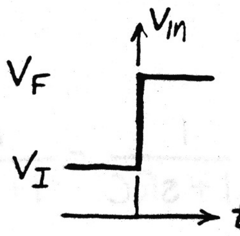
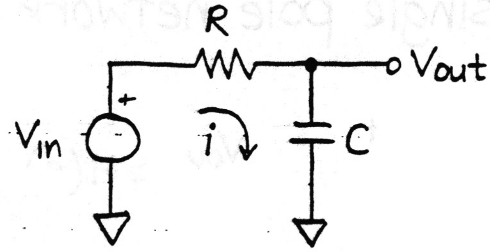
STEP RESPONSE OF FIRST ORDER SYSTEM





First order differential equation for V_{out}

$$\underbrace{V_{out} = V_{in} - iR}_{\text{KVL}} \Rightarrow \underbrace{i = C\frac{d}{dt}V_{out}}_{\text{CAPACITOR}} \Rightarrow \frac{d}{dt}V_{out} = \left(\frac{-1}{RC}\right)V_{out} + \left(\frac{1}{RC}\right)V_{in}$$

Solve differential equation; homogeneous solution: $V_{in}=0$
try solution of form $Ae^{-t/\tau}$; substitute

$$\frac{d}{dt}\left[Ae^{-t/\tau}\right] = \left(\frac{-1}{RC}\right)\left[Ae^{-t/\tau}\right]$$

$$\frac{-1}{\tau}Ae^{-t/\tau} = \frac{-1}{RC}Ae^{-t/\tau} \Rightarrow \tau = RC \left.\right\} \text{"TIME CONSTANT"}$$

Particular solution: when output is constant,
$t \to \infty$, $dV_{out}/dt \to 0$; try constant solution $V_{out} = B$
As $t \to \infty$, $V_{in} = V_F$

$$\left(\frac{-1}{RC}\right)B + \left(\frac{1}{RC}\right)V_F = 0 \Rightarrow B = V_F$$

Boundary condition: at $t=0$, $V_{out} = V_I$ (voltage across C cannot change instantaneously)

$$Ae^{-0/\tau} + V_F = V_I \Rightarrow A = V_I - V_F$$

General solution:

$$\boxed{V_{out}(t) = V_F - (V_F - V_I)e^{-t/\underbrace{RC}_{\tau}}}$$