HW 1

Ex (1.3) 2.

p	q	$p \leftrightarrow q$	$p \to q$	$q \to p$	$p \to q \wedge q \to p$	$\neg p$	$\neg q$	$\neg p \to \neg q$	$\neg p \to \neg q \wedge p \to q$
T	T	T	T	T	T	F	F	T	T
T	F	F	F	T	F	F	T	T	F
F	T	F	T	F	F	T	F	F	F
F	F	T	T	T	T	T	T	T	T

definition

All logically equivalent since they have the same truth table

Ex (1.4) 5.

p	q	$\neg p$	$p \to q$	$\neg p \wedge (p \to q)$	$(\neg p \wedge (p \to q)) \to \neg p$
T	T	F	T	F	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

when all are T $\Rightarrow$ tautology

7.

p	q	$p \to q$	$\neg q$	$p \wedge \neg q$	$(p \wedge \neg q) \to c$, $c \equiv F$	$(p \to q) \leftrightarrow (p \wedge \neg q) \to c$
T	T	T	F	F	T	T
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Ex (1.4) 12.

p	q	$\neg p$	$\neg q$	$p \to q$	$\neg q \to \neg p$	$(p \to q) \leftrightarrow (\neg q \to \neg p)$
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

Ex (1.5)

7. $(\forall y \in \mathbb{N})(\exists x \in \mathbb{N})(x = y - 7)$

For each $y \in \mathbb{N}$, there exists $y \in \mathbb{N}$ such that $x = y - 7$.

False. Counterexample: let $y = 1$, then $x = -6$ and $x \notin \mathbb{N}$.

8. $(\exists x \in \mathbb{N})(\forall y \in \mathbb{N})(x = y - 7)$

There exists $x \in \mathbb{N}$ such that for all $y \in \mathbb{N}$, $x = y - 7$.

False. Counterexample: Given $x \in \mathbb{N}$ choose $y = x$. Then $x \neq y - 7$.

Ex (1.7)

3. Statement is true. The positive integers are the natural numbers

$\neg(p \to q) \equiv p \wedge \neg q$

Negation (symbols): $(\exists x \in \mathbb{Z})(x > 0 \wedge x \notin \mathbb{N})$.

Negation (words): There exists an integer number x such that x is positive and x is not a natural number

Ex (1.7)

8. The statement is false. Counterexample $x=0$

Negation (symbles): $(\exists x \in \mathbb{Z})((x \notin \mathbb{N} \wedge (-x \notin \mathbb{N}))$

Negation (words): There exists an integer number x such that $x \notin \mathbb{N}$ and $-x \notin \mathbb{N}$. (True, $x=0$).

11. The statement is true: $|x-2|>6$ is logically equivalent to $(x-2>6 \vee -(x-2)>6)$ which is logically equivalent to $(x>6) \vee (x<-4)$.

Negation (symbles):

$$(\exists x \in \mathbb{Z})((|x-2|>6 \wedge (x \le 8 \wedge x \ge -4))$$

$\neg(p \to q) \equiv p \wedge \neg q$ $\qquad$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Negation (words):

There exists an integer x such that $|x-2|>6$ and $x \le 8$ and $x \ge -4$ (false).

We have used that

$p \wedge (q \wedge r) = (p \wedge q) \wedge r \longrightarrow$ denote by $p \wedge q \wedge r$.

Ex (1.7)

13. The statement in symbles.

Denote $\mathbb{R}^+ = \{x \in \mathbb{R} \mid x > 0\}$.

Statement $(\forall \varepsilon \in \mathbb{R}^+)(\exists k \in \mathbb{N})(\forall n \in \mathbb{N})(n > k \Rightarrow \frac{1}{n} < \varepsilon)$

Negation (symbles):

$(\exists \varepsilon \in \mathbb{R}^+)(\forall k \in \mathbb{N})(\exists n \in \mathbb{N})(n > k \wedge \frac{1}{n} \overset{\geq}{\cancel{\leq}} \varepsilon)$.

Negation (words):

There exists a positive real number ε, such that for all integer number k, there exists a natural number n where $n > k$ and $\frac{1}{n} \leq \varepsilon$.

Ex (1.7)

33. $(\exists x \in \mathbb{R})(\forall y \in \mathbb{Z})(y < x \vee y > x)$. (True, choose $x \notin \mathbb{Z}$).

Negation (symbles).

$(\forall x \in \mathbb{R})(\exists y \in \mathbb{Z})(y \geq x \wedge y \leq x)$

For all $x \in \mathbb{R}$, there exists $y \in \mathbb{Z}$ such that

$y \geq x$ and $y \leq x$.