Physics 1111 - Term C 2015
Laboratory One

The objective of the lab is to have you get some experience doing real experiments. A real experiment is one in which you do not know the answer in advance. Real experimental work has important parts that usually are not discussed in treatments of physics theory. In this course, were going to introduce some of those parts. This first experiment, which runs all of next week, introduces data analysis, dimensional analysis, experimental accuracy, and experimental design.

First, were going to introduce the notion of doing data analysis. That is, you are going to do measurements, and you are going to see that writing them down is only the front end of analyzing data. Second, you are perhaps going to see that dimensional analysis actually works, in the sense that if you do dimensional analysis sometimes you can tell what form the answer must take. Third, you are going to see experimental accuracy, the thing that lies behind the notion of significant figures. Of course, we could treat significant figures as a totally rote process with no intellectual content or meaning at all; were not going to do that here. Fourth, we are going to introduce the notion of experimental design. You will not walk into the lab and have one lab partner read mystic instructions out loud while the other lab partner presses button A, turns knob B, writes down the number on gauge C, and sacrifices to Nyarlathotep the goat in Cage D, with no idea what is going on. Instead, you are actually going to see how various choices on how you do the experiment can affect how accurate your measurements are.

Some parts of the lab appear a bit tedious. Do you really need to test all those different ways of measuring the period? Yes. You are seeing real science in action, where a major part of the experiment is getting the experiment to work as well as possible. With real science, you can spend weeks or months or years getting the instrument to work. Finally, you get to the point where you are taking real data. The measurements until then only matter because they were ways of testing the instrument. You are not going to spend weeks optimizing your experiment, but you are going to spend a day on this important art of optimizing the experimental method.

Some of these steps are a bit time consuming. We are trying to make sure that you have enough time to do each laboratory. Also, during the day the laboratory is unlocked in hours in which it is not use, so if you arent quite happy with the measurement you can walk in and try it again. The laboratory is also open for group study and use of the computers, so long as you are not in the way of people using it for a class.

The physical experiment today uses string and masses to construct a pendulum. The physics question you are trying to answer is to determine if the period of the pendulum is affected by the length of the string or the number of nuts hanging on the bottom of the spring. It turns out that the period of the pendulum is also affected by the force of gravity, but it is fairly difficult though not quite impossible to do an experiment which varies the force of gravity, so that you can measure how the period of the pendulum depends on the force of gravity. (I can offhand think of three ways to vary the force of gravity, one of which requires an extremely expensive piece of equipment, a second of which requires the services of an airplane and a competent pilot or good computer controls, and the third of which might perhaps be done with the aid of a bicycle wheel mounted to rotate in the horizontal plane.)

For this experiment, you will need a long piece of string, a set of weights, something that works as a stopwatch, a couple of braces on which to hang the string, and, if we can find some, a laser pointer. Many of you have pocket telephones or the like that in fact have a timer function. Others of you have portable devices onto which you can download an application that serves as a stopwatch. If you did as you were told and read these instructions prior to coming to class, either you would be ready with one of these, or you would know that you are going to have to borrow a piece of equipment.

As a first question, what do we mean by the period of a pendulum? The period of the pendulum is the time it takes for the weight at the end of the pendulum, the pendulum bob, to swing back and forth and return to its initial position while going in the same direction. An interesting point is that it does not matter from where you measure the period of the pendulum. You could measure the period of the pendulum starting at one end of the swing or the other, you could measure the period of the pendulum starting from the bottom of the swing, or you could measure the period of the pendulum from an arbitrary point in the middle of the swing. So long as the pendulum has swung back and forth, come back to its starting point, and is going in the same direction as it was originally, the period of the pendulum is the same. Instead of believing me, you should actually test this claim as part of your experiment.

I am now going to list several different ways you could go about measuring the period of the pendulum. You should try each of these, and include them in your report. The reason that you try each of these methods is that they are so simple that you can actually see exactly what is happening and to some extent understand why one method is more or less accurate than the others. After you have done the lab I will explain in class which method is the most accurate, in particular if you are measuring electronics voltages using an A to D converter. (You have been seeing A to D converters all your life; large numbers of instruments like many digital thermometers generate a voltage and then use a converter to convert the voltage, the analog A quantity, into a number on a screen, the digital

D quantity.)

Let us now consider several methods for measuring the period.

