently large diameter to capture all the power in the beam,

$$I_{max} = \frac{2P}{\pi w_{02}^2} = \frac{2(150)}{\pi(5.64 \times 10^{-6})^2} = 3.00 \times 10^{12} \text{ W/m}^2$$

(c) First we need to find the maximum acceptable deviation of the spot size from its value at the beam waist. The change in intensity ΔI for a given change in spot size Δw_2 (the power P being fixed) is

$$\Delta I = -\frac{2}{w_{02}} I_{max}\, \Delta w_2$$

The fractional intensity change is allowed to be at most

$$\left|\frac{\Delta I}{I_{max}}\right| = 2\frac{\Delta w_2}{w_{02}} = 0.2$$

so the maximum allowed fractional spot size change is $\Delta w_2/w_{02} = 0.1$. From Eq.(17-2) we then have

$$\frac{w^2}{w_0^2} = 1 + \left(\frac{z}{z_0}\right)^2 = (1.1)^2 = 1.21$$

Solving for z gives $z = \sqrt{0.21}\, z_0 = 0.46\, z_0$. The Rayleigh range here is

$$z_0 = \frac{\pi w_0^2}{\lambda} = \frac{\pi(5.64 \times 10^{-6})^2}{1.064 \times 10^{-6}} = 9.4 \times 10^{-5} \text{ m}$$

which makes the maximum acceptable deviation in the z direction $z = (0.46)(94\ \mu\text{m}) = 43\ \mu\text{m}$. The position of the workpiece must be maintained to within $\pm 43\ \mu$m.