Applications and Constructions of W-States

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Classical versus Quantum

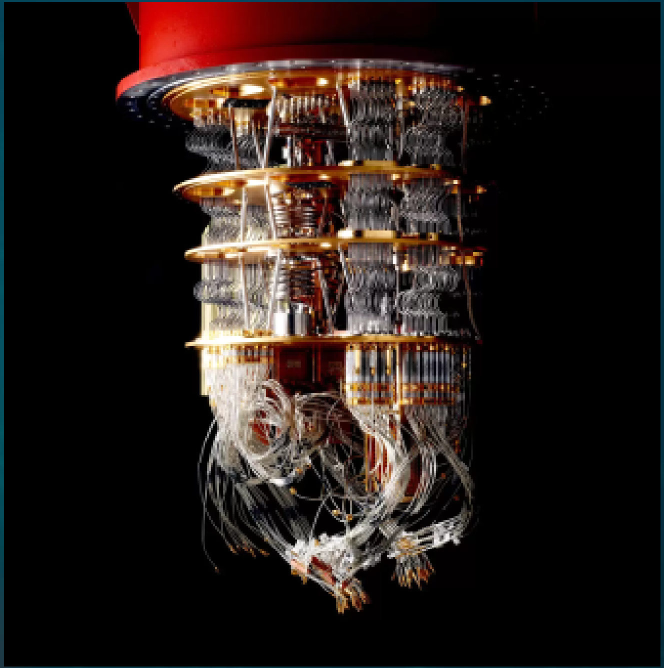
Classical Computing

- uses bits
- each bit is either 0 or 1
- can describe system of bits with a single string
    - e.g. "01011"
- completely deterministic

Quantum Computing

- uses qubits
- each qubit is in a superposition of 0 and 1, with coefficients in $\mathbb{C}$
- system of qubits described by superposition of strings
    - e.g. $\sqrt{3}/2\ |001\rangle + i/2\ |101\rangle$
- can be nondeterministic

Classical < Quantum



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- Parallelization via superpositions can lead to exponential speedup
- Crack RSA encryption, and introduce quantum encryption
- Faster simulation of atomic and molecular dynamics

Anatomy of a System of 3 Qubits

square-magnitude of state's coefficient is probability that system collapses to that state when measured

⅓ chance that system collapses to $|001\rangle$

⅙ chance that system collapses to $|100\rangle$

$$1/\sqrt{3}\,|001\rangle - 1/\sqrt{3}\,|010\rangle - i\sqrt{6}\,|111\rangle + e^{1.2i}/\sqrt{6}\,|100\rangle$$

