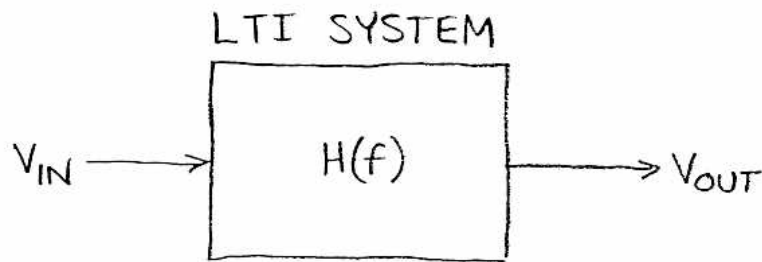


①

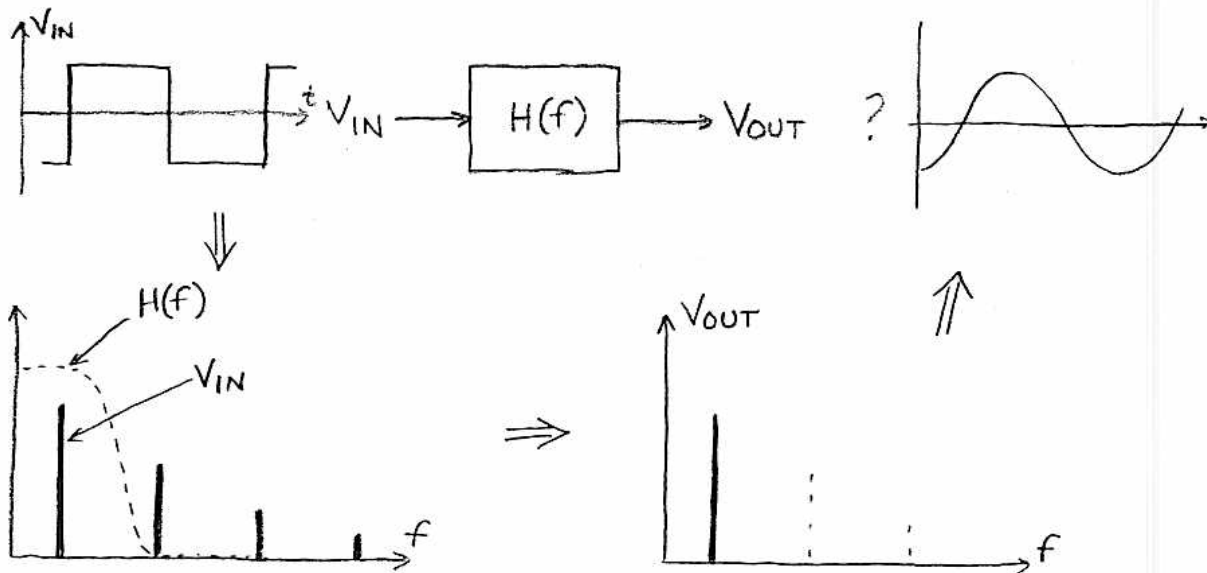
TRANSFER FUNCTION $H(f)$ ($H(s)$, $H(\omega)$, $H(j\omega)$, $H(j2\pi f)$)



MAGNITUDE : SCALE
PHASE : SHIFT

$$\underbrace{V_{OUT}(f)}_{\text{OUTPUT SPECTRUM}} = \underbrace{V_{IN}(f)}_{\text{INPUT SPECTRUM}} \underbrace{H(f)}_{\text{TRANSFER FUNCTION}}$$

EXAMPLE



(DO THIS IN LAB 5!)

(2)

FOURIER TRANSFORM: REPRESENT V AS SUM OF COMPLEX EXPONENTIALS

$$V = V_0 e^{j\omega t}$$

$$V = V_0 e^{st}$$

WHY? I-V FOR CAPACITOR

$$I = C \frac{dV}{dt}$$

CAN THIS BE AS EASY AS OHM'S LAW: $I = \frac{V}{R}$?

$$I = C \frac{d}{dt} V_0 e^{j\omega t}$$

$$I = j\omega C \underbrace{V_0 e^{j\omega t}}_V$$

$$I = (j\omega C) V$$

$$\frac{V}{I} = \frac{1}{j\omega C}$$

"IMPEDANCE": LIKE RESISTANCE, BUT COMPLEX
FREQUENCY DEPENDENT

j : "ENCODES" 90° PHASE SHIFT

$$Z_R = R$$

$$Z_R = R$$

$$Z_C = \frac{1}{j\omega C}$$

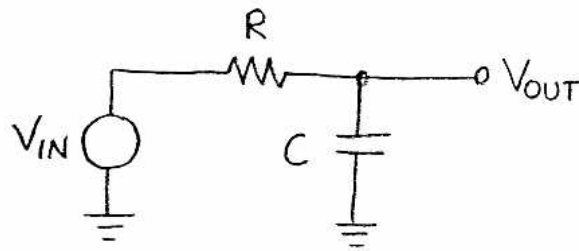
$$Z_C = \frac{1}{sC}$$

$$Z_L = j\omega L$$

$$Z_L = sL$$

ANALYSIS EXAMPLE: LOWPASS FILTER

(3)



