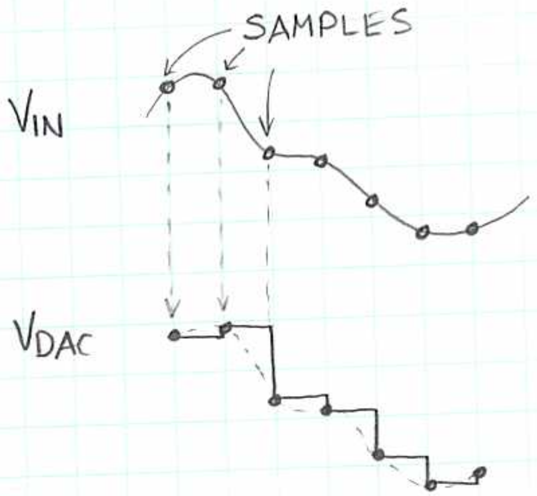
FILTER DESIGN IN THE S PLANE

WHY FILTER?

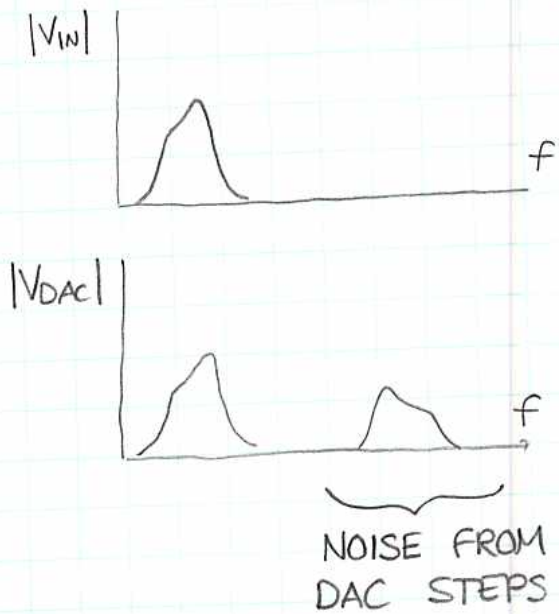
⇓

WHY IN THE S PLANE (REST OF LECTURE)

LOOK AT RECONSTRUCTION OF SIGNAL WITH DAC



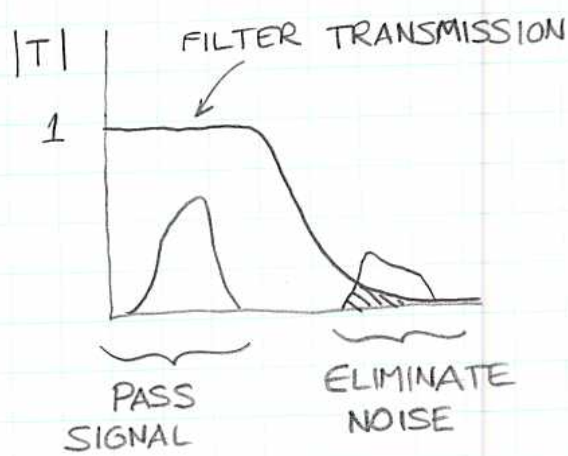
FREQ DOMAIN



TIME DOMAIN INTERPRETATION

NEED TO "SMOOTH OUT" STEPS FROM DAC

FREQ DOMAIN INTERPRETATION



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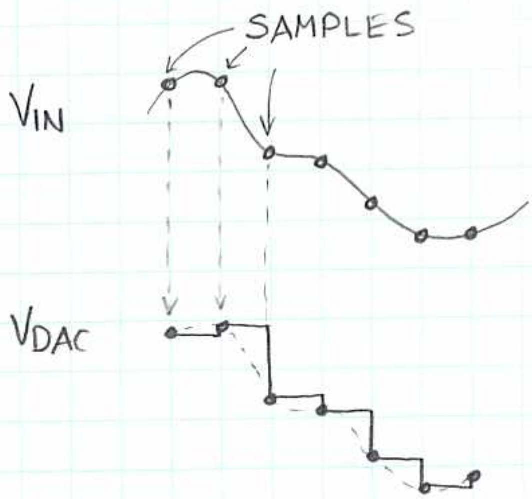