"normalized:" square-magnitude of coefficients sum to 1

"one-hot" states: exactly one 1

W-States:
Even Superpositions of One-Hot States

W-States

$1/\sqrt{2}\,(|01\rangle + |10\rangle)$ — aka the Bell state ψ^+

$1/\sqrt{3}\,(|010\rangle + |100\rangle + |001\rangle)$

$1/2\,(|1000\rangle + |0100\rangle + |0010\rangle + |0001\rangle)$

$W_n = 1/\sqrt{n}\,(|100...0\rangle + |010...0\rangle + ... + |000...1\rangle)$

Not W-States

$|01\rangle + |1\rangle$ — invalid superposition

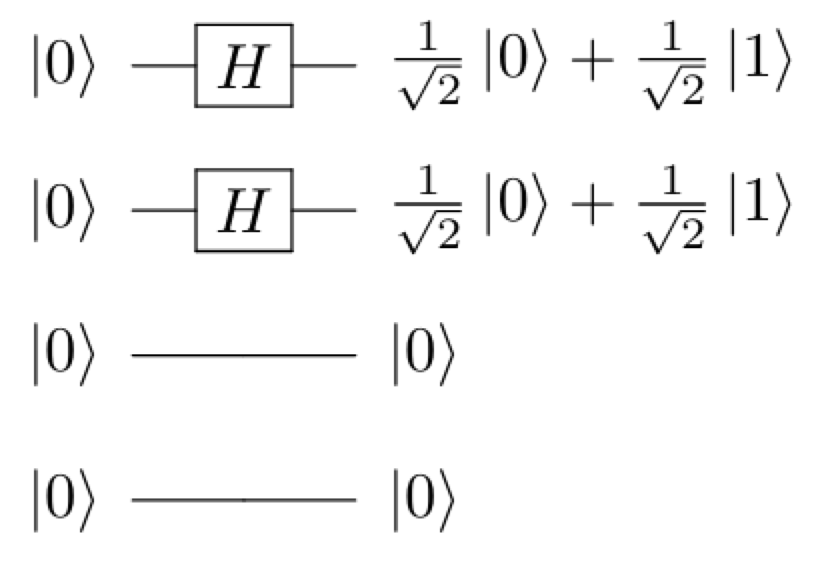
$1/\sqrt{2}\,(|000\rangle + |111\rangle)$ — "GHZ_3" state

$1/\sqrt{2}\,(|010\rangle + |100\rangle)$ — missing $|001\rangle$

$1/\sqrt{3}\,(|001\rangle - |010\rangle + |100\rangle)$ — nonuniform phase

Construction of W_4: Create Superposition

input register: all qubits in state |0⟩



gates applied to qubits

Hadamard gates transform |0⟩ to a superposition of |0⟩ and |1⟩

system state is tensor product of individual qubits

State of System

|0000⟩

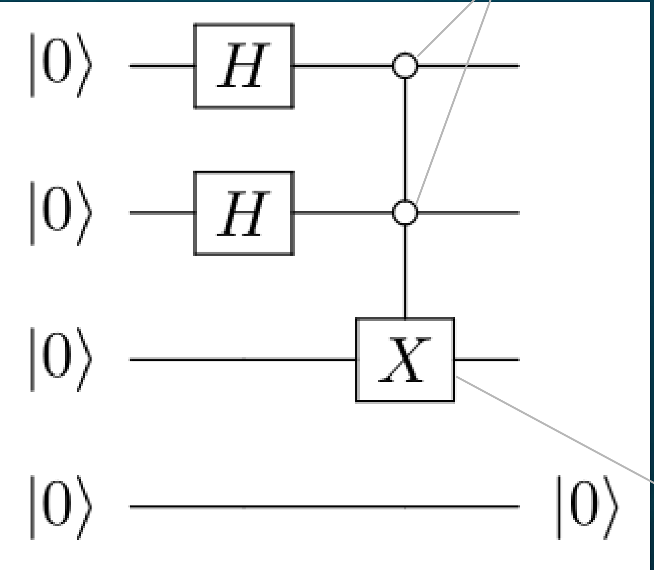
↓

½|0000⟩ + ½|0100⟩ + ½|1000⟩ + ½|1100⟩

automatically got two one-hot states

Construction of W_4: 0000 ⟶ 0010

qubits acting as "control"



