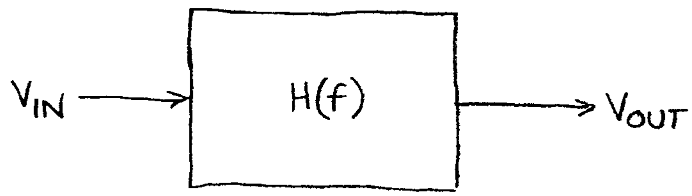
TRANSFER FUNCTION $H(f)$ $\big(H(s), H(\omega), H(j\omega), H(j2\pi f)\big)$

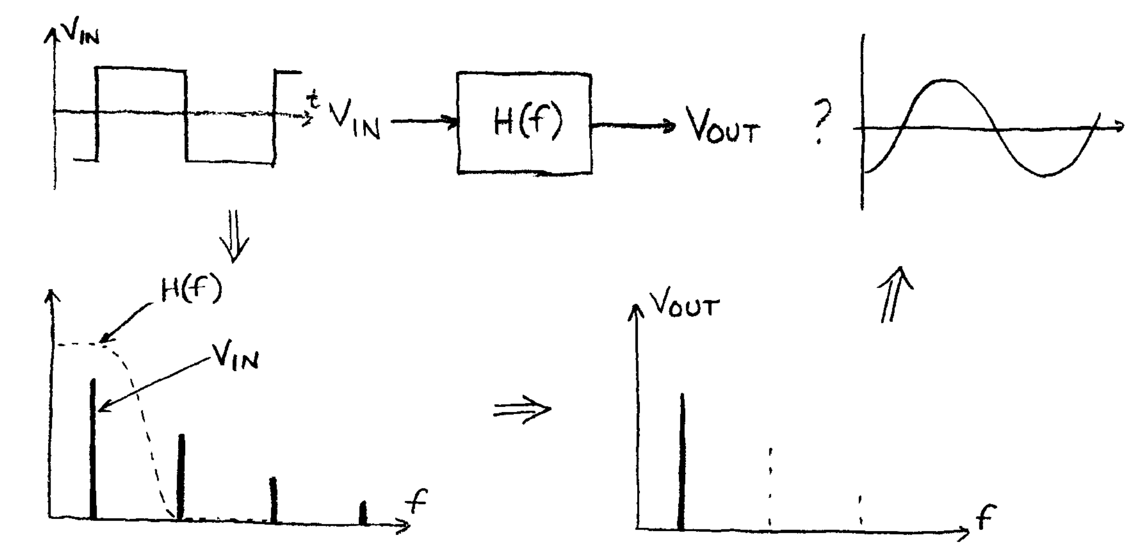
LTI SYSTEM



MAGNITUDE : SCALE
PHASE : SHIFT

$$\underbrace{V_{OUT}(f)}_{\text{OUTPUT SPECTRUM}} = \underbrace{V_{IN}(f)}_{\text{INPUT SPECTRUM}} \underbrace{H(f)}_{\text{TRANSFER FUNCTION}}$$

EXAMPLE



(DO THIS IN LAB 5!)

FOURIER TRANSFORM: REPRESENT V AS SUM OF COMPLEX EXPONENTIALS

$V = V_0 e^{j\omega t}$ $\qquad\qquad$ $V = V_0 e^{st}$

WHY? I-V FOR CAPACITOR

$I = C \frac{dV}{dt}$

CAN THIS BE AS EASY AS Ω'S LAW: $I = \frac{V}{R}$?

$I = C \frac{d}{dt} V_0 e^{j\omega t}$

$I = j\omega C \underbrace{V_0 e^{j\omega t}}_{V}$

$I = (j\omega C) V$

$\frac{V}{I} = \frac{1}{j\omega C}$

"IMPEDANCE": LIKE RESISTANCE, BUT COMPLEX

FREQUENCY DEPENDENT

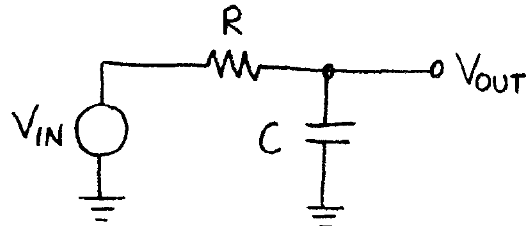
j: "ENCODES" 90° PHASE SHIFT

$Z_R = R$ $\qquad\qquad$ $Z_R = R$

$Z_C = \frac{1}{j\omega C}$ $\qquad\qquad$ $Z_C = \frac{1}{sC}$

$Z_L = j\omega L$ $\qquad\qquad$ $Z_L = sL$

ANALYSIS EXAMPLE: LOWPASS FILTER



IMPEDANCE DIVIDER

$$V_{OUT} = \frac{Z_C}{Z_R + Z_C} V_{IN}$$

$$V_{OUT} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} V_{IN} \qquad \frac{\frac{1}{sC}}{R + \frac{1}{sC}}$$

