Question:

What is the relationship between incline and acceleration of a cart travelling along a metal track in a modified Atwood's machine?

Hypothesis:

The relationship between the sin of surface incline angle and acceleration will be negative and linear. The slope of the graph of sin angle vs. acceleration will be equal to $-\frac{m_1 g(sin\theta)}{(m_1+m_2)}$.

Strategy:

- The angle of the incline in the modified Atwood's machines was varied by placing different numbers of books we put under one end of the ramp, varying by 1 book each trial. The acceleration of the cart was measured using the vernier motion detection.
- To determine the sin θ, the height of the ramp was measured for each trial and divided by the length(hypotenuse) of the ramp.
- To ensure that the slope between sin θ and acceleration was linear, we graphed the points and applied linear regression.

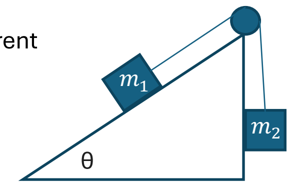

Figure 1: Modified Atwood's machine

Data:

Sine Angle	Acceleration (m/s²)
0	3.0993
.04054	2.8273
.08108	2.5807
.12162	2.3253
.16757	2.0567

The acceleration was an average of 3 trials.

Analysis:

The free body diagrams in Figure 2 show the forces on the masses in the modified Atwood's machine.

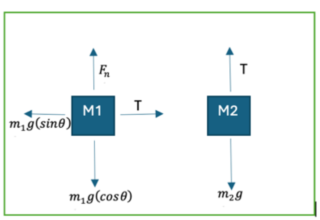

Figure 2: Free Body Diagrams

As the wheels had the ability to move freely with low resisteance, the effects of friction between the cart and the ramp were negliable. The equations below were derived seperately for each mass with the positive position of mass 1 going to the right, and the positiive position of mass 2 going down.

$$T - m_1 g(sin\theta) = m_1 a$$
$$m_2 g - T = m_2 a$$

The sum of these equations gives a new equation:

$$m_2 g - m_1 g(sin\theta) = (m_1 + m_2)a$$

Which we converted into:

$$\frac{m_2 g}{(m_1 + m_2)} - \frac{m_1 g(sin\theta)}{(m_1 + m_2)} = a$$

We derived this form to resemble the slope intercept form, y=mx+b, where acceleration a represents the dependent variable(y) and sin θ represents the independent variable(x). Because the masses were kept constant throughout our model, we were able to isolate the slope of $-\frac{m_1 g(sin\theta)}{(m_1+m_2)}$. This equation indicates a linear relationship between the sine of our angle and the acceleration of our model.

The graph between sine theta and the acceleration confirms that it is linear, showing a slope equal to -6.21 which indicates that acceleration decreases as we increase the angle.

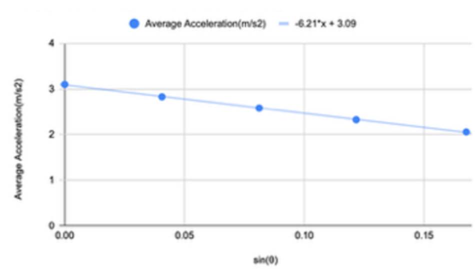

Figure 3: sinθ vs. Acceleration

The slope of the equation was 5.15% larger than expected, due to the actual calculated slope being -6.53. As the slope was too large, our measured accelerations were likely too small. One possible source of error could've been air resistance because it pushes against the cart's motion and lowers the net force. Another possible source of error could be the friction between the cart and the ramp because it slows down the acceleration.