Problem Statement:

The goal of this problem was to create an algorithm through which someone could calculate the day of the week they were born, if they didn't already know. The only tools given for the calculations were a four-function calculator and a calendar for the current month.

Process:

The beginning of our process started with us trying to find an equation that would give you the day of the week you were born for your birthday. We were unsuccessful in this because we couldn't find a single equation that accounted for changing factors like leap years and shifting days of the week as years pass. We attempted to use modular arithmetic to simulate the cycle of the 7-day week, but we noticed that it only worked for some birthdays and not others. With help from Mrs. Burns, we realized that the weekday at the start of the year was affecting this. Our next approach was to find a way to incorporate all these factors into multiple steps. We eventually broke it down into 3 main steps which accounted for those factors, which is shown below in our instructions:

STEP BY STEP INSTRUCTIONS:

Step 1: Find what day of the week of this year's January 1st was on

a. Calculate the number of days from Jan. 1 to now (Add all the days of the months prior to the current month, and then add the date of today) Then subtract 1 from this sum. You can use the table below to help you
b. Now find the remainder of this number after dividing the number by 7
c. Start from whatever day of the week today is, and count backward the same number of days as the remainder you got
d. For example, 1/1/2024 was a Monday

Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
31	28*	31	30	31	30	31	31	30	31	30	31

*29 in leap years

Step 2: Find the day of the week of January 1st was on, of the year you were born (disregard this step if you are born on this year)

a. Subtract the year you were born from the current year
b. Divide this number by 4 to find the number of leap years from your birthyear up until now. If the quotient is a decimal, round it down
c. Multiply this number of leap years by 2
d. Subtract the original number of leap years (found in step 2b) from the number you obtained in step 2a
e. Take this new number and add it to the number from step 2c (the number of leap years times 2)
f. Divide this new number by 7 and obtain the remainder
g. Start from the day of the week obtained in step 1c, and count backwards the number of days obtained in the previous step
h. The day of the week you have now is the day that January 1st fell on the year you were born

Step 3: Calculate what day your birthday was on

a. There are 365 (or 366) days in a year. Find which day your birthday is on by adding days from prior months (make sure to use 29 for February if you were born on a leap year, use 28 otherwise). Then subtract 1 from this sum.
b. Calculate this number in mod 7 (divide by 7 and find the remainder)
c. Start from the day of Jan 1st on your birthyear (for example, for 2024 it was Monday) and count forward the number of days you got in the previous step
d. You should now have the day of the week that you were born on!

<u>Why the solution works:</u>

Explanation for step 1:

The sum obtained from the addition of the days of the months and the date of today represents the total number of days from January 1st to today, but this sum also accounts for today as well. Hence, subtracting 1 from this sum will result in the number of days that needed to be added to January 1st to get today's date. The modulus of 7 of this difference shows the remainder after dividing by 7 (because a full week is 7 days). Hence, counting backwards as many times as the remainder gives the day of the week of January 1st.

Explanation for step 2:

Subtracting the current year from the birthyear will result in a number that accounts for both leap and non-leap years. Every 4 years there is a leap year, so divide the total number of years by 4 (ignoring the remainder) will obtain the number of leap years from the birthyear to the current year. Normal years have 365 days, which in mod 7 is 1; leap years has 366 days, which in mod 7 is 2. This is important because this proves that when the calendar shifts by one year, the day of the week moves also shifts by one. Transitioning out of a leap year will result in a shift of two days. Therefore, multiplying the number of leap years by two and adding the product to the number of normal years will result in how many days of the week were shifted back to reach January 1st. Mod 7 is applied to this sum to make it easier to count backwards for the day of the week, because for every 7 days deducted, the day of the week stays the same. Therefore, only the remainder of this division matters when counting backwards.

Explanation for step 3:

This step works very similarly to step 1, because we are trying to find the day of the week in a year knowing the starting day of the year instead of the converse. Finding how many days from January 1st to the birthday involves an identical process in step 1, so consult the explanation for step 1 to know why step 3a-c works. However, additional explanation for step 3 is needed as we go forward to determine the day of the week that the birthday falls on. This is because January 1st starts before or is the birthday, so adding the remainder would cycle forward through the day of the week.

<u>Extension:</u>

1. How would you develop a solution to this for a birthday in a year before 2024, that is Anno Domini (starting from 0 AD)?

This extension is developed because the rule for leap years of centuries is if they are divisible by 400. If the century is not, then it isn't a leap year. The solution for these birthyears would remain mostly the same, but when calculating the number of leap years, we would add that after dividing the difference obtained by subtracting the current year by the birth year by 4, a further subtrahend is needed. To obtain this subtrahend, divide the difference by 400 to know the number of centuries that are leap years which shall be

named quotient 1. Then, divide the original difference by 100 which results in quotient 2. Subtract quotient 2 by quotient 1 to obtain the subtrahend. Therefore, to find the number of leap years:

- Divide the difference obtained by subtracting the current year by the birth year by 4 and rounds down. This difference shall be called difference one.
- Divide the difference by 400 to obtain quotient 1
- Divide the difference by 100 to obtain quotient 2
- Subtract the quotient 1 by quotient 2 to obtain the number of normal century years (years that have 365 days) which shall be called difference 2
- Subtract difference 2 by difference 1 to obtain the number of leap years.

Everything else about the solution remains the same.

2. Would the solution for extension 1 look different if you are predicting for years after 2024?

This extension is developed because we wanted to know if our algorithm also works for future years to predict the day of the week that a future date might fall on. We predict that our solution will remain the same for steps 1 and 3. Step 2 will change slightly as we are moving to future years instead of past years, so for step 2g, we would count forward instead of backwards. This is likely the only change that is needed for our algorithm to solve for future years.