Physics 1111 - Term C 2015

Important Facts

Will these facts be on the exam? **YES!** Now will you read them? Your call. My grade.

N.B. January 14 will be used as a Lecture class. You shouldn't assume that you can cut that class because it si only a conference class.

This course has two major purposes.

First, Physics 1111 will help you learn college-level, calculus-based freshman mechanics. "College-level" means "you have to solve symbolic (no numbers) problems that aren't the same as the examples in the book". "Calculus-based" means "you need to use calculus to do exam and homework problems." Note I said 'help you learn'. I can tell you what to learn, but learning is something you do to yourself. As a corollary, I may not allow the use of calculators on exams, because there may be no number-crunching.

Second, for some of you, this course is your wake up call: You are not in high school. You are going to need to work hard. A lot of you are already awake. That's great! Some of your classmates...

Work: The WPI Plan expects an average student to work 17 hours per week in each course, including class hours. Some of you will need fewer than 17 hours per week per course; some of you will need *more*.

There is extremely good correlation between hours worked and final grade. Several years ago, I had your classmates report the hours per week they were spending on the course, *outside of class*, and lined those numbers up with the count of letter grades. The findings were:

Grade	Hours per week
A	≥ 13
B or C	9-12
NR	5-8
NR-	≤ 4

An NR- corresponds to exam grades under 20%. An A corresponds to excellent work that exceeds expectations, an outcome that oft requires more than an average effort.

Every so often, one of my advisees does very poorly. When I ask leading questions, I often get the same answer. They had been working 6-8 hours a week on each course, or less. In some cases, much less. If you believe that two hours a night of homework will cover all three courses, or if you believe that homework nights happen only five (or, Goddess forbid, four or three) days a week, your beliefs are dysfunctional. Fortunately, this dysfunction can be cured.

Basic Information: The Lecturer is G. D. J. Phillies, Professor of Physics, and Interactive Media and Game Design and Biochemistry Associated Faculties (Olin 130, X5334, WPI Email: phillies@wpi.edu; phillies@4liberty.net; http://www.wpi.edu/~phillies). **My door is always open. That means if you come by the building during the day, not while I am having lunch or teaching class, I am often here and available to talk. I read EMail regularly.** If I am (i) on the phone or (ii) talking with someone already, please respect my hand waves (Come In!, Sit Down and Wait!, or Wave-off! if I'm going to be busy for a bit.) When I am speaking with you, I will give you my full attention; please give other people the same courtesy. The students who have come by to ask questions about homework, the course, etc., report that I am actually extremely helpful and patient.

Textbook: The text is Kleppner and Kolenkow, *An Introduction to Mechanics*. The course is based on self-study, textbook reading, homework problems, lectures, conference section meetings, and laboratories, all to prepare you for the examinations.

There are three handouts, to be found at http://users.wpi.edu/ phillies/ at the fourth bullet point.

Grading: There will be three hour examinations, each worth 100 points perhaps with bonus questions. There will be 18 homework sets (each worth six points), plus a bonus problem or two (worth six points each), plus the final lab quiz (six points), for a total that will be truncated at 100 points. There will be laboratory exercises worth a total of 50 points. There are two mandatory Inventory quizzes, given as the first and last lab exercises. The lab **will** meet all 14 times.

Your final letter grade will be based on your combined performance in these three components, with an adjustment by as much as one letter grade for sharp improvement or deterioration in performance over the term. You must each perform your own assigned experiments and associated lab reports, or you will automatically receive an NR. *Someone circulated a rumor that if you flunked the course before, you could transfer your lab grades to this course.*

They were wrong. You must take both Inventory quizzes in a timely way, or you will automatically receive an NR. In all cases, I am open to legitimate excuses as to why required work was not performed.

Typically, the average final grade in my courses has been around a B. Based on past records, many PH1111 students do better. However, if most of you deserve an A, I promise that most of you will get that A. If two-thirds of you deserve an NR on an exam, then two-thirds of you will get that NR. Last term, the class did as described on the exam and earned the promised outcome. Please do better.

Special Cases: There are an extremely wide variety of issues not related to physics that may create challenges. Your relatives die. Your parents get married. Your children have emergency surgery. You spend a week in a hospital ICU for unknown causes. You have short or long term medical issues. [I did not make any of these up.]

I am happy to make reasonable accommodations. I cannot make accommodations if I do not know that you need them. The only way I can find out that there is an issue is that you tell me. Please come by and talk.

Exams: I try to write exams so people can really do well relative to the numerical class average. I do this by constructing exams to have an average in the range 45-65%, with a pass level around 50 or a bit lower. If you went to a high school that always gave really simple-minded exams, with class averages of 90% for the college-bound, you may find this change uncomfortable. I write the exams to distinguish between A, B, and C students, not to distinguish between levels of failure. In practice, students who do marginally or barely fail their exams did poorly on homework and labs, so their total grade is usually a clear fail.

Consider: If you really bomb one exam, and the class average is always 90, you have no way to get yourself out of your hole. If the class average is 60, and you do badly, you can save your position by doing far above class average on the next exam. People have done this successfully.

Exams are given during lecture hours. **There is no conflict examination.** If you are hospitalized, or have another good excuse for missing an exam, see me as soon as possible. For a legitimate excuse, I'll be happy to make arrangements.

I have attached some old exams. General statement: Exam questions are mostly like homework questions.

Homework: The syllabus lists the homework problems for the course. I will grade a statistical sample of all problems.

Homework will be collected in almost every lecture after the first. Homework placed in my mailbox or under the door of my office *before the start of lecture* will be treated as being on time. **Late homework is assigned a grade of 0.** There are rare legitimate reasons for being late with homework. If you think you have a legitimate reason for being late, speak up. I'm really pretty softhearted. Place your full name on every sheet of paper. Staple homework pages together. If your homework pages get separated because they were not stapled, it is not my problem.

In doing homework: (0) Use lined paper. (1) Write in pencil on only one side of the page. (2) Present equations in a single vertical column. (3) Present your calculations in a logical order. (4) If the equations are written in random order like a section of BASIC spaghetti code, complete with computed COMEFROM commands, expect little credit. (5) If the first character in an equation is an equals sign, the equation is wrong. (6) If the equation has more than one equals sign, the equation is wrong. (7) Write large enough that your handwriting can be read. For example, derivatives should generally spread vertically over two lines of the page.

Problems **shall** worked symbolically, with numbers plugged into equations as late steps. Solving symbolically is the correct, mature way to work problems. Some of you have not yet gained this vital skill. If you haven't learned symbolic reasoning yet, this course will work on correcting your mental defect. In each problem set, we will grade one or more problems of my choice. If you didn't do that problem, or your solution to that problem was wrong, we look at the rest of the set. (It would be nice to grade everything, but the grader budget is limited.)

The homework grading system for each graded problem is:

6 - The one graded problem is worked correctly: approach is correct, method is correct, and answer is correct.

4- Approach and method are correct, but there are minor numerical or algebraic errors. If you substitute numbers for letters at the start of the calculation, rather than working the problem symbolically and plugging in numbers at the end, and get the **right answer**, you will usually get a 4, because your method is wrong. If you get the wrong answer, you get a 0. [In rare cases there will be a homework problem that is too tedious to solve symbolically, at least if you make a bad choice of approach; there you may substitute numbers early.]