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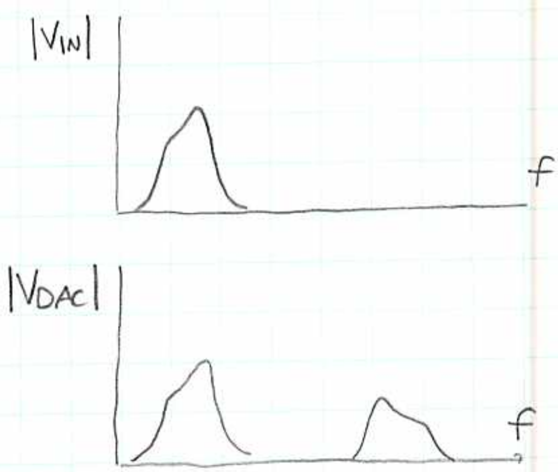
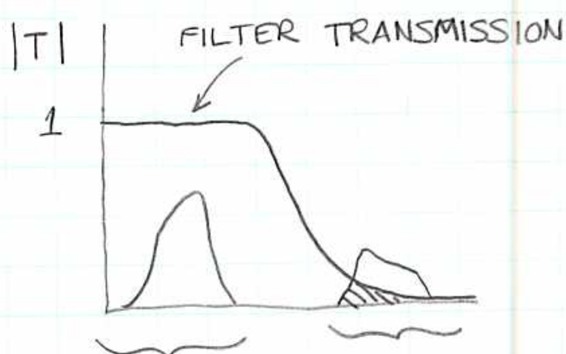