Method 1: One lab partner holds the pendulum at the outer end of the swing. The other Lab partner says start. The first lab partner lets go and the second lab partner starts her timer. After a half-dozen swings, the first lab partner grabs the pendulum as it reaches the top swing, and at the same time says stop. The second lab partner stops her timer. Repeat this process a half-dozen times, and record the elapsed time and the number of swings for each try of the experiment.

(I have just introduced an important physical distinction. A time, seen on a clock, is an absolute time, for example 2 PM. An elapsed time or time interval is the difference between two times measured on a clock. If I asked you what the period of pendulum was, 2 PM is not a reasonable answer. If I asked you what time it is, it took 20 seconds is not a rational answer. Thats because times and time intervals are completely different sorts of things.)

Method 2: the first partner holds and releases the pendulum. The second partner uses the stopwatch. The first partner waits until the pendulum has swung back and forth; when the pendulum reaches the top swing the first partner says start, and the second partner starts her stopwatch. The first partner counts off five swings; when the pendulum again reaches the top of its swing, he says stop.

Method 3: the partner with stopwatch lowers her head until she is looking along the surface of the table at an object well behind the swing of the pendulum. The height of the pendulum is adjusted so that the swing of the pendulum just clears the surface of the table. You adjust the height of the pendulum by raising or lowering the bracket holding the pendulum, not by changing the length of string, since the length of the string is a variable you are trying to study. The other partner releases the pendulum. The partner with the stopwatch times 10 or 12 passes of the pendulum bob through her line of sight. There are, of course, two passes per period.

Method 4: same as method 3, except that you put an object on each side of the path through which the pendulum swings and sight along the line defined by the two objects. Those of you who have learned how to aim a firearm will find this arrangement extremely familiar, except usually you do not place the target between the front and rear sights of the weapon.

Method 5: same as method 4, except the other Lab partner does the experiment.

Method 6: same as method 1, except the same lab partner both holds the stopwatch and lets go and grabs the pendulum bob.

Method 7: if we have found a supply of laser pointers, set a laser pointer on the table (you may have to sit it on the top of the book or something to get it to exactly the right height) so its interrupted when the pendulum swings by. Look at the point of light on the wall and use the shadow of the pendulum interrupting the laser pointer light as your time signal.

Clearly this is not an exhaustive list of different ways to do the measurement. For example, you might choose to alternate which partner is using the timer, and then average over two lab partners to get a time measurement.

DATA ANALYSIS

You now have a whole pile of numbers. What do you do with them? The objective of the above part of the experiment was experimental design. You are trying to determine what the most accurate way is to measure the period of the pendulum. There are a lot of different ways of measuring period, and all of them are not equally good. You havent tried all possible methods, but you have tried a bunch of them.

What do you mean best? Until you have some decision as to what you mean by best way to measure the period, you probably cant decide rationally which way is best. By best, I mean most accurate. The most accurate measurement is the one with the least error. In order to go any further we have to say a few words about what error is.

There are at least three sorts of error in an experimental measurement. There are fundamental conceptual errors, there are systematic errors, and there are random errors. A fundamental conceptual error is a mistake you make, because you fundamentally dont understand what is going on, and make a decision based on your lack of understanding. For example you could note that the various types of experiments were carried out at different times of day, and therefore if you cast the horoscope for each type of experiment you would be able to infer from sound astrological reasoning which of the experiments was the most accurate. As most of you realize, astrology does not work.

Catching fundamental conceptual errors is extremely difficult. In my own professional research specialty, there is a fundamental conceptual error which, as of this writing I am in the process of rooting out, which has led to large numbers of wrong experiments, and the manufacture of a machine based on a totally erroneous theory. In the meantime, literally millions of dollars of your income taxes have gone down the drain in support of this fundamental conceptual error.

You notice I did not tell you how to find fundamental conceptual errors. The best answer I can give you is that you are most likely to find things if you have an extremely deep fundamental understanding of what is going on, if in accepting a book or journal article as telling you what is going on you have actually duplicated their calculations yourself, and if you have read a large number of different sources. In addition, it is sometimes useful to take the experiment and run the system way outside its design parameters, hopefully without blowing the system up, and seeing if the failures are in the expected answers. In the end, being extremely observant and paying attention to find detail is sometimes a useful tool for finding out what is actually going on.