2 - If you didn't do the one problem, or your solution to that problem was wrong (but note below), we look at the rest of the set. If you made a serious effort on the rest of the problem set, you get a 2; otherwise, you get a 0.

0 - If you substitute numbers for letters at the start of the calculation, rather than at the end, you get no partial credit. If you get the wrong answer, you get a 0. If you do not turn in a problem set, your recorded grade counts as a 0.

If you get the right answer with no work, or with a totally wrong method, you will get a zero. Some additional points may be added or subtracted for special cases, e.g., in a large problem done mostly right, you might get a 5. At some date, the grader will start docking a point if there is no staple.

Laboratories: The lab meets every week. Laboratories are mandatory. If you do not do the lab work as prescribed, including the lab reports, you will fail the course. *Someone circulated a rumor that if you flunked the course before, you could transfer your lab grades to this course. They were wrong.* Your objective is to produce a readable lab report. Lab reports must have their text portions typed. You are encouraged to use an equation editor or LaTeX to generate your lab reports including typed equations. My web pages wpi.edu/∼phillies include a sample LaTeX file to get you started. Photographs made with cell phones, and legitimate scans of hard pencil writing, are often impossible to read. The laboratory data sheet if you prepare one **is not** a lab report.

Cheating

The Faculty's rules on academic honesty require instructors to tell students their personal attitude towards academic honesty. Mine is straightforward. This is a university. Cheating is the moral equivalent of treason. Even as a just society hangs its traitors, so also does a university expel cheaters and academic frauds from its midst. If you are caught cheating in this course, I will be asked for a recommendation on outcomes. I always recommend the maximum allowable punishment.

With respect to specific cases: During an examination, you may not consult a textbook, or written, electronic, or other notes or other external memory aids. During an examination, you may not copy any part or aspect of an answer of another student, nor may you share your answers with or provide information to another student. The physical modality of answer transfer is not significant. Violation of these examination rules is cheating.

In doing a laboratory, you must actually perform the experiments during this term, and you must submit your own laboratory report based on the data you (and your lab partner, if any) actually obtained. You may discuss data analysis with your lab partner or other students, but you must submit your own report on your experiments. Lab partners submitting physical duplicates (e.g., xeroxes, carbons, multiple printouts) of each other's lab reports will get a grade of 0 (zero) for the laboratory.

You may discuss homework problems and their solutions with your fellow students.

General Philosophy

What is physics? Physics is the study of all natural phenomena. The objective of physics is to reduce all physical behavior to a small number of principles and elementary objects. Physics explains all natural phenomena as matter and motion, as particles, fields, and forces. The original differences between physics (and its sister sciences, and engineering) and other erroneous natural philosophies arise from two understandings unique to Western science: (1) The natural world is quantitatively described by calculus and analytic geometry; (2) When you can make accurate predictions about the world, you can predict the accuracy of your predictions, too.

Physics is based on *praxis*, getting the right answers for the right reasons. A wrong method that happens to give the right answer is wrong and not worth points.

Understanding: You understand physics if you can solve new problems that are *not* the same as the problems that you have seen before. A person who tells you 'I understand the material, but I can't solve homework problems' does not understand two things. First, he does not understand the material. Second, he does not understand 'understanding'.

Notes: There are essentially no people who can learn effectively by listening raptly to a lecturer. Most of us have to take careful notes, and think about material, before we know and understand it. Certainly, I do!

You have short and long term memory. Tonight, your notes from this afternoon will be perfectly clear, because they tap short term memory. Tomorrow your short term memory will fade. Those same notes will be incomprehensible. If you took no notes, you probably didn't learn very much. As an undergraduate, I learned to recopy my day's notes onto fresh pages set into ring binders *every day*. As I did, I inserted all the marginal remarks and explanations that I remembered, but that I had not had time to write down. Your mileage may vary.

Memorization: Memorization is an extremely valuable tool for putting large bodies of fact into a useful order in your mind. No matter what your major, material in core courses is material you need to know *without looking it up*. Memorizing huge batches of randomly chosen formulae, without learning the meanings of the symbols or the conditions under which the equations are valid, will do you absolutely no good.

If someone tells you 'I don't need to memorize things, because I can always look them up', challenge them to learn a topic from a text written in a foreign language that they do not speak or read. They may use a bilingual dictionary. Korean is a fine choice of language; the Hangul script is by rational design highly logical and easy to learn. I have read technical papers in foreign languages I never studied. I guarantee that learning while reading an unfamiliar language is much easier than trying to be a competent professional when you can't remember core material.

Once upon a time, there were people who believed that creative thinking was something separate from factual learning, as though one could be creative without having material from which to create. Such people may well also believe that you can make bricks without straw, mud, or forms. Also, at some less-fortunate academic institutions, there are faculty who use their academic freedom to claim that "hierarchical learning" (e.g., you need to learn geometry before you can understand calculus) is obsolete. I will, of course, defend the academic freedom of the faculty at these other institutions to voice their totally imbecilic ideas. However, I here use my academic freedom to say that their claims are completely devoid of merit.

Homework: Homework is *almost* always the basis for learning physics. You should view homework as a self-test. The right approach to homework is to read the chapter carefully, perhaps taking notes, work through the calculations in your notes, and then do the homework *without* looking back into the chapter. The wrong approach to homework is to read the homework problem, find the book's worked example, and change the numbers. If you have to look up procedures, you are *not* studying correctly. Needing to look up the mass of the Earth's second moon is not a bad thing. If you have learned the material, you do not need to look up procedures. On the exams, 'look up procedures' is not an option. (I say "almost" because for a few of us the homework problems are an obstacle that must be overcome before serious studying can take place.)

You will not always solve every problem immediately. If you can't solve a problem, at some point go do something else. Let your subconscious work. Return to the problem later. If you always run for help after 15 minutes, you will not learn the perseverance or problem solving skills you will need when you are graduated.

Group discussion of homework is *encouraged.* That means that in this course group discussion of homework problems is *not* cheating. Other professors have different rules. Each time you take a course, learn the rules for that course.

Experimental data due to Kingsbury shows that the ideal group for group study contains exactly two people. They each solve the same two problems. They take turns showing solutions. One talks; one knows no one else is listening. Both therefore pay careful attention. What do you say? You explain each step, why you took it, and why the step is correct. The other person interrupts you and forces you to defend your thinking. Group discussions between people who each tried to solve the same problem can be very fruitful. Group discussions in which you listen while someone else announce answers are like lying in bed watching an exercise video: It may be fun. It's not exercise.

Work: Many of you have parents who work two jobs, long hours, to put you here. I help make sure you live up to your debt to them by working harder than they do.

Some of your classmates went to a high "school" where people did one or two hours of homework. Per week. For all four courses put together. There is a reason that I put "school" in quotes. Think about it.

Grade Reduction: I have had students ask me if I would promise to give them NR if they were not going to get an A in the course. The answer is No. However, I will give you the grade that you earn. If you do not turn in the last lab, you will automatically get the NR you just earned.