these qubits are now entangled; no longer have individual states

qubit being "target": NOT (X) gate only activates if control qubits are |0⟩

State of System

|0000⟩

↓

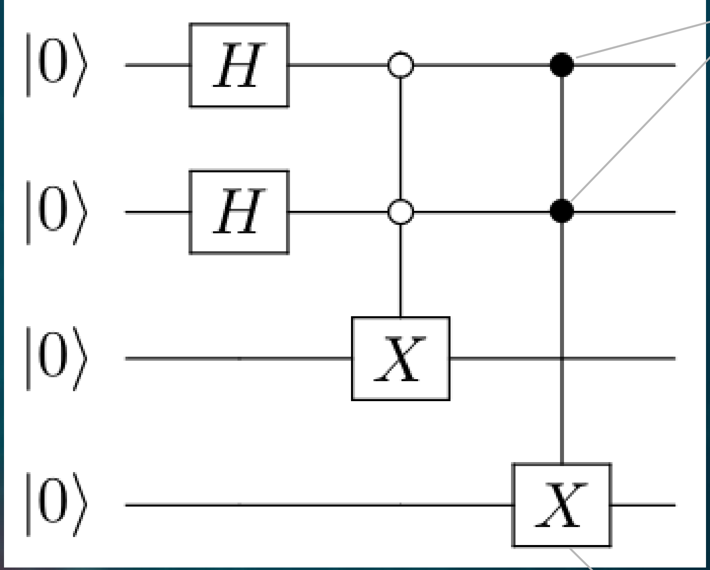
½|0000⟩ + ½|0100⟩ + ½|1000⟩ + ½|1100⟩

↓

½|0010⟩ + ½|0100⟩ + ½|1000⟩ + ½|1100⟩

✓ ✓ ✓ ✗

Construction of W_4: 1100 ⟶ 1101



qubits acting as control again; black means they must be |1⟩ to activate target

NOT gate only activates if control qubits are |1⟩

State of System

|0000⟩

↓

½|0000⟩ + ½|0100⟩ + ½|1000⟩ + ½|1100⟩

↓

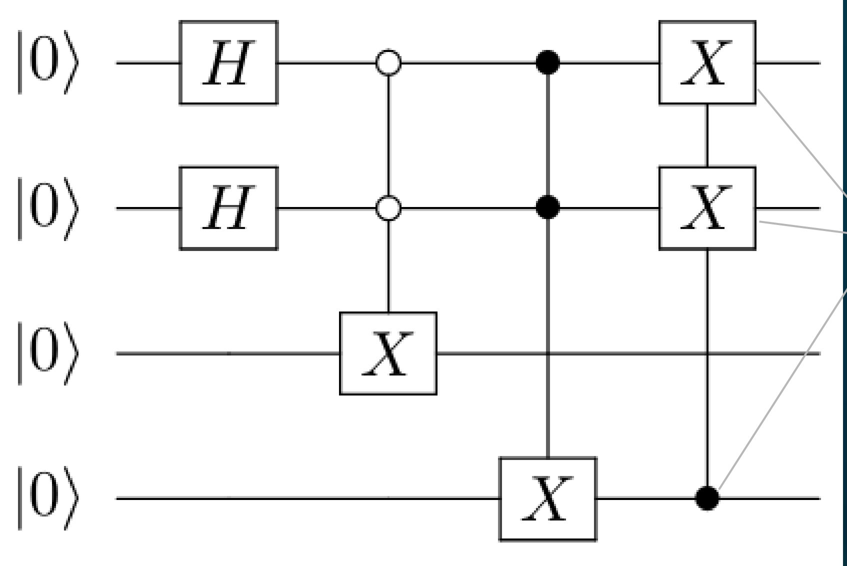
½|0010⟩ + ½|0100⟩ + ½|1000⟩ + ½|1100⟩

↓

½|0010⟩ + ½|0100⟩ + ½|1000⟩ + ½|1101⟩

✓ ✓ ✓ ✗

Construction of W_4: 1101 ⟶ 0001



one control and two targets

State of System

$|0000\rangle$

$\frac{1}{2}|0000\rangle + \frac{1}{2}|0100\rangle + \frac{1}{2}|1000\rangle + \frac{1}{2}|1100\rangle$

$\frac{1}{2}|0010\rangle + \frac{1}{2}|0100\rangle + \frac{1}{2}|1000\rangle + \frac{1}{2}|1100\rangle$

$\frac{1}{2}|0010\rangle + \frac{1}{2}|0100\rangle + \frac{1}{2}|1000\rangle + \frac{1}{2}|1101\rangle$

FINISHED

$\frac{1}{2}|0010\rangle + \frac{1}{2}|0100\rangle + \frac{1}{2}|1000\rangle + \frac{1}{2}|0001\rangle$

More About W-States

01 Difficult to Construct

For $n \neq 2^k$, need nonstandard types of gates

02 Disprove Locality

Can experimentally disprove locality without resorting to inequalities as in Bell's Theorem

03 Information Storage

Hard to disentangle -> robust way to store information

04 Unique Resource

Cannot be transformed to other LOCC classes via LOCC operations

Thank You

Questions?

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