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Epsilon Modeling Summary

Nowadays, schools are more and more frequently reaching maximum capacities and are plagued with the question of how to expand to meet rising student populations. Our team was faced with this very problem as we attempted to distribute seven new teachers within the educational departments of the Epsilon School of Mathematics and Science in order to allow for a significant student body expansion. The school aims to be able to expand their student population from 490 to 630 students for the upcoming school year in accordance with an unprecedented 140 student sophomore population increase. In order to do this, Epsilon school will be hiring seven new teachers. The task at hand was to build a mathematical model to equitably distribute these teachers among the school's departments.

To begin the model, it was important that a few assumptions be made to logically fill in any key knowledge gaps. The given information included the previous year's teacher count, student enrollments per subject – per grade, the average dropout rate, the sophomore class size increase (in terms of the previous year's senior class), and the fact that language teachers can teach two languages. With this information, it was assumed that all students take only one English class per year as the English enrollments of the previous year summed to the total school population and the English enrollments per grade had a logical size decay. The next assumption was that the dropout rates are consistent throughout all three years at the Epsilon school and the remaining assumptions were as follows: each grade attends the same proportion of classes each year, the average enrollments per student (per grade) remain constant, all teachers teach the same number of classes and the enrollments to teacher ratio from last year was ideal. With these assumptions, the model could begin to take shape.

After the assumptions were made, it was clear that the main objective was to make the enrollment to teacher ratios of the previous and upcoming school years as close to each other as possible. In order to do that, the new grade sizes had to be calculated. Using an exponential decay equation, we multiplied the previous year's senior grade size by $(100\% \text{ minus our dropout rate } (5\%) \text{ to the power of our time period } (1/3))$, which was in terms of three school years $(1 - 0.05^{1/3})$. The resulting value was rounded and equal to our senior class at graduation. Based on the given sophomore increase value, 140 students were added to this result and the sophomore

grade size for the upcoming school year was found to be 289 students. The same exponential decay function was used to figure out the grade sizes for the incoming juniors and seniors. Those values were found to be 180 and 152 students respectively, following rounding. The five percent dropout rate lessened the anticipated school population (630) by approximately nine students.

The next step was to figure out the upcoming year's individual class enrollments. Since it was assumed that the enrollment to subject ratios (the percent of students in a particular grade enrolled in a certain subject) would remain constant, a subject enrollment to total enrollment ratio was formulated to discover what fraction of all enrollments (per grade) each subject occupied. When that fraction was multiplied by the anticipated grade sizes and the average enrollments per student of the previous year (equal to the total enrollments divided by the students in the corresponding grade), the result was the upcoming year's enrollments per subject, (per grade).

Using the previous year's schoolwide enrollments per subject and teacher count, an enrollment to teacher ratio was made for each subject. Using the anticipated schoolwide enrollments per subject and the previous year's teacher count, an enrollment to teacher ratio was created with no new hires in mind. It is important to note that all language classes were considered one subject at this point in the process. The past year's enrollment per teacher ratio was subtracted from the upcoming enrollments per teacher to find the enrollment per teacher difference. The enrollment per teacher difference, represented the difference between the two years' enrollments per teacher and resultantly, which classes need the most attention. Using an active spreadsheet, teachers were added to the upcoming year's subjects with the highest corresponding enrollment per teacher differences until no teachers remained. After each teacher addition, the spreadsheet updated and presented new enrollment per teacher differences to base the next choice on. The results from this process suggested that Epsilon school add one teacher to the following subjects: art, biology, English, language, mathematics, music and social studies.

Based on those results, it was now apparent that the teacher distribution for language classes would have to be calculated. A process, similar to the previous, was carried out comparing the Spanish, French and German enrollment per teacher ratios and it was found that the new hire in the language department should result in a 1.5:1:1.5, (French:German:Spanish) teacher ratio (where $\frac{1}{2}$ of a teacher represents teaching half of the classes) but since it was

unclear which language teachers taught which classes in the previous year, nothing further could be concluded.

At this point, the model was complete and the distribution of all seven teachers had been found. One teacher would be added to art, biology, English, language, mathematics, music and social studies and the optimal language teacher distribution would be 1.5:1:1.5, (French:German:Spanish). After testing several random teacher distributions, our model was shown to work successfully to find the optimal teacher distributions for all subjects. It is crucial to note that so long as there are any assumptions, no model can be one hundred percent airtight. Within the assumptions made in the process of producing this model lie room for error and all results must be considered with that in mind. On the other hand, the model was successful in its purpose and produced equitable teacher distributions.

My experience with the team was new for me. I was proud to contribute several of my own ideas and happy to learn and help implement the ideas of those in my group. As expected, it was difficult to get started. Since the math modeling process was relatively new to the group and myself, we wanted to incorporate all variables and givens in the first calculation. As a result we hit a wall fairly quickly. After we broke the problem up, according to the math modeling process, it became much easier to make progress. I personally learned a lot about the math modeling process, mathematical cooperation and mathematical communication/presentation. The initial project development was quite difficult but as myself and the group continued to learn from our mistakes, the math modeling process became much more manageable.