Syllabus

Reading and Homework Assignments

4

The dates are the dates in which material is covered in lecture. Unless there is an exam, each homework assignment is due at the start of the next lecture, which will be 1-5 (often 23) days later. Chapters are chapters in Kleppner and Kolenkow. The homework problems are on the following pages. "FIGURE" indicates that there is a Figure to go with the problem. The figures are all on the last few pages.

I consistently use $g = 10\text{m/s}^2$ in my solutions; more accurate values exist. In Kleppner and Kolenkow, there are occasional numerical solutions to some problems. Since Kleppner and Kolenkow used a different g, their answers are wrong as homework solutions.

In homework and exam problems using Newton's Laws, if $\mathbf{F} = m\frac{d^2\mathbf{r}}{dt^2}$ is being written, it must be written using a second derivative to show the acceleration, and it must always be accompanied by a correctly labeled force diagram. **No force diagram, no credit!**

Notation: In reading a.b , a is the chapter number, and b is the part of the chapter, or it is the question number. "-" means "through", e.g., 5.1-5.5 means read Chapter 5, parts 1-5. The numbers in brackets [b] refer to the footnotes.

Lecture Date	Topic	Chapters	Graded	Other
15	Vectors	1.1-1.10,1.notes, 2.7, 2.8	2	
20	Calculus	1.1-1.10, 1.notes, 2.7, 2.8	4-7	
22	Newton's Laws	2	12-15	
26	Vector Derivatives	1.7-1.10	8-11	
28	Solving Newton's Laws	2,3	16-20	
30	Friction	3	21, 23-25	Due February 3

February 2 **Examination**

4	Springs	Handout	26-29	
6	Momentum	4.1-4.7,4.9,4.10	30-35	
9	Work	5	36-40	
11	Energy, power	4,5	41-45	
16	Collisions, stability	6.5	46-49	
18	Circular polar coordinates	1.10,1.11	50-53	Due February 20

February 18 **Examination**

20	Rotation, **L**	7,8	54-57	
23	Rotational Energy	7,8	58-61	
25	Torques	7,8	62-66	
27	Rotation problems	7,8	67-70	
2	Gravity	10	71-73	
4	Statics		74-76	Due March 5

March 6 **Examination, All Chapters**

Hints and Comments

[1] A-1 is a take home lab for those who want an experiment. A-3 is an exercise for people not clear about significant figures. A-1 and A-3 will not be graded.

[2] I will not use area or volume integrals except as results; you will not need to calculate any.

Homework Problems

A-1) In lab, you will be issued a long piece of string and several nuts. You will perform experiments to confirm the dimensional analysis suggested by this problem. The period T is the time required for the pendulum to swing back and forth. The period T might depend on the gravitational constant g (units meters/(second^2)), the length L of the string, and the mass m of the nuts. Find the combination of g, l, and m that has dimensions (time)^1. (Hints: The constants may be raised to non-integer powers, e.g., 1/3 or π.) The lab objective is to confirm $T = Kg^a\ell^bm^c$ by measuring K, a, b, and c. When would this dimensional approach fail?

A-2) Consider the vectors $\mathbf{A} = 2\hat{\imath} + 5\hat{\jmath} - 1\hat{k}$, $\mathbf{B} = -3\hat{\imath} + 1\hat{\jmath} + 2\hat{k}$, and $\mathbf{C} = +4\hat{\imath} - \hat{\jmath}$. Compute (i) $\mathbf{A} + \mathbf{B}$, (ii) $\mathbf{A} - \mathbf{B}$, (iii) $\mathbf{A} \cdot \mathbf{C}$, (iv) $\mathbf{A} \times \mathbf{C}$, (v) $\mid \mathbf{A} \mid$, (vi) $\mid \mathbf{C} \mid$, (vi) $\mathbf{A} \otimes \mathbf{B}$ and: (vii) From (iii), (v), and (vi), find the angle between $\mathbf{A}$ and $\mathbf{C}$.

A-3) Significant Figures. In the following, the numbers are known to the indicated accuracies. Do these by hand, using neat vertical columns. insert a "?" as the digit after the last digit for each number, and remember that ? when added to, subtracted from, multiplied by, or divided by or into another number is still "?" Report a correct, significant number as your final answer. I know you can do these with a calculator. That's not the point of the problem. The objective is for you to see how significant figures materialize in calculations. (a) 8.7-3.29 (b) 2.1+41.32+1.678. [Do this both by rounding at the end and also by rounding at the beginning.] (c) 2.2 x 3.784 (d) 5.682/4.2.

A-4) The position of an object as a function of time is given by $x = x_0 t \exp(-at^{1/2})$. Here x_0 and a are numerical constants in the equation for x. x_0 is not the position of the object at $t = 0$, and a is not the acceleration of the object. Discuss the behavior of the velocity as $t \to 0+$ if $a > 0$. Standard notation: $\exp(a) \equiv e^a$.

A-5 See Diagram. An object is thrown into the air (up is the +z direction; $z = 0$ is sea level) by an observer at the edge of the Dead Sea, a location well below sea level so that $z \ll 0$. A plot of the object's trajectory (height vs x) appears in the figure. At each of the five points a, b, c, d, e, (i) is v_z positive, negative, or zero; (ii) at the same points, is $a_z = d^2z/dt^2$ positive, negative, or zero; (iii) at the same points, is z positive, negative, or zero? We now repeat the same experiment five times. In each experiment, at one of the five points a small explosive charge. The parachute rapidly brings the object almost to a dead stop relative to the air. For each of the five points, while the object is being brought to a stop, is a_z positive, negative, or zero? FIGURE

A-6 OLD EXAM PROBLEM A distinguished WPI Faculty Member is loaded into a rocket ship. The ship takes off from a pit at the bottom of Death Valley, starting at an altitude 100 m below Sea level. At $t = -5$ the engines are ignited. At $t = +5$s, the ship leaves the ground. The vertical acceleration of the ship during the climb is a constant. At $t = 20$ s the ship achieves an altitude of 4000 m above sea level. What is the ship's acceleration? What is its velocity at $t = 30$ s? Solve systematically, beginning with $z = z_0 + v_0t + 0.5at^2$ and $v = v_0 + at$, not with whatever equation you may have pulled out from the book. Find z_0, v_0, and a. For credit, you must use my time and altitude origins in your calculations. To avoid undue complications, substitute numbers from the beginning. Prove your values for z_0, v_0 and a are correct by showing that they predict correctly the locations and times supplied in the problem. Clue #1) I am fond of this problem, but I keep changing which boundary conditions I supply. A few years ago, 40% of the class got this one wrong. I hope you can do better. Clue #2 You should end up with three equations in three unknowns. You can do systematic elimination to find the unknowns, or you can learn how Mathematica, Maple, or some other computer algebra program will do your work for you. Clue #3) You should have performed the check "prove your values" automatically, without needing to be told.

