# Problem of the Week #2: Happy Birthday!

October 2025

## 1 Problem Statement

Most people know their date of birth, but not everyone knows the day of the week they were born on. What if we could find this day mathematically using modular arithmetic and the general format of the Gregorian Calendar? In this problem, our goal was to find a process to help an average person find the day they were born, using their date of birth. We had some constraints concerning this problem, which included: we are calculating only for years after 1900, we can only see the calendar of the current month (October 2025). In addition, we were given a list of given dates to test our process. Using information like the number of days in each month, and the frequency of leap years (every 4 years), we set out to find a concise and effective method that calculates the day of the week using a given date.

## 2 Process

We started several different strategies before we ended up finalizing our process. At first, we considered finding the closest day to someone's birthday. The idea was to first ignore the year of the birthday and just focus on the day and month. Once the closest day and month was determined, you find the day of the week of that month and day, only this time, in the intended birth year. This worked because of the knowledge that moving one year forward or one year backward would move forward 1 day in the day of the week cycle or move backwards 1 day in this cycle. (e.g. going from Tuesday to Wednesday if moving one year forward and Tuesday to Monday if moving one year backwards). This number would change from 1 to 2 for each leap year. Using this method, it would be easy to find the day of the week for the given day and month in the intended birth year. After that, the difference in the number of days between the given day and month and the intended day and month would be calculated and would be added to the day of the week of the given day and month to find the final day of the week. This formula was successful, however it required memorizing specific birthdays or having a sheet of birthdays on hand at all times, both of which were impractical.

Next, we considered the approach of choosing a singular birthday we use as a frame of reference. Instead of using the several birthdays mentioned above, we chose the day and month of January 1st for simplicity, as every day in the year comes after this and is thus only a matter of addition. We chose the year as the next upcoming leap year (2028), and then we found out the day of the week that January 1st, 2028 was. From there, we tried to see the patterns when adding and subtracting years and the leap years, and we were able to figure out that whenever we move up a year, the respective day goes up by one, but when it was a leap year, it would go up by two. From there, we tried to generalize the number of leap years by doing

$$floor(\frac{2028 - y}{4})$$

Here, y represents the year that someone was born. We used $2028-y$ because it tells us the amount of years between the year that the person was born and our set value. Then, we divide this by four since a leap year happens every four years. By doing this, we can see how many leap years have happened in that time span, plus the progress to the next leap year. Now, by taking floor, it removes that fractional component at the end of the number, leaving only the amount of leap years in that time span. This was crucial because by finding how many leap years we have, we could figure out how many days to subtract/add when going between years.

Now, the next step was to take into account the month. For this, we noticed that January 1, 2028 was a Sunday, which we called our 0. From here, we wanted to see what day the first of any month would fall on based on our date for January 1st. To do this, we added the amount of days and then found the remainder when it was divided by 7 (mod 7). For example, January has 31 days. $31/7 = 4R3$, so 31 mod 7 = 3. Therefore, after January, the day of the first of the month is shifted ahead by three. For example, if the first of January was a Friday, the first of February would be a Monday. We used mod 7 because if we assign each day of the week to a number, it helps us use the day as a number in a mathematical formula.

By doing this process for every single month, we were able to get the respective number we needed to add based on the day of the week on the first of the month, as shown in Table 1.

| **Month** | $m_c$ **if** $y$ **does not divide 4** | $m_c$ **if** $y$ **does divide 4** |
|---|---|---|
| January | 0 | 0 |
| February | 3 | 3 |
| March | 3 | 4 |
| April | 6 | 0 |
| May | 1 | 2 |
| June | 4 | 5 |
| July | 6 | 0 |
| August | 2 | 3 |
| September | 5 | 6 |
| October | 0 | 1 |
| November | 3 | 4 |
| December | 5 | 6 |

Table 1: Corresponding numbers for each month

From here, we had to figure out one last step, incorporating the date. The way we did this is by adding the date. However, when this was done, it was always off by one day. As a result, we subtracted one from the date. The reason why the date is off by one is that each month starts on the 1st. In reality we did not care about the actual date, just the numbers of days that have elapsed within the month. Therefore, the difference in days must be found. This difference is the date - 1, as mentioned above.

