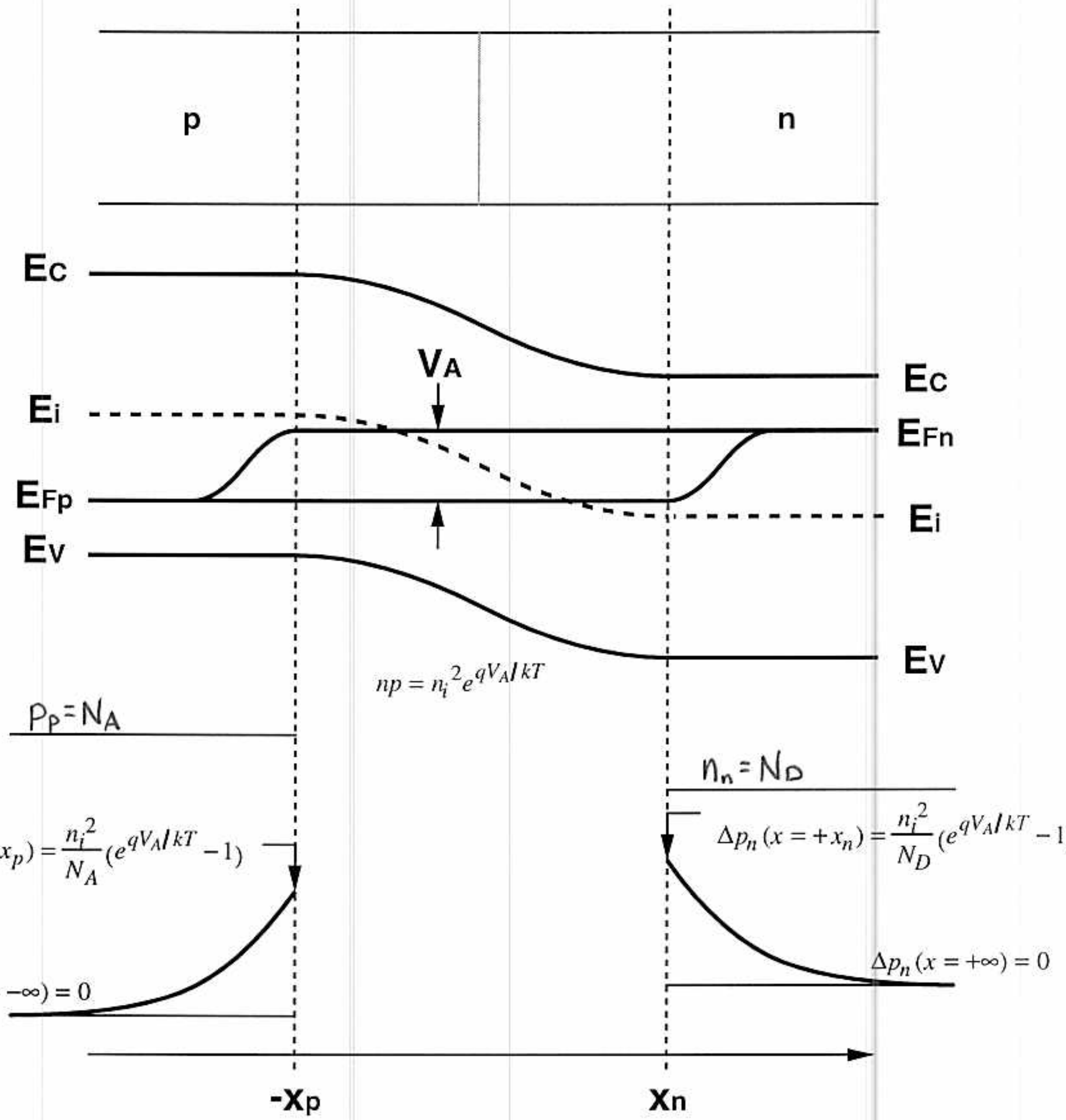
# EE3901 IDEAL DIODE EQUATION DERIVATION

Minority carrier diffusion equations (valid in quasineutral region where E field ≈0)

$$\frac{\partial \Delta n_p}{\partial t} = D_N \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_N} \qquad \frac{\partial \Delta p_n}{\partial t} = D_P \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_P}$$

Steady state (DC voltage, current: all d/dt = 0)

$$0 = D_N \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_N} \qquad 0 = D_P \frac{d^2 \Delta p_n}{dx^2} - \frac{\Delta p_n}{\tau_P}$$




"LAW OF THE JUNCTION" at edges of depletion region (assumes quasiFermi levels flat through depletion region)

$$np = n_i^2 e^{qV_A/kT}$$

Boundary conditions at edges of depletion region (from law of the junction)

$$\Delta n_p(x = -x_p) = \frac{n_i^2}{N_A}(e^{qV_A/kT} - 1) \qquad \Delta p_n(x = +x_n) = \frac{n_i^2}{N_D}(e^{qV_A/kT} - 1)$$

Boundary conditions far from depletion region (returns to equilibrium conc)

$$\Delta n_p(x = -\infty) = 0 \qquad \Delta p_n(x = +\infty) = 0$$

Solutions to differential equation subject to boundary conditions

$$\Delta n_p = \frac{n_i^2}{N_A}(e^{qV_A/kT} - 1)e^{(x+x_p)/L_N} \quad (x \le -x_p) \qquad \Delta p_n = \frac{n_i^2}{N_D}(e^{qV_A/kT} - 1)e^{-(x-x_n)/L_P} \quad (x \ge +x_n)$$