A-7 In the opening scene of the motion picture that will show up throughout these homework problems, a truck laden with explosives approaches a railroad crossing. The truck's motion is filmed and digitized by a surveillance camera. The supplied coordinates put the center of the crossing at $x = 40$. x increases from left to right. At $t = -20$ the truck driver notes that a train is approaching the crossing and floors her accelerator. At the time, the camera shows that the truck is at $x = -10$ and has a speed of 20 m/s to the right. The magnitude of the truck's acceleration is 5 m/s^2.

(a) Give equations for $v(t)$ and $x(t)$ of the truck in the seconds after the driver floors the accelerator, writing your answers in the exact forms

$$x = x_0 + v_0t + 0.5at^2$$

and

$$v = v_0 + at.$$

(b) Prove that your answers are correct by substituting $t = -20$ in these equations, and showing that you recover the initial position and velocity correctly. (c) When did the truck cross the railroad tracks? If you did the calculation correctly, you should have found two answers to the question, one of which is wrong. (d) The train crosses the intersection at $t = -17$. How long after the truck crossed the rails did the train cross the road? (e) How fast is the truck going when it reaches the intersection? [Assume SI units throughout. All numbers are accurate to the nearest hundredth of a unit. More or less immediate numerical substitution is appropriate.] [Clues: A few years ago, more than half the class got this one wrong. Be aware of signs. You should always check your answers. You should automatically have done the $t = -20$ substitution, without being told, in order to check your answer.

A-8 (a) Figure A-12a shows a 2 kg mass hanging at the end of a rope. The top of the rope is wrapped securely around a hook that is attached to the ceiling. For the 2 kg mass, draw the force diagram. *I*n complete sentences, identify each of the forces acting on the mass, including the nature of the force and the object applying it. In complete sentences, for each force in your force diagram, identify the reaction force, the object applying it, the nature of the force, and the object on which the reaction force is acting. (b) Figure A-12B shows two masses suspended by ropes; the ropes at their tops ends each go over a pulley and are connected by a spring. Repeat part (a) of this problem for the left-hand-mass and the left hand rope in the Figure. Hint: Including its own weight, there are four forces acting on the rope. The answer to this problem is quite long but does not take a lot of ingenuity to find. Hint: The phrase "complete sentence" has an exact meaning. TWO FIGURES

A-9 For the masses shown in Figure A-13 (visible on the last page of Figures), find the force diagrams for the two masses, and for each force identify the object applying it, the nature of the force, and the object on which the force is acting. FIGURE

A-10 For the masses shown in Figure A-14 (visible on the last page of Figures), find the force diagrams for the two masses, and for each force identify the object applying it, the nature of the force, and the object on which the force is acting. [Hint: In one case, the force is "external"; its nature and source cannot be determined from the problem as given.] FIGURE

A-11 A stereo speaker is suspended from the ceiling by two wires. The speaker has mass m; the tensions in the two wires are T_1 and T_2. Draw the force diagram for the speaker. In complete sentences, identify each of the forces acting on the speaker. In complete sentences, for each force in your force diagram, identify the reaction force, the object applying it, the nature of the force, and the object on which the reaction force is acting. FIGURE

A-12 One-dimensional motion. The velocity of an object as a function of time is parallel to the x axis, with $v_x = 2t^3 + 7t^2 + \cos(t) - 5$, Find the position, velocity, and acceleration of the object at $t = -1$. (Hint: there will be an unknown constant of integration someplace.) *Derive* a formula for the average acceleration, corresponding to the formula from today's class showing that the average velocity is $\bar{\mathbf{V}} = \mathbf{\Delta R}/\Delta T$. Find the average velocity and the average acceleration between times -1 and 1.

A-13 Vector motion. A particle has an acceleration $d^2\mathbf{r}/dt^2 = 3\hat{\mathbf{i}} - 5\hat{\mathbf{j}} + 6t\hat{\mathbf{k}}$. At $t = 2$ the particle is stationary and located at $\mathbf{r} = 2\hat{\mathbf{i}} - 5\hat{\mathbf{j}} - 5\hat{\mathbf{k}}$. Write the position of the particle explicitly (meaning that you solve for all the constants for which there are solutions) as a function of time (*a*s a function of time means that your answer is a function of t; if I plug in for t I get the right answer for my t. Special case issue: If the particle is stationary at all times, then $\mathbf{v} = \mathbf{0}$ is correct and is a function of time, namely it is $\mathbf{v} = \mathbf{0}t^1$.).

A-14 The Royal Army of Grand Fenwick (hint: literary reference) is testing a new low-power crossbow. On its first trial, the bolt (arrow to non-SCA types) has a speed at launch of 24.2 m/s. It is launched by an archer standing in a trench, so the launch altitude is $z = 0$. The firing field is flat. The bolt returns to earth 44.5 m from the launch location. What was the firing angle (the angle between the initial velocity vector and the horizontal)? What altitude did the bolt reach at the top of its trajectory?

A-15. The jerk $\mathbf{J}$ is the third derivative of position against time, i. e.,

$$\mathbf{J} = \frac{d^3\mathbf{r}}{dt^3}.$$

For an object performing uniform circular motion (an object moving in a circle at constant speed), so that its location satisfies

$$\mathbf{r} = r\cos(\omega t)\hat{\mathbf{i}} + r\sin(\omega t)\hat{\mathbf{j}},$$

compute $\mathbf{J}$, and find the angles that $\mathbf{J}$ forms with $\mathbf{r}$, $\mathbf{v}$, and $\mathbf{a}$.

A-16 While driving along the New York Thruway, I realized that the vehicle now visible over the top of the hill was stopped and had no brake or hazard lights showing. Worse, its passengers were outside the car blocking the other thruway lane and the emergency lane. I applied my brakes. Assuming that my Toyota with luggage had a mass of 1500 kg and that my acceleration was $-0.5g\hat{\mathbf{i}}$, that the road was smooth and level, and that wind resistance was not significant, calculate the contact force of the road on my automobile.

A-17 A pair of carts sit on a rail in contact with each other. Their masses are m_1 and m_2. The rear cart is pushed from its rear by an oscillating force $F_o(1 + \cos(\omega t))$. Give the force diagram for each of the two carts. Write the Second Law for each of the two carts. What is the acceleration of each cart as a function of time? What is the velocity of each cart as a function of time? FIGURE

A-18 Consider a child seated in a Ferris Wheel, as seen in the sketch. The tangential velocity where the child is sitting due to wheel rotation is v. For points a, b, and c, give the force diagram for the child, and compute the force that the seat is putting on the child at each point. FIGURE

A-19 (a) A rock is spun on a string in a vertical circle. When the rock is headed upward at a 45 degree angle to the horizontal, the string is cut. Give the force diagram and the equations of motion for the rock immediately before and immediately after the string was cut. (b) A rock is thrown through the air; it describes a parabola before returning to the ground. What are the force diagram, the acceleration, and the velocity of the rock when it reaches the top of its trajectory?

