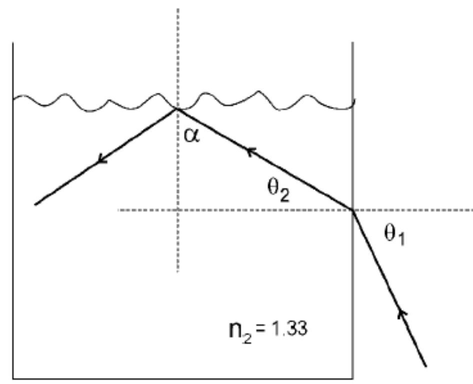
Homework #1

2.3 From the diagram below, we have



$$\sin\theta_1 = n\sin\theta_2 = n\cos\alpha = n\sqrt{1-\sin^2\alpha}$$

where $n_1 = 1$ and $n_2 = n = 1.33$. Since TIR requires that $\sin\alpha > 1/n$,

$$\sin\theta_1 < n\sqrt{1-1/n^2} = \sqrt{n^2-1} = 0.877$$

The incident angle must therefore be $\theta_1 < 61.3°$.

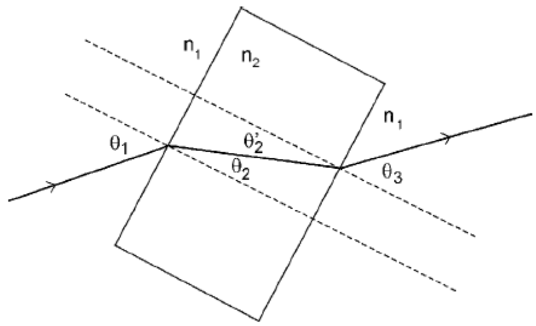
2.4 The beam intensity is

$$I = \frac{P}{A} = \frac{10^{-3}}{\pi(5\times10^{-4})^2} = 1.27\times10^3\ \mathrm{W/m^2}$$

This is related to the peak E field by $I = c\epsilon_0 E^2/2$, assuming $n = 1$, so

$$E = \sqrt{\frac{2I}{c\epsilon_0}} = \sqrt{\frac{2(1.27\times10^3)}{(3\times10^8)(8.85\times10^{-12})}} = 978\ \mathrm{V/m}$$

2.7 Consider the situation shown below, in which Brewster's condition holds for the first interface. It was already shown in problem 2.1 that $\theta_2' = \theta_2$ and $\theta_3 = \theta_1$ if the two faces are parallel. We will take $n_1 = 1$ (air) and $n_2 = n$, so Brewster's condition at



the first interface is $\tan\theta_1 = n$. What we need to show is that Brewster's condition also holds for the second interface, i.e., $\tan\theta_2' = n_3/n_2$ or $\tan\theta_2 = 1/n$. We can do this by combining the first Brewster's condition with Snell's law, to obtain the second Brewster's condition. Since Snell's law involves the sine of the angles, we first put the Brewster's condition in terms of $\sin\theta_1$,

$$\tan\theta_1 = \frac{\sin\theta_1}{\cos\theta_1} = \frac{\sin\theta_1}{\sqrt{1-\sin^2\theta_1}} = n$$

Squaring and rearranging gives

$$\sin^2\theta_1 = n^2\left(1-\sin^2\theta_1\right)$$

$$(n^2+1)\sin^2\theta_1 = n^2$$

$$\sin\theta_1 = \frac{n}{\sqrt{n^2+1}}$$

Now using Snell's law, $\sin\theta_1 = n\sin\theta_2$, we have

$$\sin\theta_2 = \frac{1}{\sqrt{n^2+1}}$$

and

$$\cos\theta_2 = \sqrt{1-\sin^2\theta_2} = \sqrt{1-\frac{1}{n^2+1}} = \frac{n}{\sqrt{n^2+1}}$$

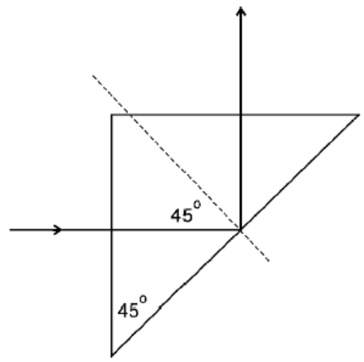
Combining the above two equations then yields

$$\tan\theta_2 = \frac{\sin\theta_2}{\cos\theta_2} = \frac{1}{n}$$

This is the condition for Brewster's angle at the second interface, which was to be proved.

A more concise derivation is to note that at Brewster's angle, $\theta_1 + \theta_2 = 90°$. Sketching a $90°$ triangle, it is easily seen that if $\tan\theta_1 = n$, then $\tan\theta_2 = 1/n$.

2.9 The critical angle for TIR at the long prism face is $\theta_c = \sin^{-1}(1/1.5) = 41.8°$. Since $\theta_1 = 45°$ (see diagram below), the condition for TIR, $\theta_1 > \theta_c$, is satisfied.



15.3 The coherence length must be greater than the path length difference, so $L_c > 33.6 - 32.9$ cm $= 0.7$ cm. Then

$$T_c > L_c/c = (7 \times 10^{-3})/(3 \times 10^{8}) = 2.33 \times 10^{-11} \text{ s}$$

and the frequency width must be less than

$$\Delta\nu < 1/T_c = 4.3 \times 10^{10} \text{ Hz} = 43 \text{ GHz}$$

The corresponding width in wavelength is

$$\Delta\lambda = \frac{\lambda^2}{c}\Delta\nu < \frac{(632.8 \times 10^{-9})^2}{3 \times 10^8}(4.3 \times 10^{10}) = 5.7 \times 10^{-11} \text{ m}$$

or $\Delta\lambda < 57$ pm.

15.5 The angular divergence (half cone angle) is $\theta \sim \lambda/D = (1.06 \times 10^{-6})/(8 \times 10^{-4}) = 1.32 \times 10^{-3}$ rad. At a height H, the beam's diameter is $\approx 2\theta H = 2(1.32 \times 10^{-3})(2.9 \times 10^4$ ft$) = 77$ ft.

15.6

$$I_{max} \sim 2B \sim 2\frac{P}{\pi\lambda^2} = \frac{2}{\pi}\frac{1.5 \text{ W}}{(488 \times 10^{-9} \text{ m})^2} = 4 \times 10^{12} \text{ W/m}^2$$

Using $I = (1/2)cn\epsilon_0 E^2$, with $n = 1$, we find

$$E = \sqrt{\frac{2I}{c\epsilon_0}} = \sqrt{\frac{2(4 \times 10^{12})}{(3 \times 10^8)(8.85 \times 10^{-12})}} = 5.5 \times 10^7 \text{ N/C}$$