IMPEDANCE DIVIDER

$$V_{OUT} = \frac{Z_C}{Z_R + Z_C} V_{IN}$$

$$V_{OUT} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} V_{IN}$$

$$\frac{\frac{1}{sC}}{R + \frac{1}{sC}}$$

$$V_{OUT} = \underbrace{\frac{1}{1 + j\omega RC}}_{H(j\omega)} V_{IN}$$

TRANSFER FUNCTION: GENERALIZED GAIN V_{OUT}/V_{IN}

$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + j\omega RC}$$

$$\frac{1}{1 + sRC}$$

"FREQUENCY RESPONSE": FIND MAGNITUDE, PHASE ($s = j\omega$)

$$\left| \frac{V_{OUT}}{V_{IN}} \right| = \left| \frac{1}{1 + j\omega RC} \right|$$

$$\angle \frac{V_{OUT}}{V_{IN}} = \angle \frac{1}{1 + j\omega RC}$$

COMPLEX NUMBER REFRESHER

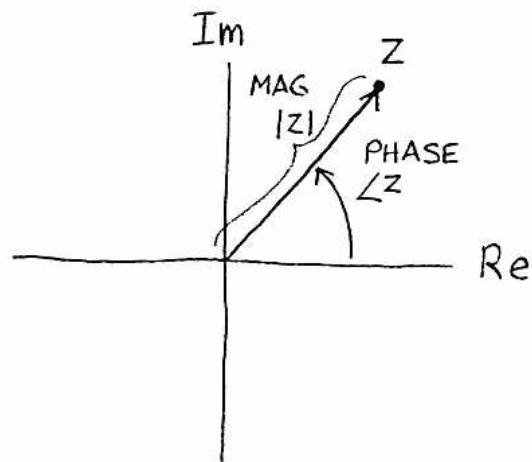
(4)

$$z = \frac{a + jb}{x + jy}$$

MAGNITUDE

$$|z| = \frac{\sqrt{a^2 + b^2}}{\sqrt{x^2 + y^2}}$$

$$|z| = \frac{|\text{NUM}|}{|\text{DEN}|}$$



PHASE

$$\angle z = \tan^{-1}(b/a) - \tan^{-1}(y/x)$$

$$\angle z = \angle \text{NUM} - \angle \text{DEN}$$

TRANSFER FUNCTIONS (USUALLY) WILL BE RATIOS OF POLYNOMIALS IN COMPLEX VARIABLE s

WE WILL LET $s = j\omega$ TO GET FREQUENCY RESPONSE

(5)

MAGNITUDE, PHASE OF STC LOWPASS

$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

$$|H(j\omega)| = \frac{\sqrt{1^2 + 0^2}}{\sqrt{1^2 + (\omega RC)^2}} = \frac{1}{\sqrt{1 + (\omega RC)^2}} = \frac{1}{\sqrt{1 + (2\pi f RC)^2}}$$

$$\begin{aligned}\angle H(j\omega) &= \tan^{-1}(0/1) - \tan^{-1}(\omega RC/1) = -\tan^{-1}(\omega RC) \\ &= -\tan^{-1}(2\pi f RC)\end{aligned}$$

EXAMPLE: $R = 7.5 \text{ k}\Omega$
 $C = 1000 \text{ pF}$

$$RC = 7.5 \mu\text{sec}$$

f	MAGNITUDE $ H(f) $	PHASE $\angle H(f)^\circ$	
1 Hz	1	-0.003°	0
10 Hz	1	-0.03°	0
100	0.9999	-0.27°	0
1 kHz	0.9989	-2.7°	-0.01 dB
10 kHz	0.9046	-25.2°	-0.87 dB
100 kHz	0.2076	-78.0°	-13.7 dB
1 MHz	0.0212	-88.8°	-33.5 dB
10 MHz	0.0021	-89.9°	-53.5 dB