NOISE FROM DAC STEPS

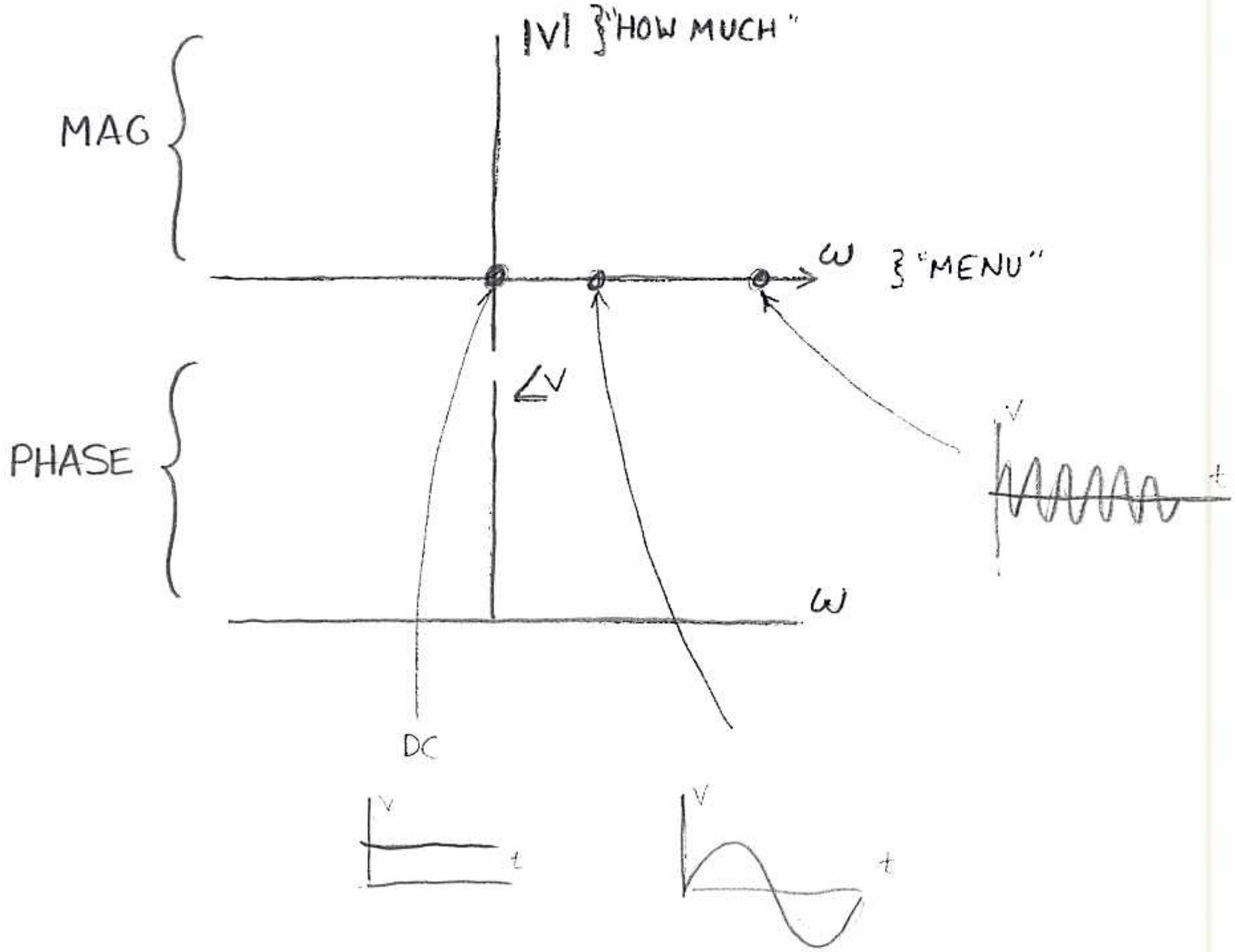
FREQ DOMAIN INTERPRETATION



PASS SIGNAL

ELIMINATE NOISE

FOURIER TRANSFORM: PLOT V AS A FUNCTION OF FREQ ω



ANY WAVEFORM CAN BE EXPRESSED AS A SUM OF SINUSOIDS

ASSUMES LINEARITY

WHY s?

STC EXPONENTIAL RESPONSE $V = V_0 e^{-t/\tau}$

NEED LOTS OF SINES TO MAKE THIS WAVEFORM!

EXPAND "MENU"

$$e^{j\omega t} \rightarrow e^{st}$$

$$s = \sigma + j\omega$$

COMPLEX FREQUENCY s

ALLOWS BOTH REAL, IMAGINARY PARTS
IN FREQUENCY

TRANSFORMS: FOURIER VS. LAPLACE

FOURIER: ω $(2\pi f)$ LAPLACE s $(\sigma + j\omega)$

SUM OF COMPLEX EXPONENTIALS

$V = V_0 e^{j\omega t}$ $V = V_0 e^{st}$

WHY? I-V FOR CAPACITOR EXAMPLE

$$I = C\frac{dV}{dt}$$

CAN WE MAKE THIS AS EASY AS OHM'S LAW $R = \frac{V}{I}$?

$$I = C\frac{d}{dt}\left[V_0 e^{j\omega t}\right] \qquad I = C\frac{d}{dt}\left[V_0 e^{st}\right]$$

$$I = Cj\omega \underbrace{V_0 e^{j\omega t}}_{V} \qquad I = Cs\underbrace{V_0 e^{st}}_{V}$$

