Thevenin Equivalent Circuits: (Material for exam - 3)

The Thevenin equivalent circuit is a two terminal output circuit that contains only one source called E_{TH} and one series resistors called R_{TH}. This circuit is used to replace a much more complex circuit consisting of one or more sources and resistors connected in various combinations around a set of output terminals.

The simple steps to convert this more complex circuit to a Thevenin Equivalent at its output terminals can be expressed as,

1.) In the original circuit, remove any load (resistor, ETC.) From its output terminals so the circuit at those terminals is an open circuit.
2.) Use KCL, KVL and Ohm's Law as needed to calculate the open circuit voltage across the output terminals. This open circuit output voltage is (E_{TH}), the Thevenin voltage source magnitude.
3.) To find the Thevenin resistance (R_{TH}) in the original circuit, follow these steps.
    a. Replace all independent voltage sources with short circuits. In other words, remove the sources(s) and put a wire in its place.
    b. Replace all independent current sources with open circuits. Remove the source(s) and leave the circuit open where the source(s) were located.
    c. Now to calculate R_{TH}, use series and parallel resistor combinations as needed to find the total resistance the circuit would have at its output terminals. This total resistance is R_{TH}.

Let's Look At A Few Sample Circuits:

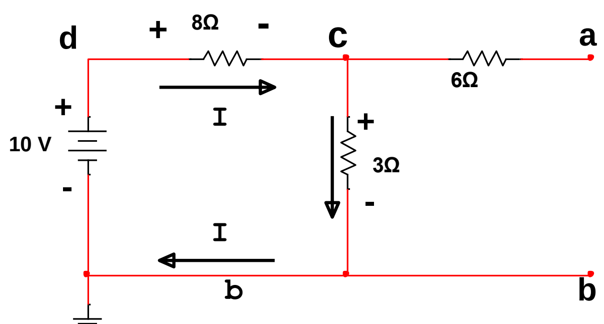


Diagram #1

First calculate V_{AB} which is E_{OC} (E open circuit). Using KVL,

$$V_{BD} + V_{DC} + V_{CB} = 0$$

$$-10 + 8I + 3I = 0 \rightarrow \quad 11I = 10$$

$$I = 0.909A$$

Using KVL at output to find V_{AB}

$$V_{AB} + V_{BC} + V_{CA} = 0$$

$$V_{AB} - 3I + 6(0) = 0 \qquad \textit{Note there is no current through the } 6\Omega \textit{ resistor.}$$

$$V_{AB} = 3I = (3)(0.909) = 2.73\ v$$

$$E_{OC} = V_{AB} = E_{TH} = 2.73v$$

Now to find R_{TH}, we replace the voltage source with a wire,

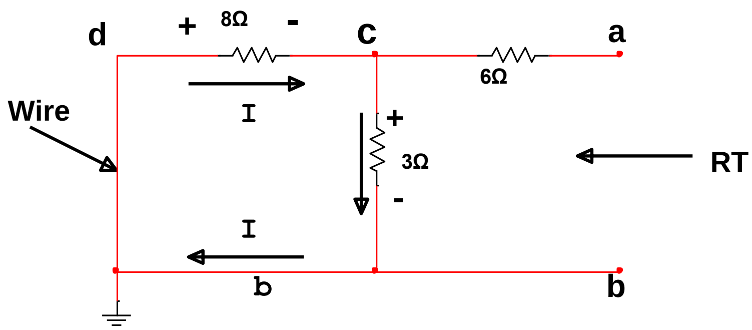


Diagram #2

$$R_{CB} = \frac{(8)(3)}{8+3} = \frac{24}{11} = 2.18\ \Omega$$

$$R_T = 6\ \Omega + R_{CB} = 6 + 2.18 = 8.16\ \Omega$$

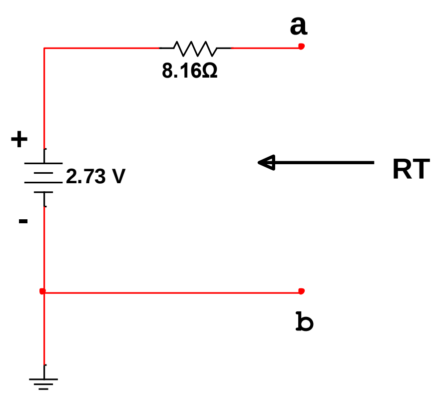


Diagram #3

This is the Thevenin Equivalent circuit of the original circuit.