At the other end of things, random errors are the scatter in the measurements from measurements to measurement, that are seen because the system is not perfect. If you time the pendulum swing five times, and have a good timer, you will discover that each occasion on which you run the experiment you get a slightly different time interval for five repetitions of the pendulum. Thats random error. You can try to improve your measurement method; that is what we were just doing. You can think about how the experiment is being done, and ask yourself which method is going to be the most accurate. That approach is very dangerous. Most people, confronted with the above list of possible ways of doing the experiment, will choose the wrong answer when asked which method is the best. There is a method that looks the best, there is a method that is the best, and these two methods are not for most people the same. I have an unfair advantage. I was tutored in how to do good experiments of this sort by Rai Weiss, my former MQP advisor, who is the fellow who designed the core of the Large Interferometric Gravitational Observatory, for which if he lives long enough he has an excellent chance of receiving the Nobel Prize. Once you understand why the correct answer is correct, it is much easier to recognize it.

So how do we measure the random error? Thats a precursor to looking for systematic errors, so we have to look at the random error first. The first step is to take your measurements for each method and to take an average of all five, or however many you took, of them. Call the individual measurements t_i. The average of the measurements we will call T.

Calculate T for each of your methods, and include T in a table in your final report. The table should have a header. It should have a column labeled by the methods. It should have a column for T.

You now have a list of methods and their determinations of the pendulum period. Your values for the pendulum period are probably not all the same. Also, you would like to be able to say how accurate your measurements of T are. Your actual measurements, the t_i, probably do not all agree exactly, the spread in the individual measurements around the average being the random error. How do you characterize the random error s size?

One answer would simply be to average all of the individual errors $t_i - T$. Do that once. If you did it correctly, the answer you obtained is exactly 0. If you did not get zero for your answer, go back and check if you remembered that $t_i - T$ as a number has a sign, plus or minus. A reasonable next step is to propose that instead of averaging $t_i - T$ you will average the magnitudes $\mid t_i - T \mid$. That does work. However, as you will see when you study statistics, in many cases it turns out to be better to average $(t_i - T)^2$, and then take the square root.

For each method, calculate the average of $(t_i - T)^2$, take the square root, and add each quantity to your table. Use a separate column. A better method has a smaller value for this quantity. Is it apparent that some of your methods are better than others? Include the answer to this question in your discussion. Choose which ever if your methods works the best for you, and proceed to the next set of experiments.

Oh, yes, the last sort of error. Systematic error. Some of your methods may be wrong in a non-random way, so that numbers obtained with that method are incorrect, e.g., the measured number is always 2% too large. The next bit of the experiments is intended to study the systematic error. For this you will need to interact with at least one neighboring lab group. For your choice of method, have each of the four of you do five measurements, calculate the average of each set of five measurements, and calculate the random error in each of the five sets of measurements. There is the key question: Do all five sets of measurements agree to within their random errors? Include your answer to this question in the discussion. Your answer should be a short paragraph. Your answer has to agree with the measurements you just did. For different groups of people, and different choices of best method, the answers may not be the same.

There is a very important lesson here, namely that if you are doing experiments, and the experimental data is good, your conclusions are supposed to agree with your experimental results, not with what you hope to find. Of course, you can be unlucky. If you expected to find a particular answer, and thanks to the random noise your data really fit the expected answer well, you will probably conclude that you proved what you set out expected to prove. Having expectations is good. They give you warnings, for example, you have forgotten to turn on one of the instruments, and the zero readings do not correspond to the expected result 42. However expectations can also be bad. I am reminded of a famous paper in my field in which the researchers did a heroic pioneering experiment and announced that they had confirmed a controversial theory created by some of their friends. There are still a lot of people who believe that theory, and there is still a lot of data that is interpreted to support that theory, but the five subsequent papers that duplicated the original experiment found a contrary result. It appears that these very good experimenters were duped by extremely unfortunately placed random noise, and their very clear understanding of what they thought the answer was supposed to look like.

SIGNIFICANT FIGURES

We are now ready to step inside the data analysis and show you what the significant figures are actually doing. Each of your results for the period of the pendulum can be written in the form $T \pm E$ in which T is the period of the pendulum and E stands for the root mean square error. You can discuss whether you should actually show E, or 2E, or something else similar after the $\pm$. The important issue at this stage is that E represents how accurately you know the number T. If you know T to ± 0.12S, then it does not make sense to report T to an accuracy much finer than 1/100 or so of a second, because you only know T to within a band that is about a quarter of a second wide. Significant figures are a crude and imprecise way of representing this lack of knowledge, namely we suppose that we know the value of a number to 1%, a tenth of 1%, a 100th of a percent, or some other (number of zeros) 1 of the basic unit.