$$V_{OUT} = \underbrace{\frac{1}{1 + j\omega RC}}_{H(j\omega)} V_{IN}$$

TRANSFER FUNCTION: GENERALIZED GAIN V_{OUT}/V_{IN}

$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + j\omega RC} \qquad \frac{1}{1 + sRC}$$

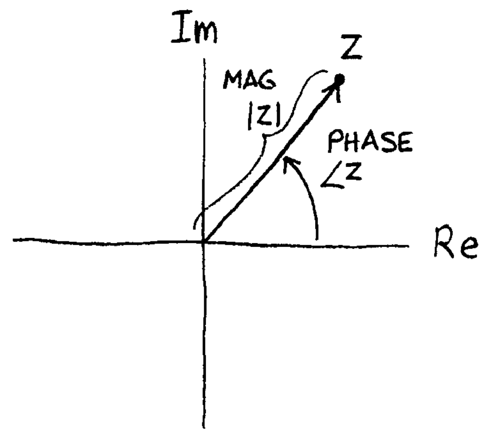
"FREQUENCY RESPONSE": FIND MAGNITUDE, PHASE $(s = j\omega)$

$$\left|\frac{V_{OUT}}{V_{IN}}\right| = \left|\frac{1}{1 + j\omega RC}\right|$$

$$\angle \frac{V_{OUT}}{V_{IN}} = \angle \frac{1}{1 + j\omega RC}$$

COMPLEX NUMBER REFRESHER

$$Z = \frac{a + jb}{x + jy}$$



MAGNITUDE

$$|Z| = \frac{\sqrt{a^2 + b^2}}{\sqrt{x^2 + y^2}}$$

$$|Z| = \frac{|NUM|}{|DEN|}$$

PHASE

$$\angle Z = \tan^{-1}(b/a) - \tan^{-1}(y/x)$$

$$\angle Z = \angle NUM - \angle DEN$$

TRANSFER FUNCTIONS (USUALLY) WILL BE RATIOS OF POLYNOMIALS IN COMPLEX VARIABLE s

WE WILL LET $s = j\omega$ TO GET FREQUENCY RESPONSE

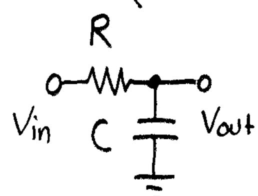
MAGNITUDE, PHASE OF STC LOWPASS

$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

$$|H(j\omega)| = \frac{\sqrt{1^2 + 0^2}}{\sqrt{1^2 + (\omega RC)^2}} = \frac{1}{\sqrt{1 + (\omega RC)^2}} = \frac{1}{\sqrt{1 + (2\pi f RC)^2}}$$

$$\angle H(j\omega) = \tan^{-1}(0/1) - \tan^{-1}(\omega RC/1) = -\tan^{-1}(\omega RC)$$
$$= -\tan^{-1}(2\pi f RC)$$