In conclusion – The Thevenin equivalent circuit is a terminal equivalent to the original circuit at its terminals. Check out the Multisim circuit on the website.

Thevenin Example Circuit: (Used to show different methods)

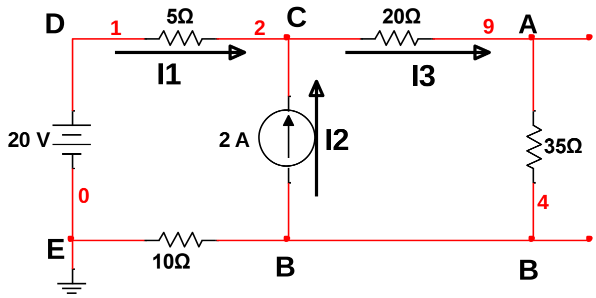


Diagram #4

Let's first solve for the open circuit voltage V_{AB}. Use KCL at node "C".

$$\sum I_{IN} = \sum I_{OUT}$$

$$I_1 + I_2 = I_3 \quad Note,\ I_2 = 2\ amps$$

So, $I_1 + I_2 = I_3$, where we have two unknowns, I_1 and I_3. This means we need another equation in terms of these two unknowns. It turns out KVL will give us this expression.

What are the possibilities for KVL equations?

Are these two equations possibilities?

$$V_{BE} + V_{ED} + V_{DC} + V_{CB} = 0 \ \ \boldsymbol{Equation\ \#1}$$

$$+ - \quad - + \quad + - \quad ?$$

$$10I_1 - 20 + 5I_1 + V_{CB} = 0$$

$$15I_1 - 20 + V_{CB} = 0$$

$$15I_1 + V_{CB} = 20$$

$$Note\ that\ this\ expression\ has\ two\ unknowns\ I_1\ and\ V_{CB}, not\ I_1\ and\ I_3$$

$$V_{BC} + V_{CA} + V_{AB} = 0 \qquad \boldsymbol{Equation\ \#2}$$

$$? \quad + - \quad + -$$

$$V_{BC} + 20I_3 + 35I_3 = 0$$

$$V_{BC} + 55I_3 = 0$$

$$This\ gives\ us\ an\ expression\ of\ I_3\ and\ V_{BC}, not\ I_1\ and\ I_3.$$

However, if we look at equation #1 and #2 we find that both equations contain V_{CB} or V_{BC}. If we recognize V_{CB} = -V_{BC} and substitute it into equation #1,

$$15I_1 - V_{BC} = 20$$

Solving for V_{BC},

$$-V_{BC} = 20 - 15I_1$$

$$V_{BC} = 15I_1 - 20$$

Now taking this and substituting this into equation #2,

$$15I_1 - 20 + 55I_3 = 0$$

$$15I_1 + 55I_3 = 20$$

We now have an expression of I_1 and I_3. If this seems to be a round-a-bout way of getting an expression of I_1 and I_3, you're right.

Note, these two equations both contain V_{CB} or V_{BC}, the voltage across the current which is an unknown. The KVL loops we took both included the current source which has been defined as I_2. Since both loops contained V_{CB} or V_{BC} and either I_1 or I_3, V_{CB} = -V_{BC} could be used to get an expression of I_1 and I_3.

However, a better solution would be to take a KVL loop around the outside components of the circuit. In other words, do not take the loop that includes a current source. The outside loop does contain I_1 and I_3 which would immediately give us our expression in terms of I_1 and I_3.

Hence,

$$V_{BE} + V_{ED} + V_{DC} + V_{CA} + V_{AB} = 0$$

$$+\,-\quad -\,+\quad +\,-\quad +\,-\quad +\,-$$

$$10I_1 - 20 + 5I_1 + 20I_3 + 35I_3 = 0$$

$$15I_1 + 55I_3 = 20$$

$$\textit{This method gets the expression we need immediately.}$$

At this point you are probably wondering why I went through the first method and then the second method which got the results we needed more directly. I did this to show that there are in this case, more than one way to derive what we need. I also wanted to show that if we follow a KVL loop that does not contain current sources but does contain the two currents we need I_1 and I_3, which yields a much simpler solution.

Now finalizing the solution to this example.

