HW 2

Ex (2.2) 3. $(\forall A,B,C)(A\cap(B\cup C)\subseteq(A\cap B)\cup(A\cap C))$

Proof. By way of contradiction, suppose $A\cap(B\cup C)\nsubseteq(A\cap B)\cup(A\cap C)$. By definition it means $\exists x\in A\cap(B\cup C)$ and $x\notin((A\cap B)\cup(A\cap C))$, therefore, $x\in A\cap(B\cup C)$ and $x\notin(A\cap B)$ and $x\notin(A\cap C)$. Hence, $x\in A\cap(B\cup C)$ and $(x\notin A$ or $x\notin B)$ and $(x\notin A$ or $x\notin C)$. ...

Case 1 $x\notin A$

Case 2 $x\notin B$ and $x\notin C$.

Ex (2.3) 8. $(\forall A,B,C)(A\subseteq B\wedge A\subseteq C\rightarrow A\subseteq B\cap C)$

Proof. By way of contradiction, suppose $A\subseteq B\wedge A\subseteq C$ and $A\nsubseteq B\cap C$. By definition implies $\exists x\in A$ and $x\notin B\cap C$, that is, $x\in A$ and $(x\notin B\vee x\notin C)$.

Case 1 $x\in A$ and $x\notin B$. Since $A\subseteq B$, $x\in B$. Contradiction

Case 2 $x\in A$ and $x\notin C$. Since $A\subseteq C$, $x\in C$. Contradiction

$\boxed{//}$

Ex (2.4) 4. $(\forall A, B, C)\left(A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)\right)$.

Proof: Let $x \in A \cup (B \cup C)$, then $x \in A$ or $x \in (B \cap C)$.

Case 1 $x \in A$. If $x \in A$ then $x \in A \cup B$ and $x \in A \cup C$ since $A \subseteq (A \cup B)$ and $A \subseteq (A \cup C)$. Hence $x \in (A \cup B) \cap (A \cup C)$.

Case 2 $x \in B \cap C$. Then $x \in B$ and $x \in C$. Hence, $x \in (A \cup B)$ and $x \in (A \cup C)$, and obtain $x \in (A \cup B) \cap (A \cup C)$

$\square$

Ex (2.5) 1. $(\forall A, B)(A = A \cap B \rightarrow A \subseteq B)$.

Proof Let $x \in A$. Since $A = A \cap B$, then

Case 1 $A \subseteq A \cap B$
Case 2 $A \cap B \subseteq A$

Ex (2.6) 1. $(\forall A, B)(A = A \cap B \rightarrow A \subseteq B)$

Pf: Let $x \in A$. Since $A = A \cap B$

Case 1 $A \subseteq A \cap B$. Then $x \in A \Rightarrow x \in A \cap B$, so $x \in B$.
We have shown that $x \in A \rightarrow x \in B$. So $A \subseteq B$.

Case $A \cap B \subseteq A$. Then $x \in A \cap B \Rightarrow x \in B$.
Always true
We have shown that $x \in A \rightarrow x \in B$. So $A \subseteq B$

$\square$

Ex 27 2. $(\forall A,B)\ \big((A-B)\cup(B-A) = (A\cup B) - (A\cap B)\big)$

Proof (Direct)

$\Rightarrow$ Let $x \in (A-B)\cup(B-A)$

Case 1 $x \in (A-B)$, that is, $x \in A$ and $x \notin B$. Since $A \subseteq A\cup B$

$x \in A \Rightarrow x \in A\cup B$. Since $x \notin B \Rightarrow x \notin (A\cap B)$

Hence $x \in (A\cup B)$ and $x \notin (A\cap B)$, so, $x \in (A\cup B)-(A\cap B)$.

Case 2: $x \in (B-A)$. Similar, swap A and B above

$\Leftarrow$ Let $x \in (A\cup B)-(A\cap B)$, that is, $x \in A\cup B$ and $x \notin (A\cap B)$

Case 1 $x \in A$ and $x \notin (A\cap B)$. Hence, $x \in A$ and $(x \notin A$ or $x \notin B)$.

Since $x \in A$, then $x \notin B$. Therefore $x \in (A-B)$.

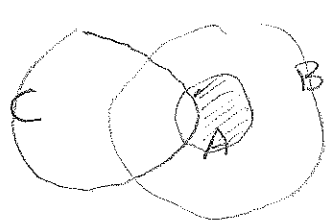
So $x \in (A-B)\cup(B-A)$

Case 2 $x \in B$ and $x \notin (A\cap B)$. Hence $x \in B$ and $(x \notin A$ or $x \notin B)$.

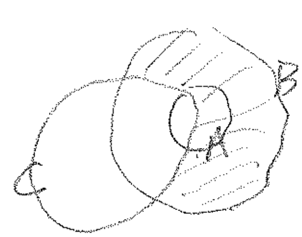
Since $x \in B$, then $x \notin A$. Hence $x \in B-A$. So

$x \in (B-A)\cup(A-B)$

Ex. 28 13. $(\forall A,B)\ (A \subseteq B \rightarrow A-C \subseteq B-C)$ ← Statement



$A-C$



$B-C$

Looks true

Proof Let $x \in A-C$, that is $x \in A$ and $x \notin C$. Since $A \subseteq B$, $x \in A$ implies $x \in B$. So, $x \in B$ and $x \notin C$ that is $x \in B-C$ and the statement is true.

Ex (2.9) $(\forall A, B)((A \cap B)^c = A^c \cup B^c)$.

Proof $\Longrightarrow$

Let $x \in (A \cap B)^c$, that is, $x \notin (A \cap B)$. Then $x \notin A$ or $x \notin B$, hence, $x \in A^c$ or $x \in B^c$, therefore $x \in A^c \cup B^c$.

$\Longleftarrow$

Let $x \in A^c \cup B^c$, that is, $x \in A^c$ or $x \in B^c$. We then have $x \notin A$ or $x \notin B$, therefore $x \notin (A \cap B)$, that is, $x \in (A \cap B)^c$.