A-20 Consider a pair of masses m and M. m lies on a ramp that makes an angle θ with respect to the horizontal. M is hanging in mid air. The two masses are linked by a string that goes over the top of a very light pulley; the tension in the string is T everywhere. Give (a) the force diagrams for m and M, (b) the Second Law equation(s) for each mass, and (c) the constraints. Compute (d) the tension in the string and (e) the acceleration of each mass. Compute the velocity of m as a function of time. If $M \gg m$, do your solutions require that mass m is moving up the ramp? Why or why not? FIGURE

A-21 A retired faculty member constructs a mansion modestly to the east of the San Andreas fault. No sooner has he moved to the mansion than the really big earthquake occurs, and all land east of the San Andreas fault begins its terminal slide to the bottom of the Atlantic ocean. In the dining room (7 m ceiling) is a chandelier of total length (from the ceiling to the bottom) 3 m. Assuming that the building's vibration isolation protected the chandelier from all vibrations of the earthquake, so that the chandelier was stationary when the slide began, and that the acceleration of the building is initially due east and horizontal at 0.1 m/s^2, calculate the deflection of the bottom of the chandelier from the vertical. You may approximate the chandelier as a point mass at the end of piece of massless string of length 3 m.

A-22 This problem is not assigned, but I will post a solution. We have two blocks connected by a rope on a long ramp that makes an angle θ with respect to the horizontal. The lower block is m_1, mass 0.2 kg and coefficient of static friction $\mu_s = 0.4$, while the upper mass m_2 has a mass of 0.1 kg and a coefficient of static friction $\mu_s = 0.6$. The angle θ is gradually increased until the two blocks begin to slide. Find the force diagrams, the equation of motion, the constraints (these are unusually complicated; be careful), the angle θ at which the slide begins, and the tension in the rope connecting the two blocks at the angle at which the slide begins. FIGURE

A-23 Consider a bag of flour sliding down an extremely long chute having angle θ with respect to the horizontal. The bag has mass M. The flour mill is gradually warming up. The temperature of the slope increases with time, leading to a coefficient of kinetic friction that depends on time as $\mu_k = A + Bt^2$, A and B being constants. Find the position of the bag as a function of position along the slope, using a coordinate system in which one axis lies parallel to the chute, as a function of time. You may assume that B is sufficiently small that the bag of flour does not come to a stop during the time under consideration.

A-24 A 200,000 kg aircraft starts its takeoff run at rest at one end of a 6000m runway. Because the engines are coming to full power, its acceleration at first increases quadratically with time, i.e., $d^2x/dt^2 = qt^2$, for the first three seconds, during which time its acceleration increases from 0 m/s^2 up to a maximum of 36 m/s^2. How far does the aircraft travel down the runway in the first six seconds? What is the thrust (the force on the aircraft due to the engines) three seconds after the aircraft begins its takeoff run/

A-25 Two blocks rest on a horizontal, stationary table. The 50 kg block rests on top of a 100 kg block. The surface between the 50 and 100 kg blocks has a coefficient of kinetic friction $\mu_k = 0.1$. The surface between the 100 kg black and the table supporting both blocks is frictionless for all practical purposes. A $100\hat{\mathbf{i}}$ N force is applied to the upper block. Find the accelerations of the upper and lower blocks. FIGURE

A-26 A mass M lies on a ramp. The ramp supports have been cleverly fitted with wheels so that the ramp may be moved to the left or the right. The ramp is frictionless. A spring of spring constant k and unstretched length ℓ is attached to the ramp at its top end and to the mass M at the bottom end. By how much is the spring stretched if the mass M and the ramp are both stationary? FIGURE

A-27 As seen in the figure, two masses m_1 and m_2 are suspended from strings. The strings drape over a pair of pulleys and connect to opposite ends of a nearly massless spring. The spring has spring constant k and unstretched length ℓ. By how much is the spring stretched from its equilibrium length? FIGURE

A-28 A right triangular block sits on a frictionless table. Resting on the frictionless hypotenuse of the block (see figure) is a mass M. The hypotenuse makes an angle θ with respect to the horizontal. A rocket motor has been attached to the back (vertical) side of the block. The rocket motor is fired, bringing the block to a constant acceleration $d^2x/dt^2 = a$; the mass M is then released so it is free to slide up or down the block. If the rocket's thrust is properly chosen, there is a value for a such that the mass slides neither up nor down the block. What is a? If a is increased, does the mass slide up or down the side of the block? Hint: You know that the vertical acceleration is zero, and the horizontal acceleration is some number, suggesting that these are the good coordinates. FIGURE

A-29 The ramp and mass from problem A-26 are now given a modest horizontal acceleration a. Once matters settle down, the mass M sits at some constant position along the ramp. Find the force diagram and the equations of motion that, if solved, would give the change in the length of the spring. Do not solve.

A-30 A particle of mass 37 kg, at rest at time 0, is subject to an external force $3\hat{\mathbf{i}} + 4t\hat{\mathbf{j}} - 3\cos(2t)\hat{\mathbf{k}}$. The particle is floating free in outer space, and is subject to no other forces. What is its momentum at the later time t?

A-31 Four masses are located at $(0, 0, 0)$, $(L, 0, 0)$, $(0, L, 0)$, and (L, L, L). The masses have mass m, m, $2m$, and $2m$, respectively. Find the center of mass of the four masses.

A-32 A thin rod made of foamed plastic has ends at $x = L$ and $x = 2L$, and a density that varies with position as $\rho = A + Bx^2$. Find the center of mass of the rod.

A-33 A 3000 kg automobile is traveling up George Street–which as may be inferred from its name is one of the more important thoroughfares in our metropolis–at 30 m/s. It encounters a second 2000 kg car exiting a driveway perpendicular to the main road, traveling at 20 m/s. The two vehicles adhere and proceed as a single unit. What is the final velocity of the unit? George Street lies in the $y - z$ place and makes an angle of 15 degrees upward with respect to the positive y axis, the y axis being horizontal. Take the second vehicle to be proceeding in the $+x$ direction.

A-34 [This is a slight variation on an old exam problem. It requires slightly more attention than it was given by many people taking the examination.] A block of wood of mass M is struck from the rear by three arrows, the arrows having masses m, $2m$, and $3m$, respectively. The block was initially stationary. The arrows each had speed v relative to the ground in the $+x$ direction. The arrows all stuck into the block. (a) What was the final velocity of the block and arrows? (b) What were the velocities of the block after being struck by the first arrow, the second arrow, and finally the third arrow? Show that your answers in parts (a) and (b) agree, or explain why they do not.

A-35) Fun problem. Will not be graded: In yet another scene from our movie, the villain, sitting at sea level in his volcanic island hideaway, fires a giant cannon at the good guy's ship. Unfortunately for the villain, he uses a cheap homebuilt atomic artillery shell of mass M, rather than a quality sabotage-resistant atomic artillery shell from Villain Supply, LLC(1). The heroine has sabotaged the artillery shell, so that at the top of its trajectory there is an internal explosion. The top of the trajectory is a distance L away from the villain. The explosion sends a section of the shell of mass $M/4$ horizontally backwards; it finally lands at the villain's feet(2). The remainder of the shell, mass $3M/4$, continues on its way until it hits the ocean some distance from the villain. To add insult to injury, the heroine has sent a coded message to the Captain of the Villain's yacht, ordering him to move his ship to a new location, where his ship will be sunk by the remainder of the shell. How far from the villain should the villain's yacht be anchored, if the $3M/4$ section of the shell is to strike the yacht? Remark: Instead of calculating out every bit of the various trajectories, a less detailed approach will get you to the right answer. FIGURE

(1) No, I did not make them up, but they are no longer on the net.