KCL $\quad \rightarrow I_1 + 2 = I_3$

KVL $\rightarrow$ $15I_1 + 55I_3$ = 20

$$15I_1 + 55(I_1 + 2) = 20$$

$$15I_1 + 55I_1 + 110 = 20$$

$$70I_1 = -90$$

$$I_1 = -1.2857\ amps$$

$$I_3 = I_1 + 2 = -1.2857 + 2 = 0.71428\ A$$

So,

$$V_{AB} = E_{oc} = E_{TH} = 35I_3 = 35(0.71428) = 25.0\ volts$$

Now calculating R_{TH} in the original circuit, replacing voltage sources with "wires" (short circuits) and current sources with open circuits.

This will yield,

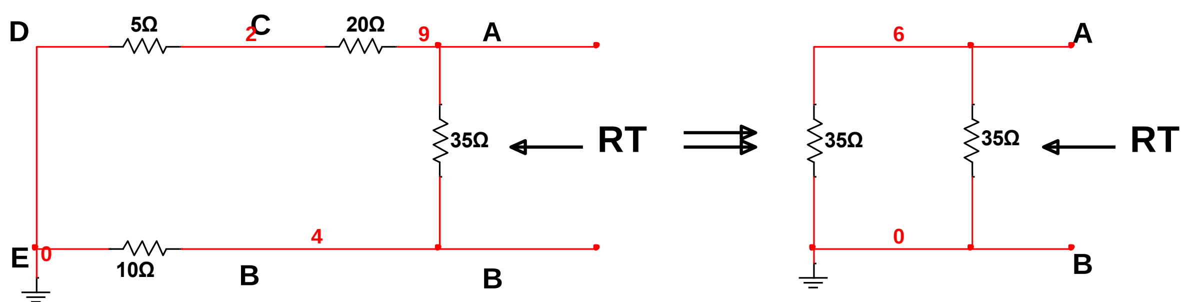


Diagram #5

$$R_T = \frac{(35)(35)}{35 + 35} = 17.5\ \Omega$$

The Thevenin equivalent of the original circuit can be drawn as,

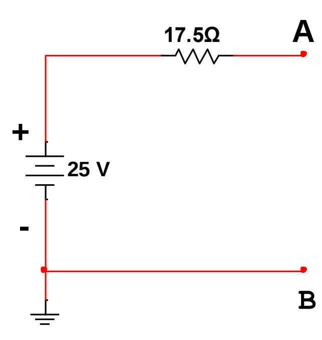
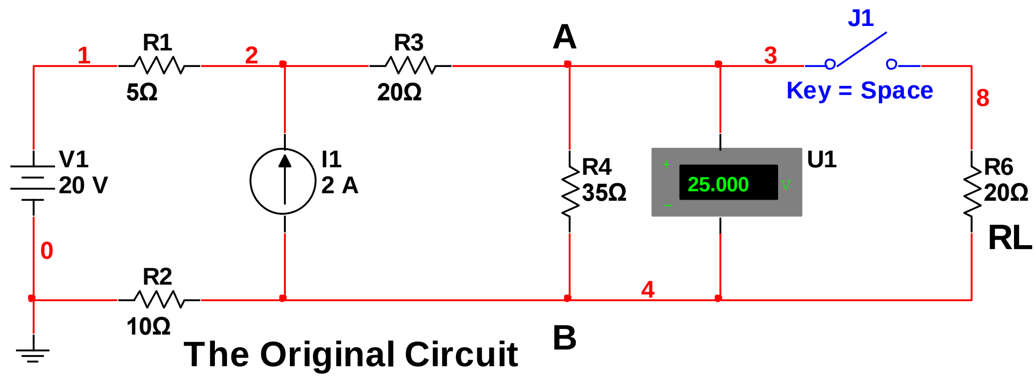
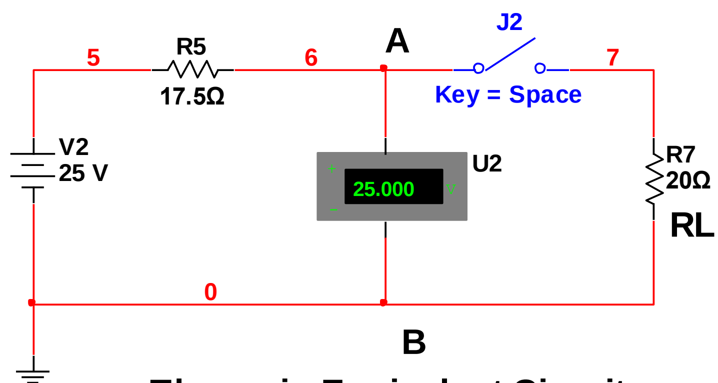