EXAMPLE: $R = 7.5\ k\Omega$
$C = 1000\ pF$



$RC = 7.5\ \mu sec$

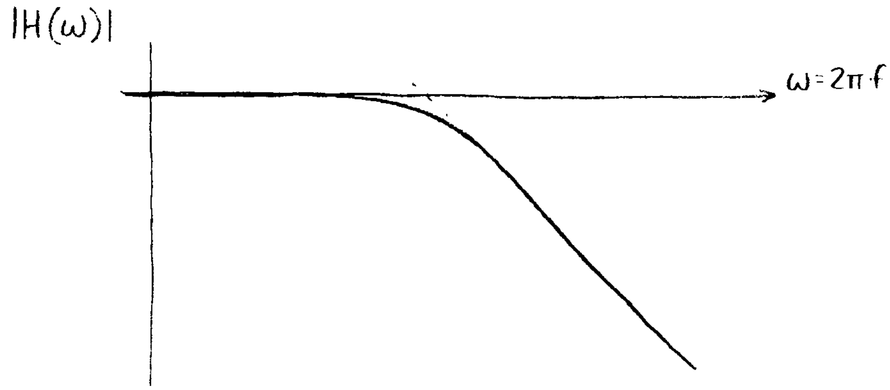
f	MAGNITUDE $\lvert H(f)\rvert$	PHASE $\angle H(f)$ °	
1 Hz	1	−0.003°	0
10 Hz	1	−0.03°	0
100	0.9999	−0.27°	0
1 kHz	0.9989	−2.7°	−0.01 dB
10 kHz	0.9046	−25.2°	−0.87 dB
21 kHz	0.707	−45.0°	−3.0 dB
100 kHz	0.2076	−78.0°	−13.7 dB
1 MHz	0.0212	−88.8°	−33.5 dB
10 MHz	0.0021	−89.9°	−53.5 dB

BANDWIDTH
f_{-3d}
"3dB FREQ"

−20 dB PER DECADE f

PLOT (LINEAR SCALE)

LOG/LOG SCALE



"BODE PLOT": USE <u>dB</u>

$$\underbrace{\left(\frac{V_2}{V_1}\right)}_{\text{VOLTAGE RATIO}} dB = 20 \log\left(\frac{V_2}{V_1}\right)$$

EXAMPLE: $A = 50{,}000$

$$20 \log(A) = 20 \times 4.7 = +94 \text{ dB}$$

"3dB FREQUENCY"

$$20 \log\left(\frac{1}{\sqrt{2}}\right) = -3 \text{ dB}$$

$$\frac{1}{\sqrt{2}} = 0.707$$

$|H(\omega)|$ HAS DROPPED TO $0.707 \times$ MAXIMUM VALUE

MAXIMUM VALUE = 1

$$|H(\omega_{3dB})| = \frac{1}{\sqrt{2}}$$

SOLVE

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+(\omega_{3dB}RC)^2}}$$

$$2 = 1 + (\omega_{3dB}RC)^2$$

$$1 = (\omega_{3dB}RC)^2$$

$$1 = \omega_{3dB}RC$$

$$\boxed{\omega_{3dB} = \frac{1}{RC}}$$

SINCE $\omega = 2\pi f$

$$2\pi f_{3dB} = \frac{1}{RC}$$

$$\boxed{f_{3dB} = \frac{1}{2\pi RC}}$$

FOR THIS EXAMPLE: $\dfrac{1}{2\pi(7.5\,k\Omega)(1000\,pF)} = 21.2 \text{ kHz}$

GENERAL FORM OF TRANSFER FUNCTION

$$H(j\omega) = \frac{[\text{DC GAIN}]}{1 + j\dfrac{\omega}{[\text{3dB FREQUENCY } \omega]}}$$

$$H(j\omega) = \frac{1}{1 + \quad RC} = \frac{1}{1 + j\dfrac{\omega}{\underbrace{1/RC}_{\omega_{3dB}}}}$$

EXAMPLE

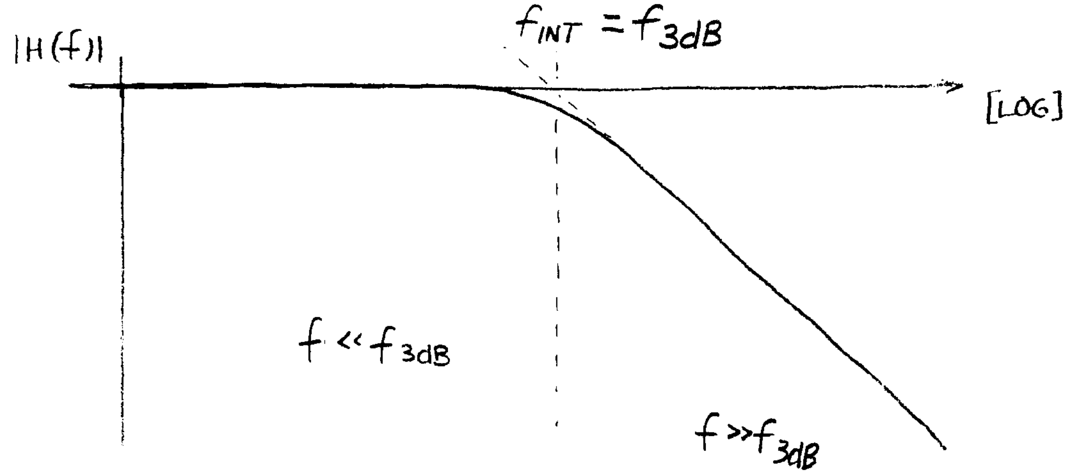
"A STC LOWPASS HAS A DC GAIN OF 20, AND A 3dB FREQUENCY OF 3 kHz. WHAT IS H(s)?"