However, we figured out that when calculating for dates after 2028, we would receive negative values from the above equation. So, we decided to use 1901 as a reference point and created a new equation:

$$y_l = floor(\frac{y - 1901}{4})$$

This would account for all the dates that should be considered in the problem (years after 1900).

## 3 Solution

Note these variables, which were used in our solution:

$y$ is the year of your date

$y_l$ is the number of leap years between 1901 and your year, found in Step 1

$y_n$ is the number of normal (non-leap) years between 1901 and your year, calculated in Step 2

$y_o$ is the total offset based on the current year, also known as the day of the First of January

$m_c$ is the corresponding value of your month, based on how much the day of the first of each month changes (if January 1 is a Monday, February 1 is a Thursday, etc...)

$d$ is the date

### 3.1 Steps

Feel free to follow along by writing your answers in the blanks below. To calculate your birthday, first write down your date of birth (MM/DD/YYYY).

_______/_______/_______

Next, split this date into variables to make future math easier:

$y$ = year from above = __________

$m$ = month from above = __________

$d$ = day from above = __________

Next, we are going to calculate the number of leap years between 1901 and your birth year, this will be important soon. First, subtract 1901 from your birth year:

$$y - 1901 = \_\_\_\_\_\_\_\_\_\_$$

Then, divide this resulting number by 4:

$$\_\_\_\_\_\_\_\_\_\_ \div 4 = \_\_\_\_\_\_\_\_\_\_$$

If you receive a decimal, don't worry, using something called a floor function in math, we can chop off the decimal part of the number, giving you a clean whole number of leap years between your birth year and 1901. Write your whole number result below:

$$y_l = \_\_\_\_\_\_\_\_\_\_$$

Next, we need to find the number of years between 1901 and your birth year that are not leap years. This will also be important in finding your birth day of the week to do this, subtract 1901 and the number of leap years we found above from your original birth year using the equation below.

$$y_n = y - 1901 - y_l = \_\_\_\_\_\_\_\_\_\_$$

Through simple mathematical reasoning, we found that each year, the first day of the year moves forward one day, and it moves forward two days for leap years (due to an extra day in the year). Therefore, due to this reasoning, in order to find the number of days moved forward since January 1st, 1901, we need to add 2 days for every leap year and 1 day for every normal year between 1901 and your birthday. So, the number of days moved forward is:

$$2 * y_l + y_n = \_\_\_\_\_\_\_\_\_\_$$

We also found through this reasoning that January 1st 1901 was a Tuesday, so to get an accurate first day of your birth year, we need to account for the two-day shift from Sunday to Tuesday, adding two to the value we found above. By doing this we receive our offset for the year:

$$y_o = \_\_\_\_\_\_\_\_\_\_ + 2 = \_\_\_\_\_\_\_\_\_\_$$

Now use Table 2 to find the corresponding number for your birthday month, which we call $m_c$. Use the second column if your birth year does not divide 4 (your year is not a leap year), and use the third column if y divides 4 (your year is a leap year).

| **Month** | $m_c$ **if** $y$ **does not divide 4** | $m_c$ **if** $y$ **does divide 4** |
|---|---|---|
| January | 0 | 0 |
| February | 3 | 3 |
| March | 3 | 4 |
| April | 6 | 0 |
| May | 1 | 2 |
| June | 4 | 5 |
| July | 6 | 0 |
| August | 2 | 3 |
| September | 5 | 6 |
| October | 0 | 1 |
| November | 3 | 4 |
| December | 5 | 6 |

Table 2: Corresponding numbers for each month (repeated for clarity)

$$m_c = \_\_\_\_\_\_\_\_\_\_$$

Next, once you have this number, you can take the day you were born in its number version and subtract one from it. This will account for the fact that months start on the 1st day, not the 0th day. Finally, add this result to the $m_c$ and $y_o$ value you received above.