Usually our error in knowing a number doesnt happen to fit in one of those bands, so significant figures are very crude representation of what is going on. A detailed and exact prescription for stating significant figures makes no sense, because the underlying intellectual basis for significant figures as much fuzzier than that.

Some attention to reality in estimating errors in various numbers make sense. I am reminded of the student laboratory, before I came here, in which students said that the temperature was known to within 1%. The temperature was being quoted in kelvins, because we were doing physical chemistry. A 1% error is 3 K or 5 Fahrenheit. There was a precision temperature control bath being used, and the student was claiming that it worked much more poorly than a typical room thermostat.

THE PERIOD OF THE PENDULUM

You are now ready to do the other half of the experiment. The actual experiment is to measure the period of the pendulum as you change the length of the string and the number of nuts tied at the end of the string. However, before you could do that, you had to do the experimental design to work out how to measure the period accurately. You have now done this, and will use your best method in the remaining experiments. (If you really dont believe that your method actually should be the best method, for example because its obvious that some other method is the best, repeat the above experiments using only the two methods of interest, and use each of them, say, 20 times. When you take the average, because you have done the experiment many more times, your determination of T will be more accurate and your determination of the root-mean-square error will be more accurate. If you choose to do the extra measurements, including your discussion and explanation of why you did the extra measurements, what your experimental results are, and which method you will now use to measure T. The grader will check if your conclusions agree with your data.)

The original question of interest was how the period of the pendulum depends on the length of the string and the mass of the bolts. You will now test this experimentally. To test the dependence experimentally you will measure the period of the pendulum for at least four lengths of string, and you will measure the period of the pendulum for at least four choices of mass the end of spring. You will then generate a graph of period against length of string and a graph of period against mass on string.

If you look back at the dimensional analysis discussion, you will find that the theoretical prediction says that T is proportional to L and to M, with L and M being raised to powers that may be integers, fractions, or real numbers. However, the data fitting scheme that you are being shown will only fit data to a straight line, finding for a straight line $Y = MX + B$ the intercept B and the slope M.

You now have two choices. One choice is to research the use of non-linear least-mean-square fitting procedures such as the simplex method. That lets you fit the curve directly. Because you are a WPI student, you are entitled to download Mathematica from the WPI website. Mathematica is an absolutely incredible software tool for doing math! I should, however, warn you that while Mathematica has a superb tutorials and support help, at the very front end there is less a learning curve and more a learning cliff. My students last year did climb the learning cliff.

The other choice is to convert $T = kM^a$, a being the unknown, into a linear equation. You all actually know how to do this, though you may not realize it yet. Take the logarithm of that equation. You will get $\log T = \log(k) + a\log(M)$, which is a linear equation in $\log T$ against $\log(M)$, with a now appearing as the slope of the hopefully straight line, and the proportionality constant k appearing as the intercept $\log(k)$. You can do a linear fit to $\log T = \log(k) + a\log(M)$ to determine k and a.

The first step is to do the actual experiment. measuring the period against the string length, and against the nut mass. Your data should appear in two properly-labeled tables.

Your next step is to generate the graphs of period against length and against mass, using log scales on both axes. You should represent each measurement as a separate point, and then indicate the average as a point, and the spread around the average as error bars. The TA will put on the board a sketch showing how to draw the error bars. Observe that in at least some cases some of your measurements will lie outside of the error bars. That is an expected result. Your final step is to do linear-least-squares fits to the measurements and determine the exponents corresponding to the string length and the pendulum bob mass. You should actually carry out the fits by hand, rather than trusting your pocket calculator, which may not have implemented precisely the algorithm that you thought it did. At the end, when you have determined the exponents of the intercepts, you should take your calculated lines and plot them on your graph to see how they compare with your measurements.

The discussion in your lab report should consider what you found for the exponents, and how your findings do or do not agree with the values you computed by means of dimensional analysis for the exponents.

A GOOD LAB REPORT

The Lab report includes the title of the lab, the names of each of the lab partners, the author of the report, and several parts. The aprts match the things that you are told to do in the above. If you read carefully through the above, you will find that I have given list of things that you need to do, for example, testing each of the ways for measuring the period of the pendulum, and confirming that the period of the pendulum is the same no matter where it is measured.

Many of your sets of measurements will lead to numbers. You report these in tables. A good Table has a name, a number, a top paragraph saying what the table contains, and the table itself, with columns and/or rows as needed properly labelled.