$$I = j\omega C\, V \qquad I = sC\, V$$

$$\frac{V}{I} = \frac{1}{j\omega C} \qquad \frac{V}{I} = \frac{1}{sC}$$

FOURIER IS "SUBSET" OF LAPLACE

LET $s = j\omega$

IMPEDANCE LIKE RESISTANCE, BUT COMPLEX

$$Z_R = R \qquad Z_C = \frac{1}{sC} \qquad Z_L = sL$$

FREQUENCY

$$Z_C = \frac{1}{j\omega C} \qquad Z_L = j\omega L$$

S-PLANE: "MAP" OF WAVEFORMS

PLOT REAL, IMAGINARY PARTS

WAVEFORM EXAMPLES

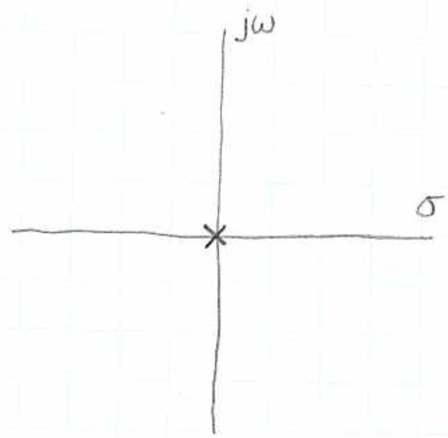
<u>DC</u>

$s = 0$

$e^{st} = e^{0t} = 1$

TIME

S-PLANE

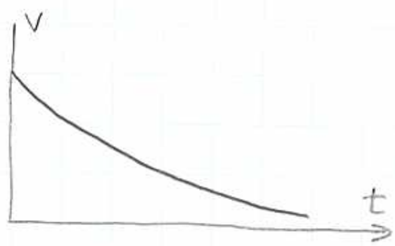


PURE EXPONENTIALS

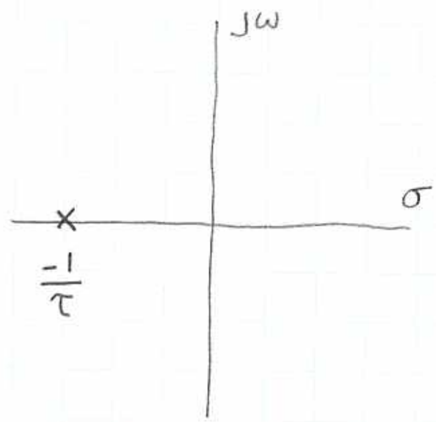
$$s = \frac{-1}{\tau}$$

$$e^{st} = e^{\left(\frac{-1}{\tau}\right)t} = e^{-t/\tau}$$

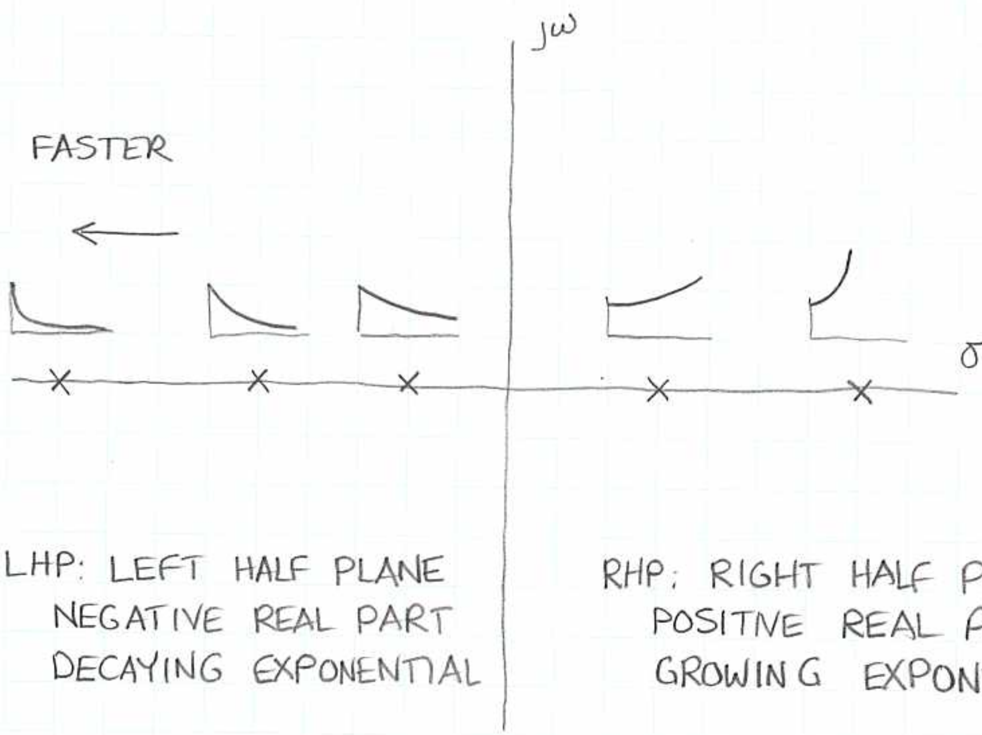
TIME



S-PLANE



REAL AXIS: EXPONENTIALS



LHP: LEFT HALF PLANE
NEGATIVE REAL PART
DECAYING EXPONENTIAL

RHP: RIGHT HALF PLANE
POSITIVE REAL PART
GROWING EXPONENTIAL

PURE SINUSOIDS (COMPLEX CONJUGATE PAIR)

