Rocket Problem

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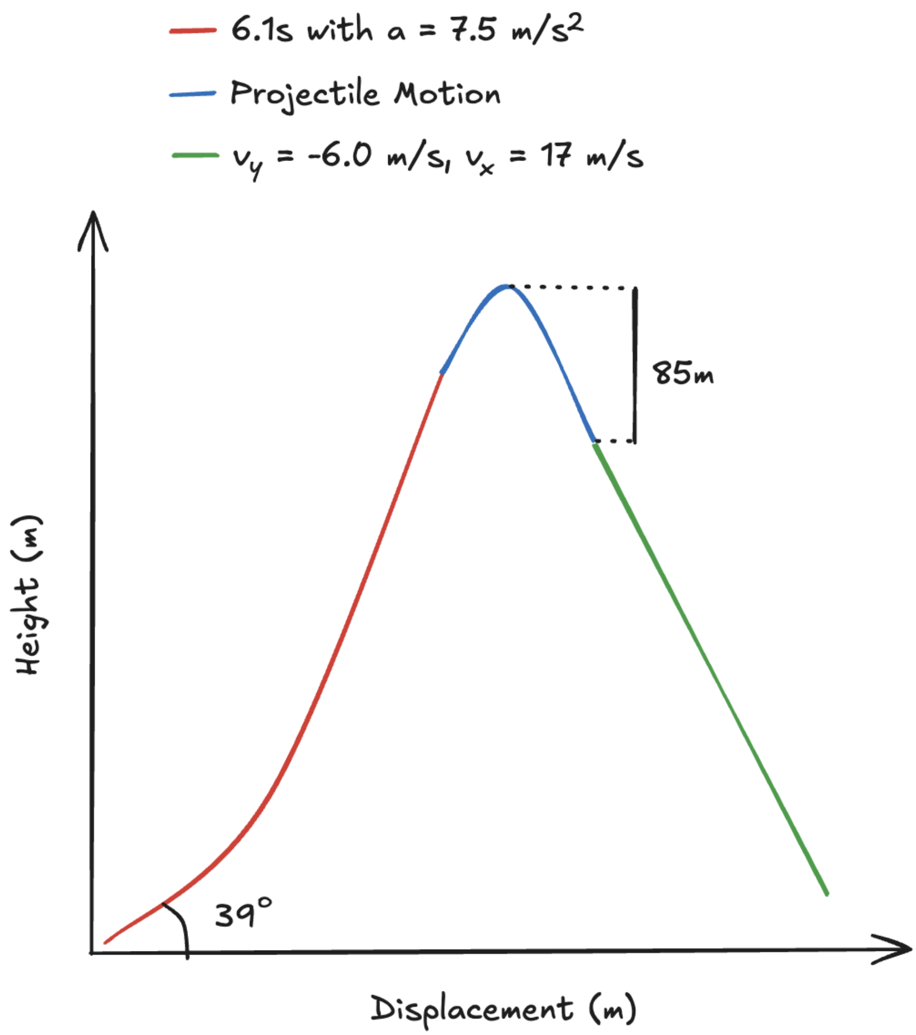

Red Section (Engine Power):

$\theta = 39°$ where θ is launch angle

$a_{engine} = 7.5\frac{m}{s^2}$ where a_{engine} is acceleration when the engine is powered

$a_{engine,x}$ is the x component of a_{engine}

$a_{engine,x} = a_{engine} \cdot cos(\theta)$

$\Rightarrow a_{engine,x} = 7.5\frac{m}{s^2} \cdot cos(39°) = 5.82859471\frac{m}{s^2}$

$t_{engine} = 6.1s$ where t_{engine} is time spent with the engine on

$v_{0,\, engine} = 0\frac{m}{s}$ where $v_{0,\, engine}$ is the initial velocity of the rocket

Δx_{engine} is the unknown, the x-displacement over the time when the engine was on

$\Delta x_{engine} = v_{0,\, engine} \cdot t_{engine} + \frac{1}{2}a_{engine,\, x}(t_{engine})^2$

$\Rightarrow \Delta x_{engine} = (0\frac{m}{s}) \cdot (6.1s) + \frac{1}{2}(5.82859471\frac{m}{s^2})(6.1s)^2 = 108.44100460m$

Miscellaneous Calculations:

$v_{f,\, engine,\, x}$ is the final x velocity after the engine section

$v_{f,\, engine,\, x} = v_{0,\, engine} + t_{engine} \cdot a_{engine,\, x}$

$\Rightarrow v_{f,\, engine,\, x} = 0\frac{m}{s} + (6.1s)(5.82859471\frac{m}{s^2}) = 35.55442774\frac{m}{s}$

$a_{engine,\, y}$ is the y component of a_{engine}

$a_{engine,\, y} = a_{engine} \cdot sin(\theta)$

$\Rightarrow a_{engine,\, y} = 7.5\frac{m}{s^2} \cdot sin(39°) = 4.71990293\frac{m}{s^2}$

Δy_{engine} is the y displacement in the engine section

$\Delta y_{engine} = v_{0,\, engine} \cdot t_{engine} + \frac{1}{2}a_{engine,\, y}(t_{engine})^2$