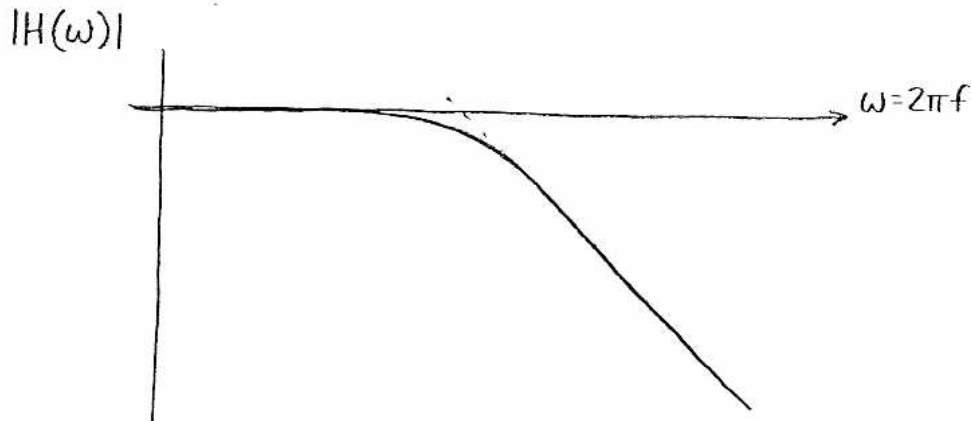
$\left. \begin{array}{l} -33.5 \text{ dB} \\ -53.5 \text{ dB} \end{array} \right\} \begin{array}{l} -20 \text{ dB} \\ \text{PER} \\ \text{DECADE } f \end{array}$

PLOT (LINEAR SCALE)

⑥



LOG/LOG SCALE



"BODE PLOT": USE dB

$$\underbrace{\left(\frac{V_2}{V_1}\right)}_{\text{VOLTAGE RATIO}} \text{ dB} = 20 \log\left(\frac{V_2}{V_1}\right)$$

EXAMPLE: $A = 50,000$

$$20 \log(A) = 20 \times 4.7 = +94 \text{ dB}$$

"3dB FREQUENCY"

⑦

$$20 \log\left(\frac{1}{\sqrt{2}}\right) = -3 \text{ dB}$$

$$\frac{1}{\sqrt{2}} = 0.707$$

$|H(\omega)|$ HAS DROPPED TO $0.707 \times \text{MAXIMUM VALUE}$

MAXIMUM VALUE = 1

$$|H(\omega_{3\text{dB}})| = \frac{1}{\sqrt{2}}$$

SOLVE

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + (\omega_{3\text{dB}} RC)^2}}$$

$$2 = 1 + (\omega_{3\text{dB}} RC)^2$$

$$1 = (\omega_{3\text{dB}} RC)^2$$

$$1 = \omega_{3\text{dB}} RC$$

$$\boxed{\omega_{3\text{dB}} = \frac{1}{RC}}$$

SINCE $\omega = 2\pi f$

$$2\pi f_{3\text{dB}} = \frac{1}{RC}$$

$$\boxed{f_{3\text{dB}} = \frac{1}{2\pi RC}}$$

GENERAL FORM OF TRANSFER FUNCTION

⑧

$$H(j\omega) = \frac{[\text{DC GAIN}]}{1 + j \frac{\omega}{[\text{3dB FREQUENCY } \omega]}}$$

$$H(j\omega) = \frac{1}{1 + RC} = \frac{1}{1 + j \frac{\omega}{1/RC}} \omega_{3dB}$$

EXAMPLE

"A STC LOWPASS HAS A DC GAIN OF 20, AND A 3dB FREQUENCY OF 3 KHz. WHAT IS $H(s)$?"

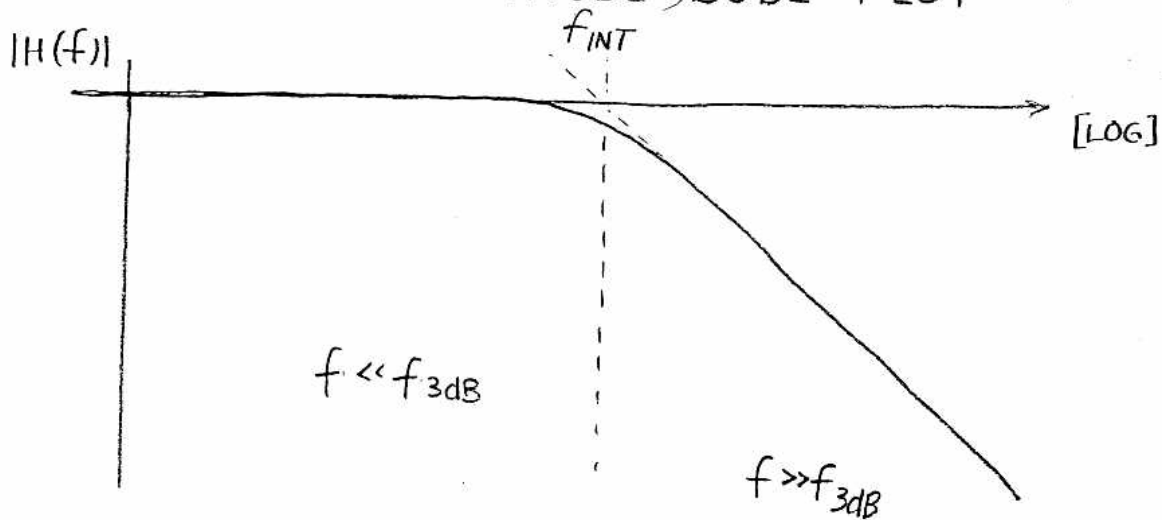
$$H(j\omega) = \frac{20}{1 + j \frac{\omega}{2\pi(3\text{kHz})}}$$