$s_1 = j(+\omega) \qquad s_2 = j(-\omega)$

$e^{s_1 t} = e^{j\omega t} \qquad e^{s_2 t} = e^{-j\omega t}$

EULER RELATIONSHIP (COMPLEX NUMBERS)

$e^{j\theta} = \cos\theta + j\sin\theta$

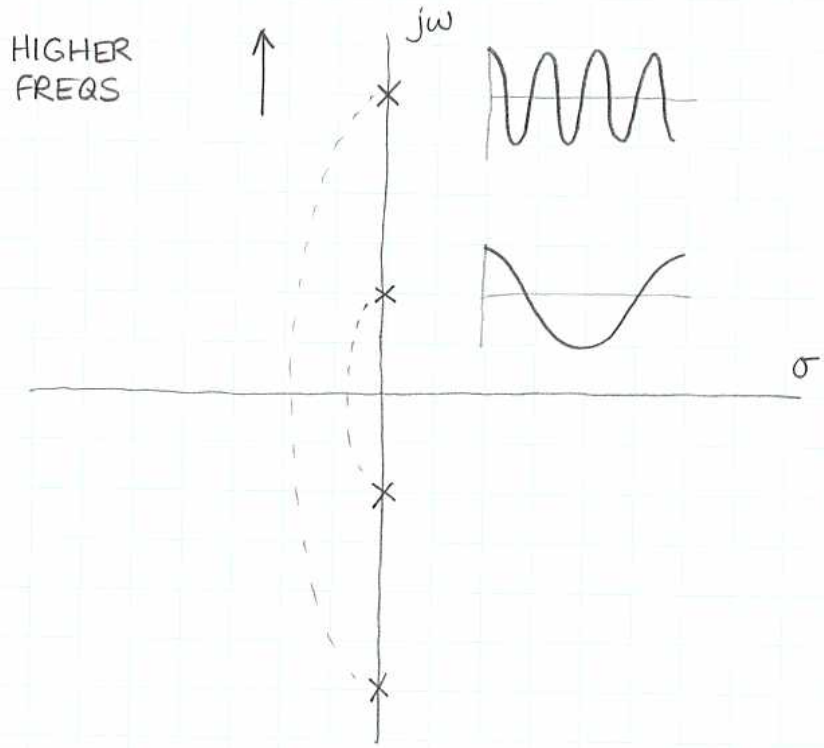
$e^{j\omega t} = \cos\omega t + j\sin\omega t \qquad e^{-j\omega t} = \cos\omega t - j\sin\omega t$

ADD BY SUPERPOSITION

$$e^{s_1 t} + e^{s_2 t} = \cos\omega t + \underbrace{\cancel{j\sin\omega t} + \cos\omega t - \cancel{j\sin\omega t}}_{\text{NEED CONJUGATE TO MAKE IMAGINARY PART GO AWAY}}$$