$\Rightarrow \Delta y_{engine} = (0\frac{m}{s}) \cdot (6.1s) + \frac{1}{2}(4.71990293\frac{m}{s^2})(6.1s)^2 = 87.81379407m$

$v_{f,\, engine,\, y}$ is the final y velocity after the engine section

$v_{f,\, engine,\, y} = v_{0,\, engine} + t_{engine} \cdot a_{engine,\, y}$

$\Rightarrow v_{f,\, engine,\, y} = 0\frac{m}{s} + (6.1s)(4.71990293\frac{m}{s^2}) = 28.79140789\frac{m}{s}$

Blue Section (Projectile Motion):

$\Delta y_{projectile,\, max}$ is the maximum height displacement reached by the rocket in the projectile stage (displacement after the engine stage)

$g = -9.8\frac{m}{s^2}$ is acceleration due to gravity

$v_{y,max} = 0\frac{m}{s}$ is the velocity at the maximum height, which is $0\frac{m}{s}$.

$(v_{y,\, max})^2 = (v_{f,\, engine,\, x})^2 + 2g\Delta y_{max}$

$\Rightarrow (0\frac{m}{s})^2 = (28.79140789\frac{m}{s})^2 + 2(-9.8\frac{m}{s^2})\Delta y_{max}$

$\Rightarrow \Delta y_{projectile,\, max} = \frac{-(28.79140789\frac{m}{s})^2}{2(-9.8\frac{m}{s^2})} = 42.29312083m$

$v_{f,\, projectile}$ is the final velocity of the rocket after the projectile motion period

$\Delta y_{down} = -85m$ where Δy_{down} is the downward displacement after the rocket reaches its peak height

$(v_{f, projectile})^2 = (v_{y, max})^2 + 2g\Delta y_{down}$

$\Rightarrow (v_{f, projectile})^2 = (0\frac{m}{s})^2 + 2(-\ 9.8\frac{m}{s^2})(-\ 85m)$

$\Rightarrow v_{f, projectile} = \ -\ 40.81666326\frac{m}{s}$ [Negative result because the rocket is falling downward after the 85m descent]

$t_{projectile}$ is the total time spent in the projectile section

$v_{f, projectile} = v_{f, engine, x} + gt_{projectile}$

$\Rightarrow -\ 40.81666326\frac{m}{s} = 28.79140789\frac{m}{s} + t_{projectile} \cdot (-\ 9.8\frac{m}{s^2})$

$\Rightarrow t_{projectile} = \frac{-40.81666326\frac{m}{s} - 28.79140789\frac{m}{s}}{-9.8\frac{m}{s^2}} = 7.10286440s$

$\Delta x_{projectile}$ is the <u>unknown</u>, the x displacement during the projectile section

$\Delta x_{projectile} = t_{projectile} \cdot v_{f, engine, x}$

$\Rightarrow \Delta x_{projectile} = 7.10286440s \cdot 35.55442774\frac{m}{s} = 252.53827917m$

Miscellaneous Calculations:

$\Delta y_{engine, projectile}$ is the total y displacement between both the engine section and projectile motion section

$\Delta y_{engine, projectile} = \Delta y_{engine} + \Delta y_{projectile, max} + \Delta y_{down}$

$\Rightarrow \Delta y_{engine, projectile} = 87.81379407m + 42.29312083m - 85m = 45.10691490m$

Green Section (Parachute):

$v_{parachute, x}$ is the horizontal velocity

$v_{parachute, x} = v_{f, engine, x} - 17\frac{m}{s}$ [$17\frac{m}{s}$ wind in the westward direction, whereas rocket is traveling eastward]

$\Rightarrow v_{parachute, x} = 35.55442774\frac{m}{s} - 17\frac{m}{s} = 18.55442774\frac{m}{s}$

$\Delta y_{parachute} = \ -\ 45.10691490m$ is the downward distance traveled by the rocket in the parachute section, which is the negative of the total displacement so far because the rocket lands back on the ground to a total Δy of $0m$.

$t_{parachute}$ is the time spent in the parachute section

$v_{parachute, y} = 6\frac{m}{s}$ where $v_{parachute, y}$ is the constant downward velocity in the parachute section

$\Delta y_{parachute} = t_{parachute} \cdot v_{parachute, y}$

$\Rightarrow -\ 45.10691490m = t_{parachute} \cdot (-\ 6.0\frac{m}{s})$

$\Rightarrow t_{parachute} = \frac{-45.10691490m}{-6.0\frac{m}{s}} = 7.51781915s$

$\Delta x_{parachute}$ is the <u>unknown</u>, the x displacement over the parachute section

$\Delta x_{parachute} = t_{parachute} \cdot v_{parachute, x}$

$\Rightarrow \Delta x_{parachute} = 7.51781915s \cdot 18.55442774\frac{m}{s} = 139.48883218$

Total Displacement:

Δx is total displacement

$\Delta x = \Delta x_{engine} + \Delta x_{projectile} + \Delta x_{parachute} = 108.44100460m + 252.53827917m + 139.48883218m$
$= 500.5m$

The rocket traveled 500.5 meters in the Eastward direction.