ALTERNATIVE (SINCE $\omega = 2\pi f$)

$$H(jf) = \frac{20}{1 + j \frac{f}{(3\text{kHz})}}$$

ASYMPTOTES OF MAGNITUDE BODE PLOT

⑨



TRANSFER FUNCTION REWRITE

$$|H(f)| = \frac{1}{\sqrt{1 + (f/f_{3dB})^2}}$$

$$f \ll f_{3dB}$$

$$|H(f)| \approx 1$$

$$= 0 \text{ dB}$$

$$f \gg f_{3dB}$$

$$|H(f)| \approx \frac{1}{\sqrt{(f/f_{3dB})^2}}$$

$$\approx \frac{f_{3dB}}{f}$$

IN dB

$$20 \log \left(\frac{f_{3dB}}{f} \right)$$

f INCREASE BY X10 ("DECADE")

log DECREASE BY 1

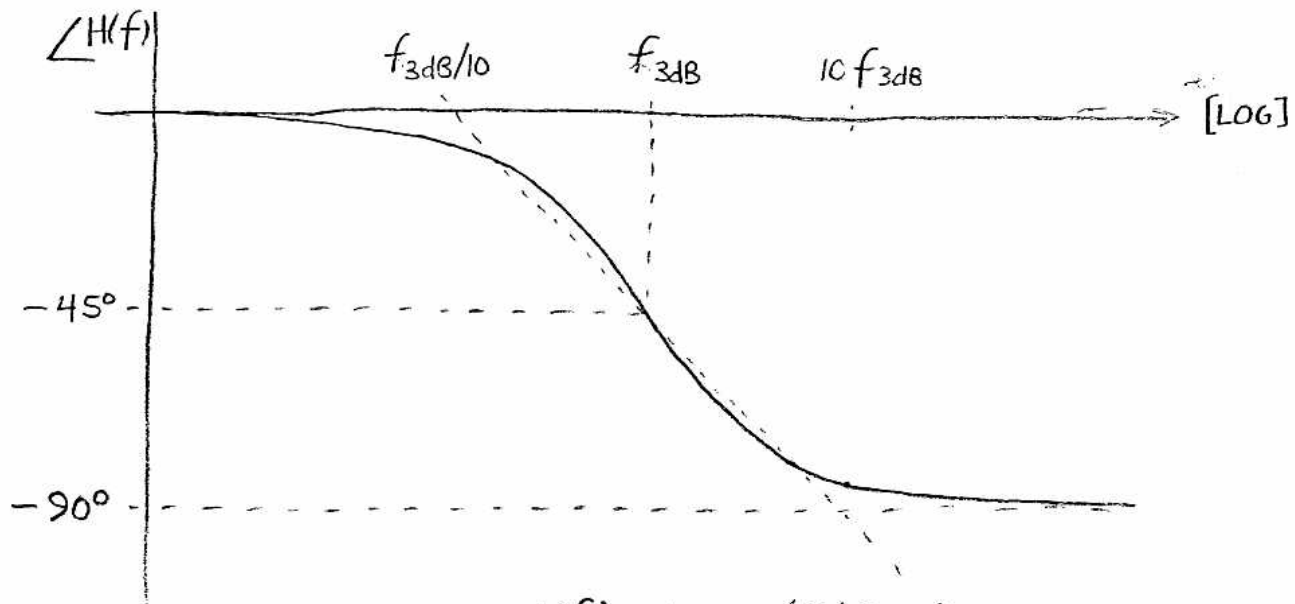
dB "-20 dB PER DECADE"

INTERSECTION

$$1 = \frac{f_{3dB}}{f_{INT}} \Rightarrow f_{INT} = f_{3dB}$$

PHASE BODE PLOT

(10)



$$\angle H(f) = -\tan^{-1}(f/f_{3dB})$$

$$f \ll f_{3dB}$$

$$-\tan^{-1}(0)$$

$$= 0^\circ$$

$$f \gg f_{3dB}$$

$$-\tan^{-1}(\infty)$$

$$= -90^\circ$$

AT 3dB FREQUENCY

$$-\tan^{-1}(f_{3dB}/f_{3dB}) = -\tan^{-1}(1) = -45^\circ$$