Diagram #6

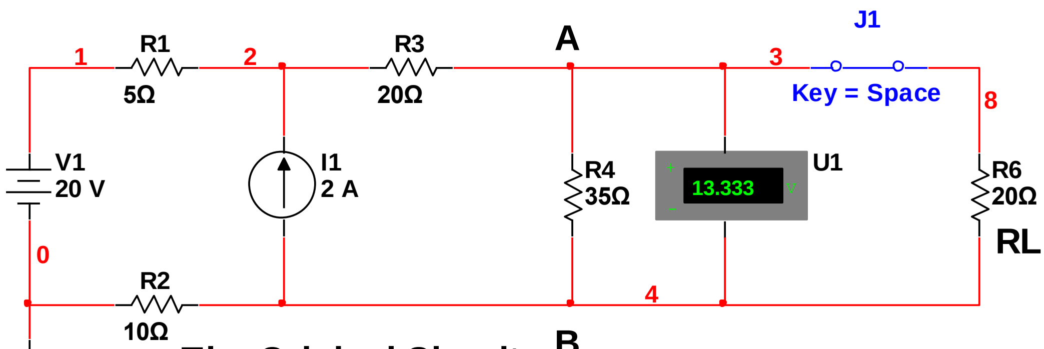
The next two pages are multisim results showing open circuit, 20 ohm load and when the output was shorted.



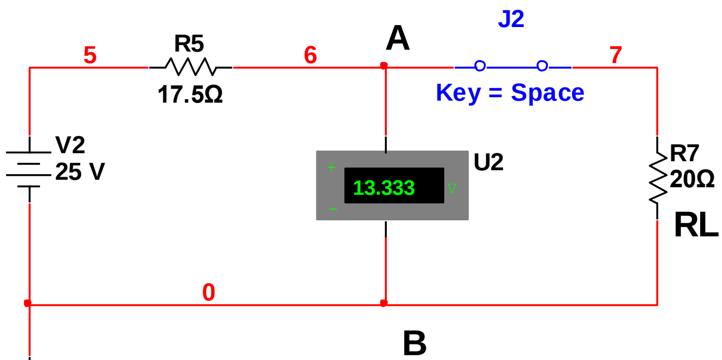
The Original Circuit



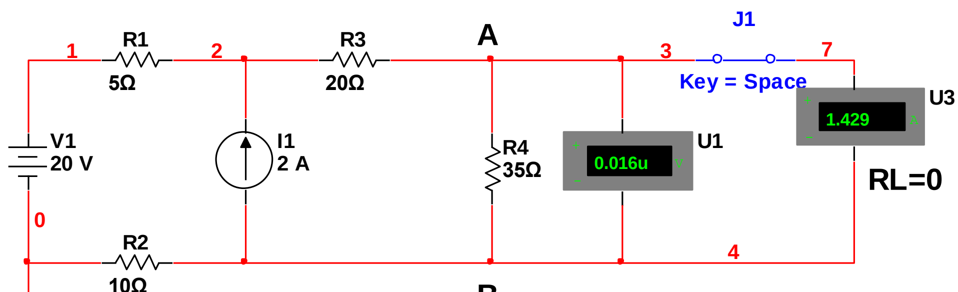
Thevenin Equivalent Circuit



The Original Circuit

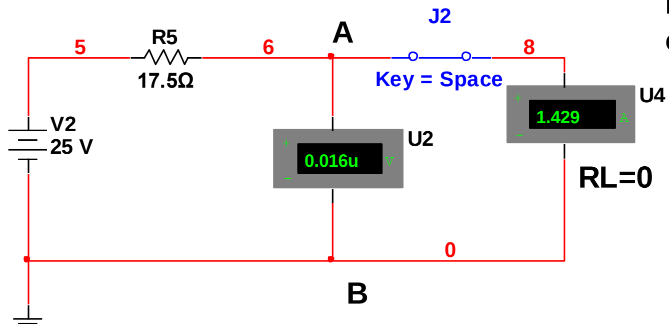


Thevenin Equivalent Circuit



The Original Circuit

RL=0 Yields each circuits output short circuit current.



Thevenin Equivalent Circuit