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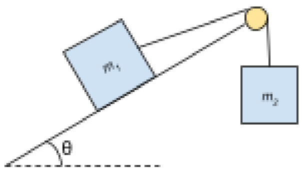
Question

How do differences in the incline between two masses affect the acceleration of the cart in a modified Atwood's machine?

Hypothesis

When m_1 is on an inclined plane, then the acceleration would be greater towards the mass that is not inclined than if both masses were hung straight down. The acceleration would be proportional to the sine of the angle of incline, with higher angles causing a slower descent down the inclined plane.

Strategy



We will have the cart on one side and a counterweight hanging down, and measure their masses, which are m_1 and m_2 respectively. Their mass shouldn't affect our experiment, but we will still take these measurements for redundancy. We will start them with an incline θ = 30° as a control. We will measure the acceleration in this scenario over three trials. Then, we will change the incline θ to 45°, measuring the acceleration at this angle (over three trials once again). Finally, we will run three trials with θ = 15° to get another data point for acceleration. We can compare the average acceleration at each incline to see if the incline had an effect on the acceleration, and to measure their relationship.

Variables

The incline angle θ is the value that is being varied in our experiment. The value being measured is acceleration.

Data

Acceleration for Three Trials at Each Incline Angle

Incline Angle (degrees)	Trial 1 Acceleration (m/s^2)	Trial 2 Acceleration (m/s^2)	Trial 3 Acceleration (m/s^2)
0	3.951	3.578	3.983
15	2.579	2.637	2.595
30	1.135	1.185	1.177
45	0.06859	0.07254	0.0759

We recorded three trials at different incline angles (θ). On top of our proposed measurements of a 30°, 45°, and 15° measurement, we also took a 0° measurement of acceleration to have additional data. On the three columns right of the incline angle are the measured accelerations (in m/s^2) for each of three trials. For these trials, we measured a cart weight (m_1) of 286.8g and a counterweight (m_2) of 210g.

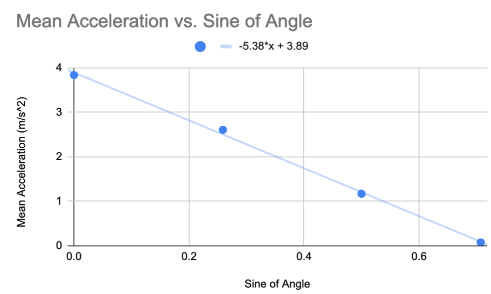
Analysis & Conclusion

We started by taking the mean of our three acceleration trials, which yielded the following values:

Mean Acceleration at Each Incline Angle

Incline Angle (degrees)	Mean Acceleration (m/s^2)
0	3.837333333
15	2.603666667
30	1.165666667
45	0.072343333

We then made a chart of the sine of each incline angle and compared it to the mean acceleration, giving us this plot:



The acceleration demonstrates a linear relationship with the sine of the angle, which we measured to follow the line y = -5.38x + 3.89 (the precise slope is about -5.3846 and the intercept is 3.8931). Here, the slope tells us the relation between the acceleration and θ, while the intercept tells us the acceleration if m_1 was on a flat surface. This proves the hypothesis that the acceleration and the incline have a sinusoidal relationship, and it demonstrates our claim that the acceleration up the slope is greater at higher angles (the positive direction in the graph shows acceleration down the slope). We can see how accurate our measurements were also, by calculating *g* (the acceleration of gravity) from our data. We know that $F = ma$ so $a = \frac{F}{m}$. Substituting our tension force for the string, we get $a = \frac{m_2 g - m_1 g sin\theta}{m_1 + m_2}$. This can be expanded like so: $a = \frac{m_2 g}{m_1 + m_2} - \frac{m_1 g}{m_1 + m_2} sin\theta$. This means that our coefficient slope of -5.3846 is $-\frac{m_1 g}{m_1 + m_2}$, meaning $g = \frac{5.3846(m_1 + m_2)}{m_1}$. Since we measured m_1 and m_2, we can solve for *g* by substituting our values: $g = \frac{5.3846(0.21 + 0.2868)}{0.2868} = 9.33 \frac{m}{s^2}$. Knowing that the actual value of *g* is 9.8 m/s^2, we only had a 4.8% error in our experimental value for *g*. Using our y-intercept, we use a similar process to find $g = \frac{3.8931(m_1 + m_2)}{m_2} = \frac{3.8931(0.21+0.2868)}{0.21} = 9.21 \frac{m}{s^2}$, which has a 6.0% error from the true value of 9.8 m/s^2. These low errors reaffirm the accuracy of our measurements. However, the presence of the slight error indicates that improvements could still be made. Possible sources may include reduced acceleration due to friction between the cart and the track and slightly imprecise measurements of acceleration (the cart was moving quite fast, so there may not have been enough time for highly precise measures of acceleration).