

Question:

What is the relationship between the incline and acceleration of a cart traveling along a metal track in a modified Atwood's machine?

Hypothesis:

The relationship between surface incline angle and acceleration will be linear. The slope of the graph of incline vs. acceleration will be equal to $-\frac{m_1 g (\sin \theta)}{m_1 + m_2}$.

Strategy:

- The hanging mass (m_2) in the modified Atwood's machine was the various number of washers from a paper clip tied to the string. The ramp was propped up at angles of 9.646, 6.986, 4.651, 2.323, and 0 based on the number of textbooks
- The angle of the incline was constantly changed by stacking different numbers of textbooks under one end of the track. The acceleration was measured using the Vernier motion detector
- The mass of m_2 , 0.15 kg, and the mass of the cart, m_1 , is 0.301 kg, was kept constant in order to see the progression of the acceleration across an increased angle
- The approximate angle measures were found by taking the proportion of the incline's base length to its hypotenuse. We take the answer and find take its inverse sign to find the angle measures

Data:

The hanging mass (m_2) and the mass of the cart (m_1) are 0.15 and 0.301 kg, respectively.

| Angle Measure (Degrees) | Sin(theta) | Average Acceleration(m/s ²) |
|-------------------------|------------|---|
| 0 | 0 | 3.0993 |
| 2.232 | 0.041 | 2.8273 |
| 4.651 | 0.081 | 2.5807 |
| 0.986 | 0.122 | 2.3253 |
| 9.646 | 0.168 | 2.0567 |

Analysis:

The free body diagrams in Figure 2 show the forces on the masses in the modified Atwood's machine.

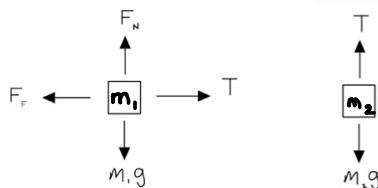


Figure 2: Free Body Diagrams

Friction is not very significant because the cart's wheels spin freely. The equations provided below are based on the free body diagrams, and the positive motion for m_1 is defined to the right while positive motion m_2 is defined down for the hanging mass.

$$T - m_1 g (\sin \theta) = m_1 a$$

$$m_2 g - T = m_2 a$$

The sum of these equations gives a new equation and then we isolate for the variable, a:

$$m_2 g - m_1 g (\sin \theta) = (m_1 + m_2) a$$

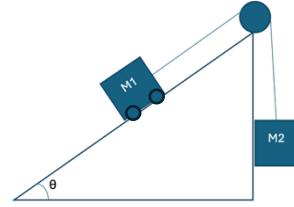


Figure 1: Modified Atwood's machine

$$a = -\frac{m_1 g (\sin \theta)}{m_1 + m_2} + \frac{m_2 g}{m_1 + m_2}$$

Taking this acceleration equation, we were able to derive an equation for the slope because our acceleration equation is equivalent to $y = mx + b$. A linear relationship can be shown by our independent variable and our constant, $-\frac{m_1 g (\sin \theta)}{m_1 + m_2}$ and $\frac{m_2 g}{m_1 + m_2}$, respectively.

A graph of the sin theta of the incline vs. the average acceleration data for this experiment shows that it is linear, and that the slope is equal to -6.5386 m/s^2 .

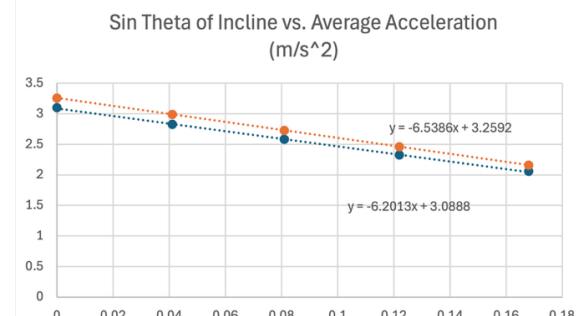


Figure 3: Sin Theta of Incline vs. Average Acceleration

The actual slope of the average acceleration is -6.2013 m/s^2 , which means that the average acceleration found from the formula data is 5.44% larger than expected. This indicates that when we were recording data, something caused the acceleration to slow down. Likely sources for this discrepancy could be the friction from the wheels of the cart because any friction would reduce the acceleration. Another source could have been the air resistance as the cart traveled up the ramp because any form of drag on the cart would reduce the acceleration as well.