Nicholas Giza
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B Group

Investigation question

How does the acceleration of a cart on a level, frictionless plane, fixed to another cart rolling down an angled plane, below the first plane, correlate to the angle of the lower plane?

Hypothesis

Prior to testing, it was hypothesized that as the angle of the plane increased, so would the acceleration of both carts. It was thought that this relationship could be modeled with the equation $a = 4.9\sin\theta$, where a is the acceleration of either cart and θ is the plane angle. This was justified by the force body diagrams where equations for each cart were combined and simplified.

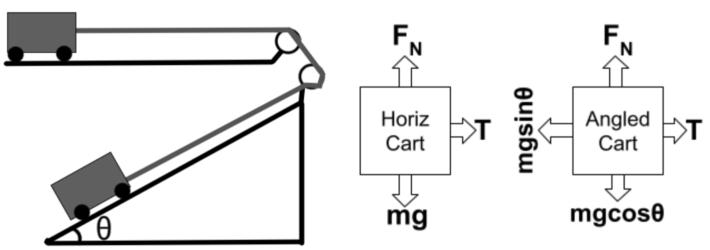

Fig. 1 Model of the apparatus and force body diagrams

Deriving for acceleration

1. $m_{horiz}{}^{*}a = T$
2. $m_{angled}{}^{*}a = m_{angled}{}^{*}(g)\sin\theta - T$
3. $m_{angled}{}^{*}a = m_{angled}{}^{*}(g)\sin\theta - m_{horiz}{}^{*}a$

(Since $m_{horiz} = m_{angled}$)

4. $a = (g)\sin\theta - a$
5. $a = 0.5(g)\sin\theta = 4.9 m/s^2 {}^{*}\sin\theta$

Strategy

In order to test this situation, a model had to be constructed and tested according to the research question (Fig. 1). A horizontal plane was placed on two stands, approximately three feet tall and the apparatus was leveled. Next, another plane was placed below and 40. cm away from the horizontal plane, angled on a stand of similar size and anchored at the bottom. A pulley was then attached to the end of each plane where a string was run, connecting two carts, one on each plane. The cart on the horizontal plane was turned on, pulled back until the cart below was approximately level with the horizontal plane and released. Velocity in terms of time was recorded by embedded cart probes for five angles and could be used to determine acceleration in further analysis.

Data

θ (°)	v_1 ($\frac{m}{s}$)	v_2 ($\frac{m}{s}$)	t_1 (s)	t_2 (s)
75	0.171	1.307	2.10	2.38
60.	0.32	1.218	2.34	2.58
45	0.407	1.426	1.98	2.300
30.	0.565	1.083	0.38	0.62
15	0.405	0.792	2.62	3.02

*Cart masses = 0.275kg

Analysis

In order to find the acceleration of the cart, the velocities and corresponding times had to be converted using the equation $a = (v_2 - v_1)/(t_2 - t_1)$. Once the accelerations for each angle were found, they were graphed against $\sin(\theta)$ in order to produce a linear graph and verify the hypothesis. This is justified by the anticipated equation $a = 4.9\sin\theta$ where 4.9 becomes the slope and a can be expressed in terms of $\sin(\theta)$ with a linear slope. A line of best fit was calculated where $a = 4.42(\sin\theta) - 0.09$ (Fig. 2).

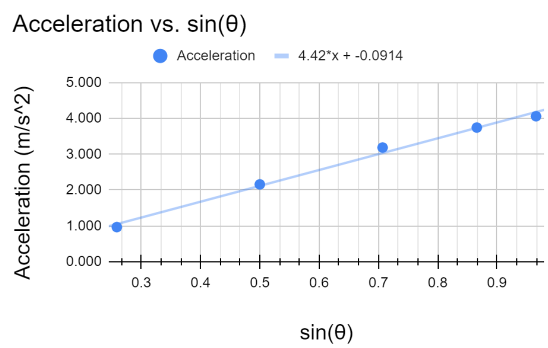

Figure 2: Graph of acceleration vs sin(θ)

This equation is close to the projected equation of $a = 4.9\sin\theta$, verifying the hypothesis. The difference in slope (which was projected to be *½g or 4.9)* can be explained by the presence of friction. While the hypothesis and problem statement did not account for friction, it was a considerable factor in testing. This slope was 10.% lower than expected, which is acceptable because friction lowers the net force and in turn, lowers acceleration. The other two possible sources of error include the fact that only one trial was conducted per group and when θ=75°, the rear wheels of the angled cart tended to lift off of the plane. Testing errors were the cause for a y-intercept as a line of best fit for imperfect data will not necessarily pass through the origin.