Current density equation

$$J_N = qD_N \frac{d\Delta n_p}{dx} \qquad J_P = -qD_P \frac{d\Delta p_n}{dx}$$

lug differential equation solution into current density equation

$$J_N = q\frac{D_N}{L_N}\frac{n_i^2}{N_A}(e^{qV_A/kT} - 1)e^{(x+x_p)/L_N} \quad (x \le -x_p) \qquad J_P = q\frac{D_P}{L_P}\frac{n_i^2}{N_D}(e^{qV_A/kT} - 1)e^{-(x-x_n)/L_P} \quad (x \ge +x_n)$$

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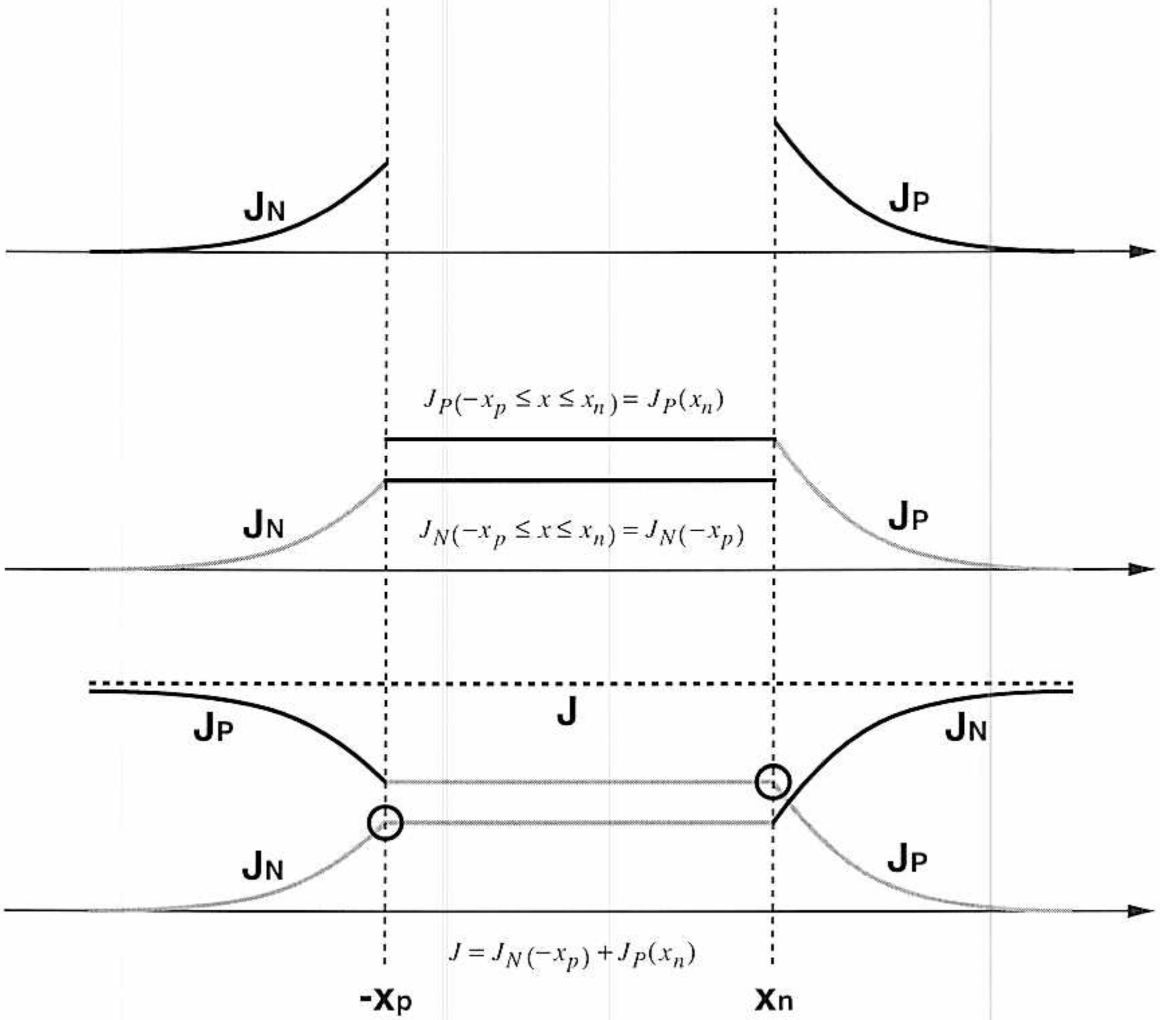
Plug differential equation solution into current density equation

Assume no R-G in depletion region

Steady state: total J constant throughout

J given by sum of hole, electron components at edges of depletion region




Evaluate current density expressions at edges of depletion region

$$J_N(-x_p) = q\frac{D_N}{L_N}\frac{n_i^2}{N_A}\left(e^{qV_A/kT}-1\right) \qquad J_P(x_n) = q\frac{D_P}{L_P}\frac{n_i^2}{N_D}\left(e^{qV_A/kT}-1\right)$$

Total current density

$$J = q\left[\frac{D_N}{L_N}\frac{n_i^2}{N_A} + \frac{D_P}{L_P}\frac{n_i^2}{N_D}\right]\left(e^{qV_A/kT}-1\right)$$

Total current I=JA
Current density x junction area A

$$\boxed{I = qA\left[\frac{D_N}{L_N}\frac{n_i^2}{N_A} + \frac{D_P}{L_P}\frac{n_i^2}{N_D}\right]\left(e^{qV_A/kT}-1\right)}$$