Introduction

Purpose:

The purpose of this lab is to design and perform an experiment which analyzes the conservation of energy in a spring-based system.

Researchable question:

How does increasing the stretch distance of a rock elastic system from the equilibrium point effect the maximum height the weight will reach when released?

Hypothesis:

If the initial height of a reverse bungee increases, then the maximum height the rock reaches will decrease proportionally with the square of the stretch distance.

Methodology

Procedure:

1. The apparatus was built by wrapping a rock in rubber bands and duct tape.

2. A chain of 7 rubber bands was then created by tying them together.

3. The rock and the chain of rubber bands was then attached to each other by using a paper clip.

4. A dowel taped on a table with some overhang so the end of the bungee chain can be taped 5 cm away from the table.

5. A meter stick was taped to the side of the table so that the numbers increased from bottom to top and it was touching the ground. An additional ruler was also taped above the meter stick in the same fashion, but the bottom starts at the top of the meter stick.

6. Neena then holds the rock so that there is no slack and no pull in the bungee.

- 7. Julia records the height.
- 8. Neena pulls the bungee down to various heights so that the bottom of the rock is at the indicated heights.

9. Akhi counts 3, 2, 1 and starts recording in slow motion video at 2.

10. Neena lets go of the rock and Akhi stops the recording when the rock reaches the max height.

11. Akhi reads the max height the rock reaches, and Julia records the data.





Summary Table

h	xi	X _{AVG}	SD	%RSD	Xt	%err
(cm)	(cm)	(cm)	(cm)	of x _{AVG}	(cm)	of x
55.0	73.0	63.5	0.7	1.1	62.3	2.0
40.0	72.0	75.6	0.9	1.2	63.1	19.9
30.0	72.0	83.2	2.0	2.4	69.7	19.3
25.0	72.0	88.2	2.7	3.1	74.8	18.0
20.0	72.0	96.6	1.2	1.3	80.9	19.4
15.0	72.0	102.7	1.3	1.3	88.2	16.4
10.0	72.0	113.8	3.8	3.4	96.6	17.8
			AVG	2.0	AVG	16.1



The group is testing the apparatus.

Constants and Equations

m = 77.0 g = 0.0770 kg g = 9.80 m/s² k = 0.034 N/cm

$$x_f = \frac{\frac{1}{2}k(x_i - h)^2 + mgh}{mg}$$
(see appendix)

Height Graph

Final Height vs Initial Height Measured Height Theoretical Values 120 110 $y = 0.0151x^2 - 2.0513x + 131.42$ 100 Finial Height, x (cm) $R^2 = 0.9938$ 90 80 +-----70 ******* $y = 0.0233x^2 - 2.2812x + 117.15$ 60 R² = 1 50 15 25 35 45 55 5 65 Initial Height, h (cm)

Energy Analysis



TEi	TE _f	%Е	
(J)	(J)	TE _i to TE _f	
0.47	0.48	2.0	
0.48	0.57	19.9	
0.53	0.63	19.3	
0.56	0.67	18.0	
0.61	0.73	19.4	
0.67	0.77	16.4	
0.73	0.86	17.8	
	AVG	16.1	

The total energy in the system increased by an average of 16.1% after the rock reaches its maximum height.

Analysis

- The data indicates that there is a high precision since the average %RSD is 2.0%.
- There is a low accuracy since the average |%err|, being 16.1%, is higher than 10%.
- The mathematical model for the measured values is very strong with an R2 value of 0.9938.
- The graphs show that the lower the initial height, the higher and the final height will be. When the initial height is 0 cm, the final height will be 131.42 cm based on the y-intercept of the measured data. The apparatus is limited by the equilibrium point, so it cannot be started at a lower height.
- Eventually when the height increases beyond the equilibrium point, the max height will increase again because it will start with only gravitational potential energy and end with that same energy. This is reflected in the equation because of the squared height. After hitting a minimum height, where the pull of gravity equals the force of the spring, the height will increase again, so the final height will never be the ground.

Sources of Error

- The rock was irregularly shaped so measuring the rock from the bottom was difficult to keep consistent.
 - This may have resulted in measuring of the height of the rock to be higher than it was, increasing the final height of the rock.
- The rubber bands were not straight when tied together, so there may still have been some pull when measuring the equilibrium point of the bungee.
 - This would also increase the final energy because there was more initial potential energy than what was calculated.

Conclusion

The original hypothesis was correct because as the initial height increased, the maximum final height decreased. However, if the initial height passes a minimum point were the initial height equals the point where the pull of gravity is the same as the force of the spring, the maximum height will increase again. The difference seen in the actual and measured heights can be attributed to the low accuracy of the model. In the case of the measured values, they were all higher than the predicted values. Therefore, the calculated energy in the system increased at the maximum height.

Future Extensions

- Different rocks with different masses can be used to test the effects of mass on the maximum height of the rock.
- Different elastics can be used to determine the effects of the k constant on the maximum height.
- The equilibrium point can be increased to measure longer stretch distances.

Appendix

Calculating K



X 1	X ₁ X ₂		F
(cm)	(cm)	(cm)	(N)
28	20	8	0.518
28	15	13	0.801
48	30	18	1
48	20	28	1.35
48	10	38	1.65
58	8	50	1.98

The slope of the force vs stretch distance graph is the k constant which in this case means that the k constant is 0.034 N/cm.

Deriving x_t

$$\sum_{i=1}^{n} E: PE_{si} + PE_{gi} = PE_{gf}$$

$$\frac{1}{2}k(\Delta x)^{2} + mgh = mgx_{t}$$

$$\frac{1}{2}k(x_{i} - h)^{2} + mgh$$

$$= mgx_{t}$$

$$x_{t} = \frac{\frac{1}{2}k(x_{i} - h)^{2} + mgh}{mg}$$