$$H(j\omega) = \frac{20}{1 + j\dfrac{\omega}{2\pi(3\,kHz)}}$$

ALTERNATIVE (SINCE $\omega = 2\pi f$)

$$H(jf) = \frac{20}{1 + j\dfrac{f}{(3\,kHz)}}$$

FIRST ORDER LOWPASS TRANSFER FUNCTION
ASYMPTOTES OF MAGNITUDE BODE PLOT



TRANSFER FUNCTION REWRITE

$$|H(f)| = \frac{1}{\sqrt{1 + (f/f_{3dB})^2}}$$

$f \ll f_{3dB}$

$$|H(f)| \approx 1$$
$$= 0 \text{ dB}$$

$f \gg f_{3dB}$

$$|H(f)| \approx \frac{1}{\sqrt{(f/f_{3dB})^2}} \approx \frac{f_{3dB}}{f}$$

IN dB

$$\underbrace{20 \log\left(\frac{f_{3dB}}{f}\right)}$$

f INCREASE BY X10 ("DECADE")

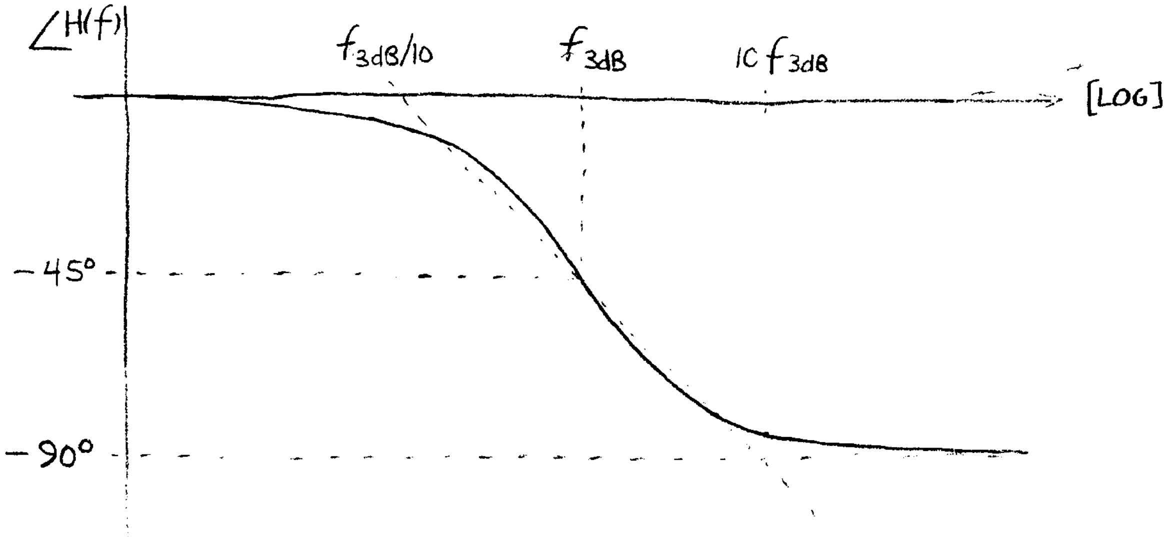
log DECREASE BY 1

dB "−20 dB PER DECADE"

INTERSECTION

$$1 = \frac{f_{3dB}}{f_{INT}} \Rightarrow \boxed{f_{INT} = f_{3dB}}$$

PHASE BODE PLOT



$$\angle H(f) = -\tan^{-1}(f/f_{3dB})$$

$f \ll f_{3dB}$

$-\tan^{-1}(0)$

$= 0°$

$f \gg f_{3dB}$

$-\tan^{-1}(\infty)$

$= -90°$

AT 3dB FREQUENCY

$$-\tan^{-1}(f_{3dB}/f_{3dB}) = -\tan^{-1}(1) = -45°$$