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Math Modeling

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Birthday POW

1. Problem Statement: The seven-day week is used everywhere, and the day you are born on is believed to shape your individual personality. Using just a list of famous birthdays, their corresponding days of the week, and a calendar for the current month, can you develop a way to find the day of a week a person was born on from the date given? The person finding their birthday only has access to a four-function calculator and does not have an extensive math background.

2. Process: We immediately realized repeating pattern in weeks and months, which told us that we would likely be using skills learned in our modular arithmetic unit. We started by assigning (mod 7) numbers to each day of the week, setting Monday to 0 and incrementing until Sunday was set to 6. We then found the first day of each month as a number, setting January 1st to 0. We then took the remainder of the number corresponding to the first of each month when dividing by 7, allowing us to set it equal to a day of the week, assuming January 1st was a Monday (since we set this date to a 0, which corresponds with Monday). Next, we derived a formula for the first day of the year. Below is our initial attempt at a formula:

$$f(x) = \left(x + \left\lfloor \frac{x}{4} \right\rfloor\right) mod\ 7$$

It took the year as an input and returned the number that corresponded to the day of the week that January 1st fell on for that year, assuming 1900 was 0. To develop this formula, we first noticed that every 365 days, our day of the week was incremented by one. That is, if one year starts on Monday (which we labelled as 0), the next year started on a Tuesday (which we labelled as a 1). We also needed to account for leap years. We used the floor function (this function simply rounds decimal numbers down) applied to x/4 for this purpose. This function increments by one every four years, which accounts for the extra increment to the day of the week on leap years. Finally, we summed up the amount of leap day increments and yearly increments, taking the modulo by seven to get a day-number. We knew, given one of the celebrities' birthdays, that in 1971 January 1st fell on a Friday.

Thus, we plugged 1971 into our formula and when we got Wednesday, we thought we had to add 2 to both x's to offset the equation from our assumption of January 1^{st} ,1900 being Monday. However, some assumptions that we made for our original formula were false. We did need to subtract two from the year to account for the offset of January 1^{st}, 1900. However, we needed to subtract *one* from our floored expression since a leap year's affect only affects January 1^{st} of the next year. As an example – 2008 was a leap year, but January 1^{st} of 2008 was before the leap day. The effects of the leap day only should be accounted for by January 1^{st}, 2009. This gave us a new formula:

$$f(x) = \left((x-2) + \left\lfloor \frac{x-1}{4} \right\rfloor\right) \bmod 7$$

Finally, we broke this equation up into a readable way, as denoted in the solution. Finally, using our month table, made earlier when we found the offset of the first of each month, we told the user to add their corresponding month value, the day of the month they were born on, and the year offset they got using the above formula. After doing this, we knew that taking the mod 7 of this number would get them the day of the week they were born, according to the day table.

3. Solution:

1. First take the year you were born and subtract one.
2. Divide that number by 4 and round down to the nearest whole number.
3. Add that number to the year you were born, then subtract two.
4. Take the remainder when you divide your number by 7.
    a. If you are unsure about how to take the remainder, take the decimal part of the number when you divided by 7, and then multiply this decimal by 7.
5. Find the number corresponding to your birthday month on the month table and add this to your previous result.
    a. If your birthday is on a leap year and you were born after February, add 1 to your number. A leap year is a year that is a multiple of 4.
6. Add the day of the month you were born to your number.
7. Take the remainder when you divide your number by 7.
8. Check this remainder with the day table – this should give you the day of the week of your birthday.

Month table:

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sept	Oct	Nov	Dec
6	2	2	5	0	3	5	1	4	6	2	4

Day Table:

Mon	Tues	Wed	Thurs	Fri	Sat	Sun
0	1	2	3	4	5	6

We know our solution is correct because we tested the process on all our birthdays, as well as our XYZ group's birthdays and a couple of people from the list given. All the birthdays returned were accurate.

4. Extensions:

One extension that we considered is to revise our method to provide days of the week for dates before the 20th century and after the 21st. In order to do this, we need to account for the leap year rules that state that there is no leap year every 100 years, but there is a leap year every 400. To account for this, we need to make some small changes.

First, we must add additional steps between step two and step three. First, we add this step:

> Take the year you were born, subtract one, and divide this by 100. Round this number down and **subtract** it from your previous result.

This accounts for the rule that there is no leap year every 100 years. Next, we must account for the fact that there *is* a leap year every 400 years by adding another step:

> Take the year you were born, subtract one, and divide this by 400. Round this number down and **add** it from your previous result.

Testing these new rules, we find that now, we are one day behind where we should be. We can fix this with a modification to step three: instead of subtracting two, we subtract one. These steps essentially change our January 1st formula to this:

$$f(x) = \left((x-1) + \left\lfloor\frac{x-1}{4}\right\rfloor - \left\lfloor\frac{x-1}{100}\right\rfloor + \left\lfloor\frac{x-1}{400}\right\rfloor\right) \text{ mod } 7$$

Now, our process works for *any* date, from the distant past to the distant future.