$$= 2\cos\omega t$$

IMAGINARY AXIS: SINUSOIDS



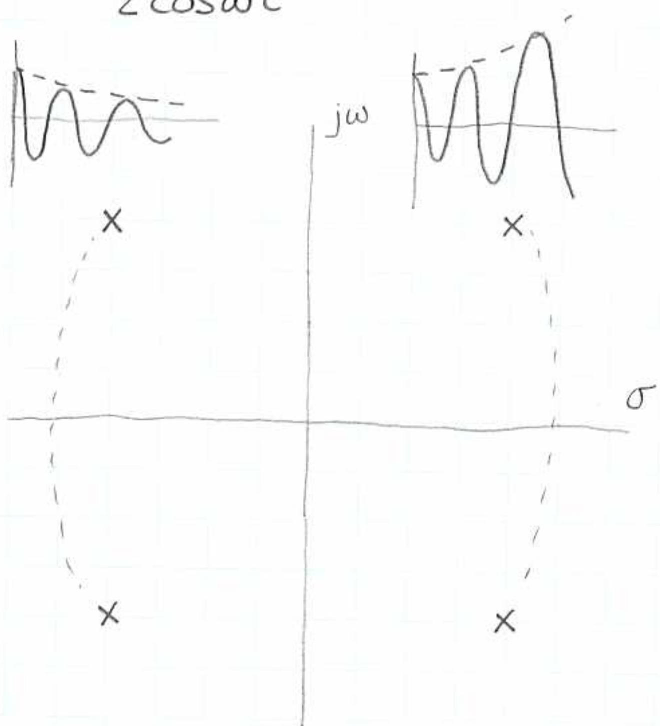
REAL AND IMAGINARY PARTS

$s_1 = \sigma + j\omega$ $\qquad$ $s_2 = \sigma - j\omega$

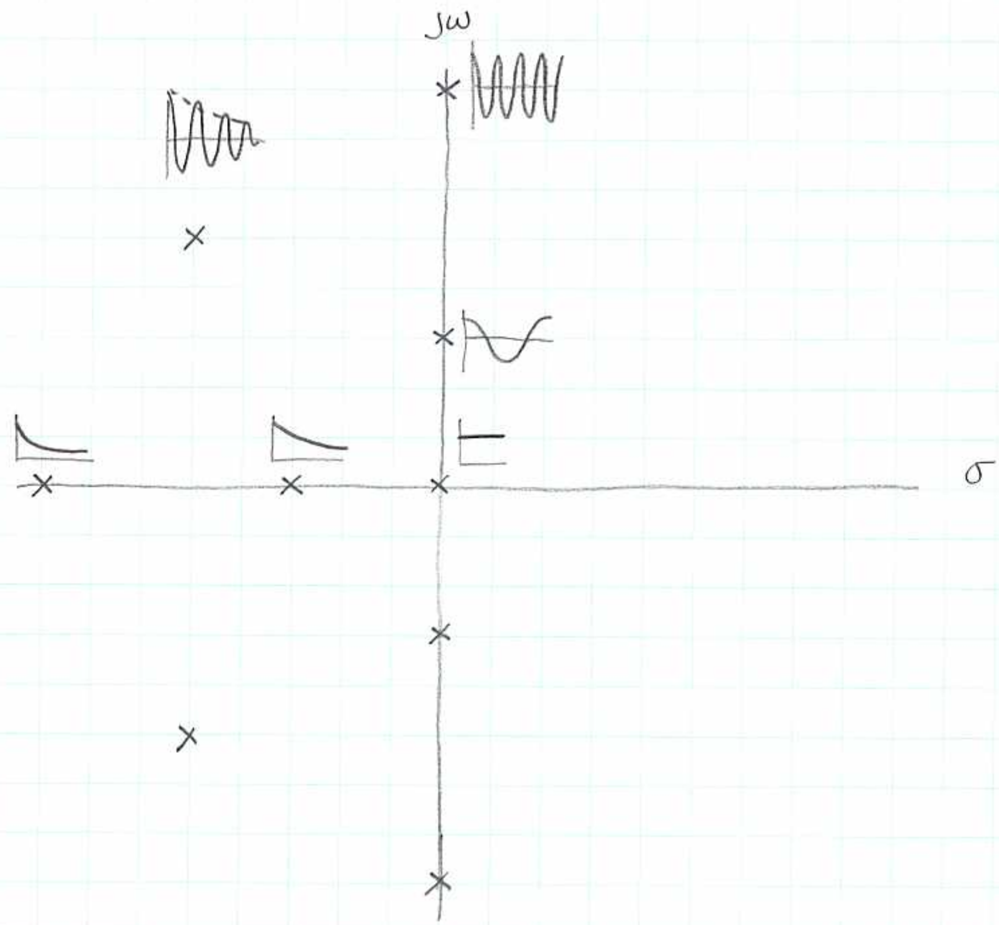
$$e^{s_1 t} = e^{(\sigma + j\omega)t}$$
$$= e^{\sigma t} e^{j\omega t} \qquad = e^{\sigma t} e^{-j\omega t}$$

ADD

$$e^{\sigma t}\underbrace{\left(e^{j\omega t} + e^{-j\omega t}\right)}_{2\cos\omega t}$$



S-PLANE SUMMARY



FORMS OF TRANSFER FUNCTION

$$T(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \cdots + a_0}{s^n + b_{n-1} s^{n-1} + \cdots + b_0}$$