(2) This is the section with the atomic warhead, set to detonate when it returns to sea level.

A-36 A particle of mass 98 kg is subject to an external force $(3 + 2x)\hat{\mathbf{i}}$ as it moves from $x = 15$ to $x = 10$ along the x-axis. What is the change in the kinetic energy of the particle due to this move? Could the initial (at $x = 15$) kinetic energy of the mass have been zero?

A-36A Repeat A-20 but solve using energy conservation.

A-37 A 10 kg box starts at the top of a frictionless inclined plane and slides down it. The incline makes a 50 degree angle with respect to the horizontal. Identify each force acting on the box, and calculate the work done on the block by that force after the box has slid 3 m along the plane. What is the total work done on the box? If the box began its slide from rest, how fast is it moving after it has moved 3 m along the plane. If the box began its slide with an initial down slope speed of 6 m/s, how fast is it moving after it has moved 3m along the plane?

A-38 A mass M rests on a horizontal table. It is initially stationary at $x = 0$. It is touching an initially uncompressed spring, fixed at its far (left) end. An outside force P pushes leftward on the mass, compressing the spring very slowly by a distance ℓ. At the end of the compression step, M is again stationary. During the compression step, compute the work done on M by the spring, by the outside force, and by the total force. What is the change in the kinetic energy of M during the compression step? The outside force is then removed. The mass M is pushed to the right by the spring, starting at $x = -\ell$ and ending at $x = 0$. What is the total work done on M while it is being pushed to the right? What is the kinetic energy of M when it reaches $x = 0$? [Graders have been warned to check carefully for multiple, canceling, sign errors.] FIGURE

A-39 This question has been cancelled.

A-40 A 10 kg box starts at the top of an inclined plane and slides down it. The incline makes a 50 degree angle with respect to the horizontal. The coefficient of kinetic friction is $\mu_k = 0.3$, Identify each force acting on the box, and calculate the work done by that force after the box has slid 3m along the plane. What it the total work done on the box? If the box began its slide from rest, how fast is it moving after it has moved 3m along the plane? If the box began its slide with an initial down slope speed of 6 m/s, how fast is it moving after it has moved 3 m along the plane? (We have the same situation as in A-37 but now there is also friction.)

A-41 In the space between two flat parallel plates, the electric field vector $\mathbf{E}$ is uniform and perpendicular to the plates. An electric field produces a force $\mathbf{F}_e = q\mathbf{E}$. In the electrical system as sketched, the electrical field is $\mathbf{e} = 250\hat{\mathbf{j}}$ in SI units. The plates are a distance L apart. Take the plates to be at $y = 0$ and $y = L$. A tiny, neutrally buoyant helium balloon bearing a charge $q = 1 \cdot 10^{-3}$ in SI units is released (perhaps not from rest) at point (0.5, 0.6, -.8); it moves to point (1.0, 1.5, 2.2). Both points are between the two plates. As a result of this move: What is the change in the potential energy of the balloon? What work is done on the balloon by the electrical field? FIGURE

A-42 Two children, masses m_1 and m_2, sit on opposite sides of a teeter-totter at distances ℓ_1 and ℓ_2, respectively, from the center. Find their total gravitational potential energy as a function of the angle θ between the teeter-totter arm and the horizontal. Find the values of ℓ_1 and ℓ_2 (there may be more than one pair of these) for which the gravitational potential energy is independent of θ. FIGURE

A-43 A non-linear spring produces a force along the x-axis $F_x = -3x - 7x^3$, where one end of the spring cannot move and x is the location of the other end of the spring. Find the potential energy of the spring when it is stretched or compressed. Take $U = 0$ at $X = 0$.

A-44 A mass M lies at the top of a frictionless ramp having length L and inclined at angle θ to the horizontal. The mass is released. It slides down the ramp until it comes into contact with an initially uncompressed, unstretched spring having spring constant k and length ℓ. The mass then sticks to the spring. Find the locations at which the spring brings the mass to a stop. A reasonable calculation finds two answers. Why? What is their physical meaning? FIGURE

A-45 A mass m sits on top of a frictionless sphere having radius R. A vagrant breeze perturbs the mass, which begins to slide down the slope. It moves faster and faster, until at some point the contact force between the mass and the sphere goes to zero and the mass's trajectory takes it away from the sphere. The position of the mass is defined by an angle θ with respect to the vertical through the center of the sphere. At what value of θ does the mass cease to move along the surface of the sphere?

A-46 A somewhat exotic automobile, when the accelerator is applied, approaches its maximum speed exponentially, so that its speed is given by

$$v = v_0(1 - \exp(-\Gamma t))$$

where v_0 and Γ are constants and t is the time since the accelerator was first applied. The automobile has mass m. Calculate the power being supplied as the car accelerates toward v_0. You may treat the problem as one-dimensional. Hint: The power depends on t.

A-47 In yet another scene from the motion picture, the director stages a car crash, complete with smoke, loud noises, acrobatics, and high explosives hidden at the impact point. Entering stage left we have the villain's 3000 kg convertible SUV traveling at $30\hat{\mathbf{i}} + 40\hat{\mathbf{j}}$ m/s and the heroine's 1000 kg convertible sports car traveling at $50\hat{\mathbf{j}}$ m/s. The quoted masses of the vehicles include the masses of the people in the cars. The two vehicles run into each other. A half-dozen of the villain's henchmen (mass, 700 kg) jump from his car to the heroine's car; the explosives are then detonated. At scene close, the sports car with seven people on board is exiting the collision point at $100\hat{\mathbf{j}}$ m/s, while the villain's vehicle departs at $23\hat{\mathbf{i}}$ m/s. What was the total work done on the two vehicles and their contents during the scene, which you cannot approximate as a collision?

A-48 The electrostatic potential energy between two charges in a sodium chloride solution (at a very low approximation, two protein molecules in dilute solution) may be written as a *screened* Coulomb potential

$$U(R) = \frac{k q_1 q_2 \exp(-\kappa R)}{R} \tag{1}$$

where k is a remarkably complicated constant, q_1 and q_2 are the electrical charges on the two protein molecules, κ is a constant called the "Debye (inverse) length", and R is the distance between the two protein molecules. What is the magnitude of the electrical force between the two molecules?

A-49 Consider the totally elastic collision between two billiard balls. To simplify the math, all motion is parallel to the x-axis. The balls both have mass m. The initial velocities of the two balls are $v_{1i}\hat{\mathbf{i}}$ and 0; the final velocities are $v_{1f}\hat{\mathbf{i}}$ and $v_{2f}\hat{\mathbf{i}}$. For a totally elastic collision, compute the two final velocities. (Aside: there are actually two answers. Explain.) FIGURE

A-50 It is once again time to boot up a slightly older computer. The flash boiler is started at $t = 0$. At $t = 30$ seconds, the flash boiler has worked up a sufficient head of steam, and a valve is thrown, supplying steam to the engine that spins the 2048 bit drum memory. The drum, intially at rest, accelerates at 15 rad/s^2 up to its maximum 120 radians/second. Assuming that the drum started at $\theta = 0$, calculate the angular position and velocity of the drum at $t = 70$ seconds. For credit, do all calculations using my origin for the time coordinates.

