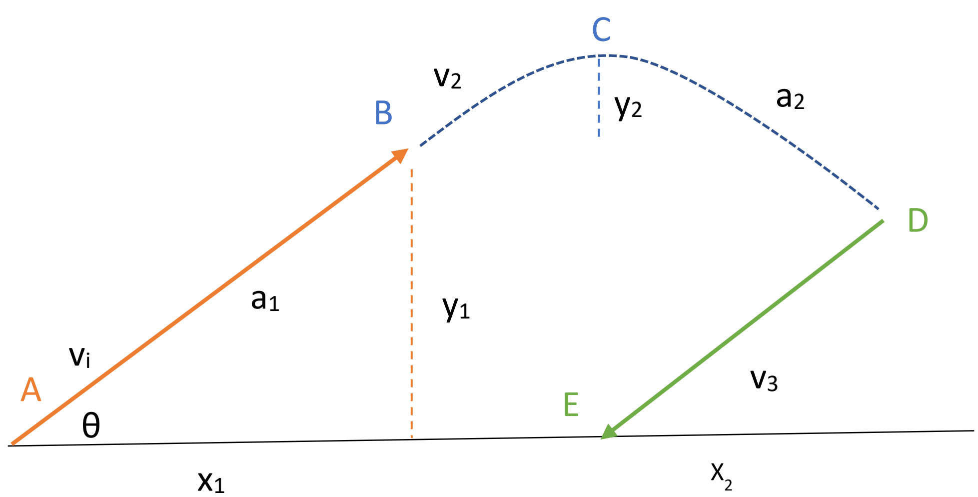
Neena Xiang September 22, 2019 Section K

Uber Rocket Problem

One breezy afternoon Algebra Alex decides to launch Hamster Huey into the air using a model rocket. The rocket is launched over level ground, from rest, at a 43-degree angle above the East horizontal. The rocket engine is designed to burn for 7.9 seconds while producing a constant net acceleration of 4.5 m/s^2 for the rocket. Assume that the rocket travels in a straight-line path while the engine burns. After the engine stops the rocket continues in projectile motion. A parachute opens after the rocket falls 83 meters from its maximum height. When the parachute opens after the rocket instantly changes speed and descends at a constant vertical speed of 7 m/s. A horizontal wind blows the rocket, with parachute, from East to West at a constant speed of 12 m/s. Assume the wind affects the rocket only during the parachute stage. Calculate the x-displacement of where the rocket lands from the initial x-position.

Diagram



Givens

$\theta_i = 43°$
$t_{AB} = 7.9$ s
$v_i = 0$ m/s
$a_1 = 4.5$ m/s^2
$a_2 = -9.8$ m/s^2
$y_2 = 83$ m
$v_{x4} = -12$ m/s
$v_{y4} = -7$ m/s

Strategy

1. Find the velocity of the rocket when the engine stops burning. Then find the velocity in the x and y directions.

v_2:	Y-Dir:	X-Dir:
$v_2 = a_1 * t_1 + v_i$	$v_{y2} = \sin(\theta) * v_2$	$v_{x2} = \cos(\theta) * v_2$
$v_2 = 4.5 * 7.9 + 0$	$v_{y2} = \sin(43) * 35.55$	$v_{x2} = \cos(43) * 35.55$
$\underline{v_2 = 35.55 \text{ m/s}}$	$\underline{v_{y2} = 24.35 \text{ m/s}}$	$\underline{v_{x2} = 26.00 \text{ m/s}}$

2. Using the x and y velocity that was found in the previous step, find the x and y displacement when the rocket reaches point B which is when the engine stops burning.

EQ3 X-Dir:	EQ3 Y-Dir:
$x_1 = \frac{1}{2}(v_{xi} + v_{x2}) * t_{AB}$	$y_1 = \frac{1}{2}(v_{yi} + v_{y2}) * t_{AB}$
$x_1 = \frac{1}{2}(0 + 26.00) * 7.9$	$y_1 = \frac{1}{2}(0 + 24.345) * 7.9$
$\underline{x_1 = 102.70 \text{ m}}$	$\underline{y_1 = 95.77 \text{ m}}$

3. Find the max height of the rocket by using the EQ2 to calculate the time it takes for reach that height, and EQ3 to find the value of the height

Time to reach max height:

$$v_C = a_2 * t_{BC} + v_{y2}$$
$$0 = -9.8 * t_{BC} + 24.25$$
$$-24.25 = -9.8 * t_{BC}$$

t_{BC} = 2.474 s

Value of max height:

$$y_{max} = \frac{1}{2} * a_2 * {t_{BC}}^2 + v_{y2} * t_{BC} + y_1$$
$$y_{max} = \frac{1}{2} * -9.8 * 2.474^2 + 24.25 * 2.474 + 95.77$$

y_{max} = 125.759 m

4. Using the found max height, find the rocket remains in projectile motion. Then after finding that time, calculate the x displacement from point A to point D.

Time in projectile motion:

$$y_D = \frac{1}{2} * a_2 * {t_{BD}}^2 + v_{y2} * t_{BD} + y_1$$
$$y_{max} - y_2 = \frac{1}{2} * a_2 * {t_{BD}}^2 + v_{y2} * t_{BD} + y_1$$
$$125.76 - 83 = \frac{1}{2} * -9.8{t_{BD}}^2 + 24.25 * t_{BD} + 95.77$$
$$0 = -4.9 * {t_{BD}}^2 + 24.25 * t_{BD} + 53.01$$

~~t = -1.6417 s~~ and t = 6.5897 s

Time cannot be negative so the only valid answer is 6.5897 seconds, and the -1.6417 seconds can be ignored.

X displacement at point D:

$$x_D = \frac{1}{2} * a_x * {t_{BD}}^2 + v_{x2} * t_{BD} + x_1$$
$$x_D = 0 + 26.00 * 6.5897 + 102.66$$

$$x_D = 274.03\ m$$

5. Find the time it takes for the rocket from point D to land at point E by using the height of the rocket at point D and the y-velocity when the parachute opens. That time and the x-velocity when the parachute opens can then be used to find the final x-displacement of the parachute.

Time in projectile motion:

$$y_E = \frac{1}{2} * a_3 * {t_{DE}}^2 + v_{y3} * t_{DE} + y_D$$
$$y_E = \frac{1}{2} * a_2 * {t_{DE}}^2 + v_{y2} * t_{DE} + y_{max} - y_2$$
$$0 = 0 + (-7) * t_{DE} + 125.76 - 83$$
$$0 = -7 * t_{DE} + 42.759$$
$$-42.759 = -7 * t_{DE}$$

$$t_{DE} = 6.1043s$$

X displacement at point D:

$$x_f = \frac{1}{2} * a_x * {t_{DE}}^2 + v_{x3} * t_{DE} + x_2$$
$$x_f = 0 + (-12) * 6.1043 + 274.03$$

$$\boxed{x_f = 200.7\ m}$$