$T(s)$ FILTER TRANSMISSION

$H(s)$ GENERAL TRANSFER FUNCTION

POLYNOMIALS IN S
a_i, b_i REAL NUMBERS

n ORDER OF THE NETWORK

$m < n$ FOR REAL SYSTEM

BETTER FORM FOR TRANSFER FUNCTION:
SHOW ROOTS EXPLICITLY

$$T(s) = \underbrace{K}_{\substack{\text{DC} \\ \text{GAIN} \\ (s=0)}} \frac{\left(1-\frac{s}{z_1}\right)\left(1-\frac{s}{z_2}\right)\cdots\left(1-\frac{s}{z_m}\right)}{\left(1-\frac{s}{p_1}\right)\left(1-\frac{s}{p_2}\right)\cdots\left(1-\frac{s}{p_n}\right)}$$

$p_1, p_2, \ldots, p_n$ POLES: VALUES OF S FOR WHICH DENOMINATOR $\rightarrow 0$
$\Rightarrow |T| \rightarrow \infty$
OUTPUT FOR 0 INPUT
"NATURAL MODE OF A SYSTEM"

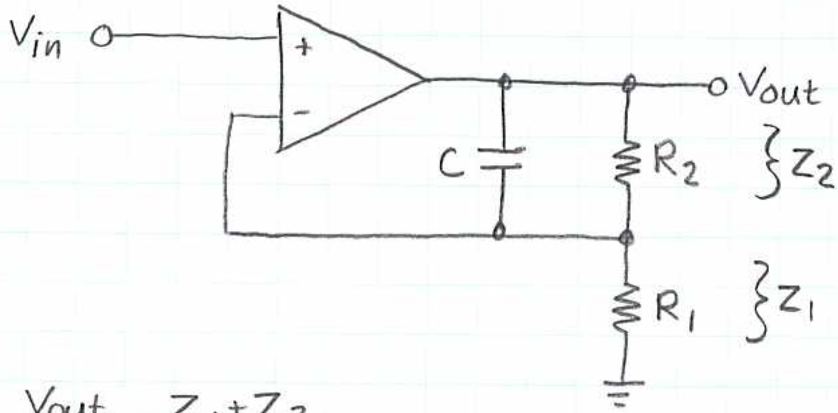
$z_1, z_2, \ldots, z_m$ ZEROS: VALUES OF S FOR WHICH NUMERATOR $\rightarrow 0 \Rightarrow |T| = 0$
NO OUTPUT FOR ANY INPUT

POLES, ZEROS

PLOT THESE SPECIAL VALUES OF S
IN THE S PLANE

ANOTHER WAY TO SPECIFY TRANSFER FUNCTION

EXAMPLE



$$T(s) = \frac{V_{out}}{V_{in}} = \frac{Z_1 + Z_2}{Z_1}$$

$$= \frac{R_1 + \overbrace{\dfrac{R_2}{1 + sR_2C}}^{Z_2}}{R_1}$$

MASSAGE INTO STANDARD FORM

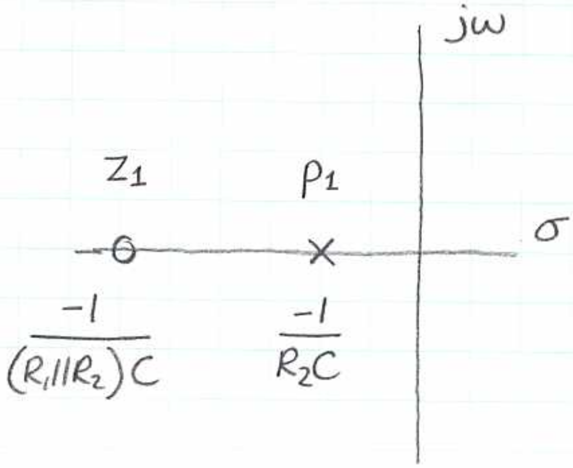
$$= \frac{R_1 + R_2 + sR_1R_2C}{R_1(1 + sR_2C)}$$

KNOW DC GAIN

$$T(s) = \left(\frac{R_1 + R_2}{R_1}\right) \frac{1 + s\overbrace{\left(\dfrac{R_1R_2}{R_1 + R_2}\right)}^{R_1 \| R_2}C}{1 + sR_2C}$$

ZERO: $z_1 = \dfrac{-1}{(R_1 \| R_2)C}$

POLE: $p_1 = \dfrac{-1}{R_2C}$



BODE PLOT (MAGNITUDE)