A-51 A pin is stuck into a rigid rod. The pin is not perpendicular to the rod. At a particular moment in time, the rod lies along the line $-\hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and has angular velocity $\vec{\omega} = -2(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$. (Yes, the angular velocity is parallel to the rod, which is rotating around its long axis.) The pin intersects the rod at the origin, and at time 0 has its head at $3\hat{\mathbf{i}} + \hat{\mathbf{j}}$. (a) What is the angular rotation rate ω of the rod? (b) What is the velocity of the head of the pin? Hint: The standard error is to fail to notice that $\vec{\omega}$ is *not* written in the form $\omega\hat{\omega}$. Look carefully and you will find why.

A-52 The *beanstalk* is a hypothetical spaceship launching device composed of an asteroid and a long (e.g., 50,000 km) rope that is tied to the earth's equation at one end and the asteroid at the other. The asteroid is not in orbit around the earth; it is being held in place by the rope, like a stone being whirled around at the end of a long string. The rope is under a large tension T. An elevator starts at time zero, climbing up the rope from the earth's surface. In circular polar coordinates, the position of the elevator follows $r = r_0 + 0.5 a_r t^2$. Its angular velocity is a constant $\dot{\theta} = \omega$, ω being the angular rotation rate of the earth in radians/second. In polar coordinates, find the elevator's acceleration as a function of time.

A-53 In yet another scene from our motion picture, the heroine has been trapped in a steel cage on a giant merry-go-round. The villain is spinning the merry-go-round faster and faster, so that $\omega = \alpha t$. Also, the cage is gradually being winched from the center to the outside edge of the merry-go-round, where dire consequences will ensue; the radial position of the heroine depends on the as $r = r_0 + v_r t$. Compute the heroine's velocity and acceleration in circular polar coordinates.

A-54 We have three masses spaced along a rigid massless rod that initially lies on the x-axis. Mass 1 is 1 kg and starts at $x = 0$. Mass 2 is 2 kg and is +1 m from mass 1 along the x axis. Mass 3 is 4 kg and is + 1 m along the x-axis from mass 2, meaning it is not 0 or 1 m from the origin. The masses are spun around an axis occupied by the z-axis, revolving at 2 radians/second. What is the angular momentum (hint: It is a vector) of the three masses and their rod?

A-55 A mass m is at the mobile end of a rigid, nearly massless, pivoted rod of length ℓ. The rod extends perpendicular to the axis of rotation out to the mass. The axis of rotation lies on the z-axis, from which the angular momentum is to be calculated. The mass m incorporates an electrically driven propeller which applies a force $\mathbf{F}$ that lies in the plane of rotation and perpendicular to the rotating rod. (a) The fan is gradually sped up in speed, so that it supplies a force whose magnitude is $F_o \exp(st)$, where F_o and s are constants determined by the fan rotation. What is the angular momentum of the mass relative to its rotation axis as a function of time? (b) Find the angular velocity of the mass m around the rotation axis as a function of time. FIGURE

A-56 Prove

$$\frac{d}{dt}(\mathbf{A} \times \mathbf{B}) = \frac{d\mathbf{A}}{dt} \times \mathbf{B} + \mathbf{A} \times \frac{d\mathbf{B}}{dt}$$

by (1) writing the cross product in term of the scalar components, (2) taking the derivatives by using the rule for derivatives of products having scalar multiplication, and (3) collecting terms. Warning: You are not proving the derivative-of-product rule you learned in calculus. For example, your proof probably assumed that multiplication is commutative, and the cross product is not commutative, so you have probably not ever proved this before.

A-57 A space station consists of two very heavy objects, M_1 located at $(0, 0, 0)$ and M_2 located at $(\ell, 0, 0)$. The two objects are linked by a very light boom having length ℓ. A rocket motor applies a force of magnitude a in the $-\hat{\mathbf{j}}$

direction to M_2. What is the torque applied to the space station, if the torque is calculated relative to the location of (A) M_2, (b) M_1, or (c) the space station's center of mass?

A-58 A uniform rod lies in the x-y plane. It rotates about one end, the axis of rotation being the z axis, at ω radians per second. The rod has mass M and density M/ℓ along its length. What is the angular momentum $d\mathbf{L}$ of an infinitesimal piece of rod? By integrating, determine $\mathbf{L}$ of the entire rod.

A-59 Consider the rod of the previous problem. Assume $M = 2$ kg, $\ell = 3m$, $g = 10$ m/s^2. If the angular acceleration $d\omega/dt$ is 5 radians/second^2, compute the external torque on the rod.

A-60 A rigid rod is attached at one end perpendicular to a rotating axis. The linear density of the rod increases linearly with distance from the axis, starting at 3 kg/m at the axis, and increasing to 5 kg/m at the outermost end of the rod, which is 2 m from the axis. The rod rotates in the $x - y$ plane. A torque is applied to the rod, such that the angular velocity of the rod increases by 4π radians/second over two seconds. What is the torque $\vec{\tau}$ applied to the rod?

A-61 A mass m slides down a frictionless ramp inclined at 30 degrees to the horizontal. The mass begins stationary at a height H directly above the origin. Find the block's linear and angular momenta with respect to the origin as functions of the height h of the block.

A-62 Two circular disks mounted on the same axis are cemented together so that they rotate as a unit. Their combined moment of inertia is I. Their radii are R_1 and R_2, respectively. A rope is wrapped around each of them, with one end hanging down to support masses m_1 and m_2 as seen in the figure. Find the force and torque diagrams, the equations of motion, and the constraints. Find the ratio of the masses such that the disks will not accelerate in either direction. For the general case that the two masses do not balance each other, find the angular acceleration and the tension in each rope. FIGURE

A-63 A HARD PROBLEM; WILL NOT BE GRADED AND WORTH NO POINTS: [Some years ago, a slight variation of this was an exam problem. Class performance left room for improvement.] In yet another scene of the film that was a stock of old exam questions, the heroine is dropped from a helicopter onto the back of a dirigible. The studio did not have a dirigible handy. The dirigible mockup is a long plastic cylinder mounted between two towers. The cylinder rotates around its own center of mass; its vertical motion can be neglected. After a short period of time, the dirigible rotates so that the heroine is at the bottom of the dirigible. What is the angular velocity of the heroine as she passes the bottom? Put the origin of the coordinate system on the central axis of the dirigible. Do not try to simplify your final expression.

The heroine has a mass m and is traveling at speed V when she impacts the dirigible. The dirigible has mass M, radius R, and moment of inertia MR^2. Until the heroine lands on it, the dirigible is stationary. The heroine lands a distance l to the left of the center of the dirigible. (**Hints!** This is not a collision problem. A near-infinite force holds the center axis of the dirigible in place, so that the dirigible rotates freely, but its center axis does not move. The linear momentum of heroine + dirigible is therefore not conserved. However, the bearings are frictionless, so the only external torque on the actress+dirigible system is exerted by gravity. No one is making you answer the questions in the order that I am asking them!) FIGURE

(a) What is the kinetic energy of the heroine just before she hits the dirigible?