$$(d-1) + m_c + y_o = \_\_\_\_\_\_\_\_\_\_$$

Finally, to convert this number to a value between 0 and 6 for our day, we find the remainder when the sum is divided by 7, also known and finding the number *mod 7*.

$$\_\_\_\_\_\_\_\_\_\_ \bmod 7 = \_\_\_\_\_\_\_\_\_\_$$

This final answer represents the day of the week corresponding to your date of birth. Use Table 3 to find which day corresponds with your number. Congratulations on finding your solution!

| **Number** | **Corresponding Day** |
|---|---|
| 0 | Sunday |
| 1 | Monday |
| 2 | Tuesday |
| 3 | Wednesday |
| 4 | Thursday |
| 5 | Friday |
| 6 | Saturday |

Table 3: Days of the week corresponding to numbers 0–6

In short, this is our equation

$$(2+(2*\lfloor\frac{y-1901}{4}\rfloor+(y-1901-\lfloor\frac{y-1901}{4}\rfloor)) + m_c + (d-1)) \bmod 7$$

Note that $\lfloor x \rfloor$ is the same as floor($x$)

# 4 Extensions

After solving the problem, we considered certain extentions, which could be used to expand the scope of the problem.

## 4.1 Extension 1: Actual Leap Years

Our first extension considers any birthday before the 1900s. This extension would change our formula slightly due to the actual definition of a leap year. A leap year is any year that is divisible by 4 e.g. 1904, 1908, 1912, etc.. The number of days in a leap year is 366 as the number of days in February is increased by 1 changing from the usual 28 to the increased 29. However, for any year that is a multiple of 100, it only counts as a leap year if it is also divisible by 400. For example, 1600 and 2000 are leap years while 1700, 1800 and 1900 are not.

## 4.2 Extension 2: Doomsday

Our second extension involves the idea behind John Conway's Doomsday Rule. The core of this idea centers around "Doomsdays", dates which have the same days every year. This could be 4/4, 6/6, 8/8, etc. From here, by taking the last two digits of the year, you divide it by 12 to get the quotient and remainder. The remainder is then divided by 4, and the quotient of that is also saved. Now, we add the remainder and quotient of the first one with the quotient of the second one and then the respective century number. These steps are done because this theory goes based on the simple model that everything repeats after 12 years, with each year being a bit different at the turn of the century based on leap year patterns between the years that are multiples of 100 or 400. Therefore, the formula for the doomsday index is

$$(anchor + \lfloor \frac{y}{12} \rfloor + y \bmod 12 + \lfloor \frac{y \bmod 12}{4} \rfloor) \bmod 7$$

where $y$ represents the last two digits of the year and the anchor is the special number we add based on the century we are in. Now, armed with this information, it is relatively easy to deduce the closest number to said date and do the mental gymnastics required to solve it. For example, if we were to try and find the date of October 8th, 2025, we could use October 10th, 2025 since it is a doomsday. If the formula is used for our current year, the last two digits are 25. As a result, we do $25 \div 12 = 2R1$ and then $1 \div 4 = 0R1$. Now if we add up the numbers, we get $2 + 1 + 0 = 3$. Now, adding the anchor, or a special number which changes every century, it becomes $3 + 2 = 5$. From here, this number is to be taken mod 7 and the result tells us the doomsday day of the week that every single doomsday date falls under. For this year, all Doomsday dates, including October 10th, 2025, are on a Friday. From this, we would have to move backwards 2 days to get to October 8th, 2025, making it a Wednesday.

## 4.3 Extension 3: Reversal

Our last extension involves reversing the problem. Instead of being given an individual's birthdate and having to solve for the day of the week, a year (e.g., 2025) and a day of the week (e.g., Sunday) will instead be given. From this reversal, the problem could be approached in two ways.

- Find all the dates in a specific month (e.g., October) that fall on that day of the week.
- Find the total number of days within the given year that fall on that day of the week.