Preamble

I recently had to cut a pizza in four, so I made a first cut through the center and was about to make a perpendicular cut when I was asked why I didn't make the other cuts parallel to the first one. I took this as a joke and proceeded in the usual way, but afterwards I wondered how I could have gotten four equal pieces by making parallel cuts. The more general problem is, how can one cut a pizza into N pieces of the same area by making cuts that are all parallel to each other?

On the next two pages you will find a simple set of instructions for doing this. All you need is a string, a marker, a ruler and a T-square.

I will admit that I wouldn't want to eat a pizza cut in this way; the pieces in the middle would be too long and thin and the ones at the ends would have more crust and less of the good stuff than I'd want. But I thought it might make for some laughs and also some good-natured ribbing of all the math nerds out there on Pi day.

How to cut a pizza into N equal pieces by cuts parallel to a diameter

1. Copy the top graph on the next page and enlarge it so that the length of its horizontal axis is one-fourth of the circumference of the pizza you wish to cut.

2. If N is even, draw N/2 equally spaced lines parallel to the horizontal axis, with the last just grazing the top of the figure (see illustration at the bottom left, which is for N = 12). If N is odd, draw N lines in the same manner as before and then erase all the odd lines (or simply avoid drawing them); see illustration at the bottom right, which is for N = 7.

3. Drop perpendiculars from all the points where the parallel lines intersect the curve to the horizontal axis (this has also been done in the two illustrations).

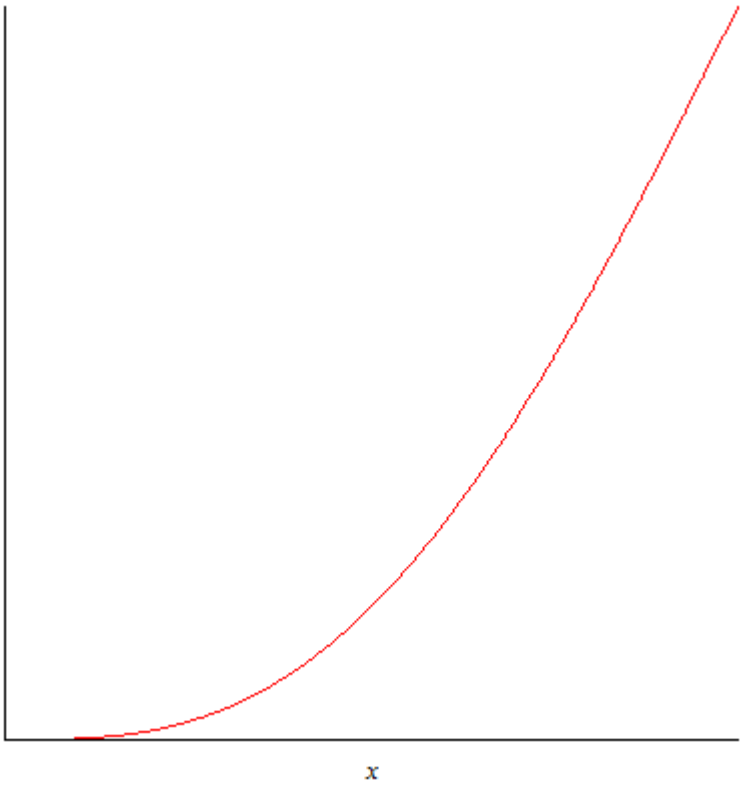
4. Take a piece of string equal in length to the horizontal axis, stretch it out along this axis and mark off the feet of all the perpendiculars on it.

5. Wrap the string around the rim of the pizza, placing its left end at the 12 o'clock position and its right end at 3 o'clock and put marks on the rim of the pizza next to the locations of all the marked points on the string. Then pivot the string about the 3 o'clock position until the end that was at 12 o'clock goes to 6 o'clock, and again mark off all the neighboring points on the rim. Repeat the process twice more, first pivoting about 6 o'clock and then about 9 o'clock until the entire rim is marked.

6. The points on the rim should come in N-1 horizontally separated pairs. Slicing along the lines joining these pairs (N-1 slices in all) should give the desired division.

If you have marked out a string for the case N = 12, lumping the neighboring segments together in groups of 4,3 or 2 will allow you to get divisions into N = 3,4 or 6 pieces with no additional work.

For those who are curious, the curve in the top plot is $A(\theta) = 2\theta - \sin(2\theta)$ for θ going from 0 to $\pi/2$. You can, of course, do all the constructions described above using a computer program, with the length of the horizontal axis chosen appropriately.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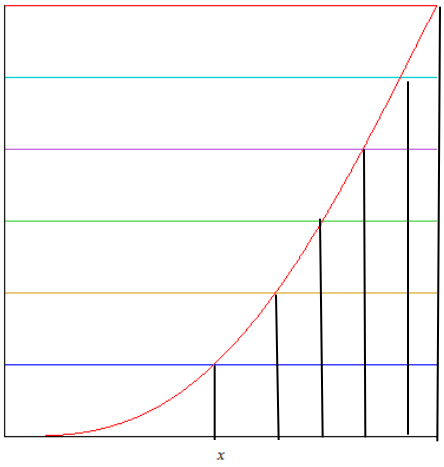
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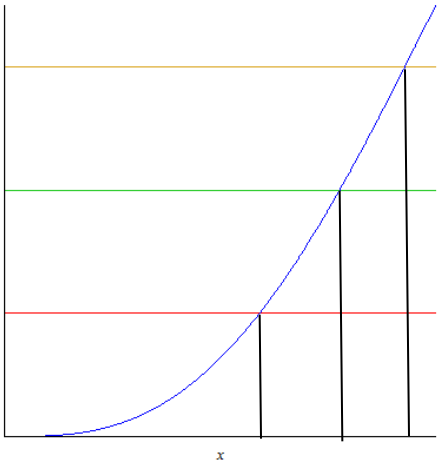
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