(b) What is the linear momentum of the heroine just before she hits the dirigible?

(c) What is the potential energy of the heroine just before she hits the dirigible? Take the center of the dirigible to have height 0.

(d) What is the angular momentum of the heroine just before she hits the dirigible?

The heroine hangs on to the dirigible when she hits. The dirigible starts to rotate around its own central axis, which is firmly mounted and does not move, while she hangs on.

(e) What is the kinetic energy of the heroine just after she hits the dirigible?

(f) What is the linear momentum of the heroine just after she hits the dirigible?

(g) What is the potential energy of the heroine just after she hits the dirigible? Take the center of the dirigible to have height 0.

(h) What is the angular momentum of the heroine just after she hits the dirigible?

(i) Draw a force diagram for the heroine just after she lands on the dirigible.

(j) The dirigible rotates freely. What is the angular rotation rate of the dirigible just after the heroine hits it? FIGURE

A-64) BONUS PROBLEM: WORTH ONE HOMEWORK EXERCISE. SUBMIT ON A SEPARATE SHEET OF PAPER. This problem is short, but quite difficult if you do not see how to do it. A pencil of length L and mass M is resting sharpened point down on a totally frictionless surface. The air current set up by a passing butterfly disturbs its equilibrium and the pencil falls over. What is the speed of the pencil's center of mass just before the pencil falls flat into the table? FIGURE

A-65 Lab Problem. YOU WILL NEED THE ANSWER TO THIS PROBLEM FOR YOUR LAB THIS WEEK. A uniform rod of length ℓ and mass M hangs from a pin at its top end. It makes an angle θ with respect to the vertical. Find (i) the moment of inertia of the rod, (ii) the torque on the rod, and (iii) (from (a) and (b)) the equation of motion for the rod. (iv) Find the angular acceleration of the rod. (v) For small angles θ find the frequency and period of oscillation. A nut of mass m is now attached to the rod, a distance x from the top end; find the period of small oscillations as a function of m and x. Solve this problem also for a pendulum having the shape of a long thin pie wedge. FIGURE

A-66 A rod of length L is pivoted at one end. At equilibrium it hangs vertically downward. At some moment, it makes an angle θ with respect to the vertical. The rod is not uniform; its density depends on distance ℓ from the pivot as $\rho = \mu\ell^3$. ρ is the density per unit length, so that the mass of a differential section of the rod of length $d\ell$ is $\mu\ell^3 d\ell$. Find (a) the gravitational torque on the rod, (b) the moment of inertia of the rod, and (c) assuming the pivot is frictionless, the angular acceleration of the rod (a correct answer is linear in $\sin(\theta)$). For small displacements, find the frequency of oscillation.

A-67 A yo-yo is a classic child's toy. It consists of a slotted wheel at the end of a string. Part of the string is wound around inside the slot. The string makes rolling contact with the slot. If the string is held at the other end, and precisely the right tension is applied, the top end of the string remains stationary and the yo-yo simply rolls down the string. Find the tension T in the string if the top end of the string is stationary and the yo-yo is unrolling the string that has been rolled around it. Assume that the yo-yo has mass M, radius R and moment of inertia $0.5MR^2$. FIGURE

A-68 A long cylinder (see figure) lies in a circular groove within which it rotates freely. The uniform cylinder has mass M and radius R. A rope wrapped repeatedly around the cylinder then goes down to a block of mass m. The mass lies on an inclined plane that makes an angle θ with respect to the horizontal. Find the force and torque diagrams, the constraints, the angular acceleration of the cylinder, and the tension in the rope. FIGURE

A-69 An outraged student at a competing university takes his 10 kg textbook to the stop of his 20-story (70m) dormitory, walks to the edge of the roof, holds the book 0.5 m forward, and drops the book. The book falls to earth 70 m below. "Forward" is in the x direction; "up" is the y direction. Find the angular momentum of the book relative to the student as a function of its altitude h above the ground. FIGURE

A-70 Return to Problem A-69. Calculate the torque on the book, with respect to the location of the student, due to the gravitational force on the book. Beginning with $\vec{\tau} = d\vec{L}/dt$, calculate the elapsed time of the fall as a function of the height h of the book above the ground. The objective of this problem is to use the torque/angular momentum approach to solve the problem, so if you write s= -0.5 g $t^2 + H$ you are completely missing the point of the question and will get no points. FIGURE

A-71 Find the first four terms in the series expansions for the Earth's gravitational potential and the earth's gravitational force, centering the expansion at the radius R of the earth, and calling the expansion variable x. Interesting math feature: If you did the expansions right, they are not convergent for $x/R > 1$, no matter what R you chose. "Why" is a sophisticated math question.

A-72 The Black Star passes (literary reference). One of the cosmic catastrophes that could render the earth uninhabitable is the passage through the solar system of another star, a body that would significantly perturb the earth's orbit so that we froze or fried to death. For the sake of hypothesis, WPI astronomers have recently detected a star traveling at speed V whose straight-line trajectory would take it a distance $r_\perp$ from the sun. Alas, the Black Star's trajectory will be perturbed by the sun's gravity, so that it will actually pass a distance r_m from the sun. Noting that the gravitational potential is GMm/r where M is the solar mass, m is the black star mass, G is the gravitational constant, and r is the distance between the sun and the black star, find r_m in terms of the other variables. FIGURE

A-73 A solid sphere having radius R has had removed from its middle two spheres of radius $R/2$. The sphere is located at the origin. The two empty spherical volumes have their centers on the z axis at $+R/2$ and $-R/2$. Find the gravitational potential energy created by this object for a mass m located along the x-axis. Do not attempt to reduce your answer to a simpler form. Comment: There are three or four ways to work this problem. Some use more brute force. Some use more cleverness. FIGURE

A-74 A large circular pipe lies on a log ramp. A rope is tacked to the top of the pipe and proceeds horizontally to the ramp. The coefficient of static friction is large enough that the pipe does not move in the indicated situation. Find the force applied by the rope to the pipe, the force applied by friction to the pipe, and the normal force from the ramp on the pipe. FIGURE.

A-75 Some years ago, this was an exam problem. An impressive number of people forgot that force is a vector, not a scalar, and got no credit as a result. A painter sits in a bosun's chair as indicated. She holds herself in position by holding the cable in one hand as shown in the sketch. The chair and painter are stationary. The painter has a mass of 100 kg; the chair has a mass of 20 kg. (a) Draw the force diagram for the painter. (b) Draw the force diagram for the bosun's chair. (c) Find the tension in the cable. (d) Find the normal force $\vec{N}$ that the painter applies to the chair. FIGURE

A-76 The sign at a local cocoa store is a block of mass M attached to the end of a horizontal rod of length L. The other end of the rod is fastened to a wall with a large hinge. A rope is fastened to the wall at a distance H above the hinge; the rope is also fastened to the rod at a distance x from the wall. The mass of the rod is negligible. Find the tension in the rope and the force on a hinge. FIGURE