

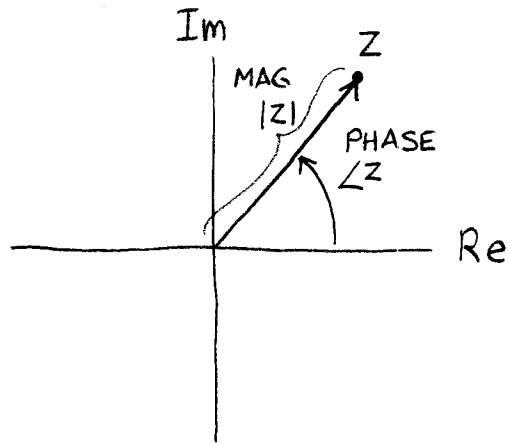
COMPLEX NUMBER REFRESHER

$$z = \frac{a + jb}{x + jy}$$

MAGNITUDE

$$|z| = \frac{\sqrt{a^2 + b^2}}{\sqrt{x^2 + y^2}}$$

$$|z| = \frac{|\text{NUM}|}{|\text{DEN}|}$$



PHASE

$$\angle z = \tan^{-1}(b/a) - \tan^{-1}(y/x)$$

$$\angle z = \angle \text{NUM} - \angle \text{DEN}$$

MAGNITUDE, PHASE OF STC LOWPASS

$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

$$|H(j\omega)| = \frac{\sqrt{1^2 + 0^2}}{\sqrt{1^2 + (\omega RC)^2}} = \frac{1}{\sqrt{1 + (\omega RC)^2}} = \frac{1}{\sqrt{1 + (2\pi f RC)^2}}$$

$$\begin{aligned}\angle H(j\omega) &= \tan^{-1}(0/1) - \tan^{-1}(\omega RC/1) = -\tan^{-1}(\omega RC) \\ &= -\tan^{-1}(2\pi f RC)\end{aligned}$$

EXAMPLE: $R = 7.5 \text{ k}\Omega$
 $C = 1000 \text{ pF}$

$$RC = 7.5 \mu\text{sec}$$

f	MAGNITUDE $ H(f) $	PHASE $\angle H(f)^\circ$	
1 Hz	1	-0.003°	0
10 Hz	1	-0.03°	0
100	0.9999	-0.27°	0
1 kHz	0.9989	-2.7°	-0.01 dB
10 kHz	0.9046	-25.2°	-0.87 dB
100 kHz	0.2076	-78.0°	-13.7 dB
1 MHz	0.0212	-88.8°	-33.5 dB
10 MHz	0.0021	-89.9°	-53.5 dB

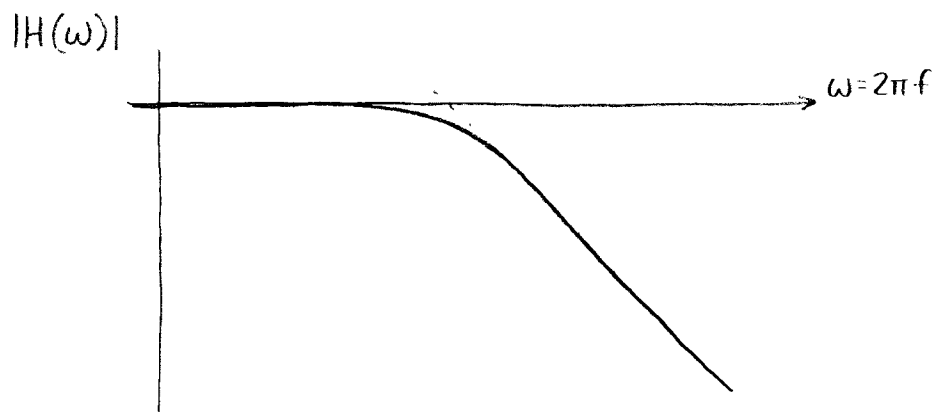
}

-20 dB
PER
DECADE f

PLOT (LINEAR SCALE)



LOG/LOG SCALE



"BODE PLOT": USE dB

$$\underbrace{\left(\frac{V_2}{V_1}\right)}_{\text{VOLTAGE RATIO}} \text{ dB} = 20 \log\left(\frac{V_2}{V_1}\right)$$

EXAMPLE: $A = 50,000$

$$20 \log(A) = 20 \times 4.7 = +94 \text{ dB}$$

"3dB FREQUENCY"

$$20 \log\left(\frac{1}{\sqrt{2}}\right) = -3 \text{ dB}$$

$$\frac{1}{\sqrt{2}} = 0.707$$

$|H(\omega)|$ HAS DROPPED TO $0.707 \times$ MAXIMUM VALUE

MAXIMUM VALUE = 1

$$|H(\omega_{3\text{dB}})| = \frac{1}{\sqrt{2}}$$

SOLVE

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + (\omega_{3\text{dB}} RC)^2}}$$

$$2 = 1 + (\omega_{3\text{dB}} RC)^2$$

$$1 = (\omega_{3\text{dB}} RC)^2$$

$$1 = \omega_{3\text{dB}} RC$$

$$\boxed{\omega_{3\text{dB}} = \frac{1}{RC}}$$

SINCE $\omega = 2\pi f$

$$2\pi f_{3\text{dB}} = \frac{1}{RC}$$

$$\boxed{f_{3\text{dB}} = \frac{1}{2\pi RC}}$$

GENERAL FORM OF TRANSFER FUNCTION

$$H(j\omega) = \frac{[\text{DC GAIN}]}{1 + j \frac{\omega}{[\text{3dB FREQUENCY } \omega]}}$$

$$H(j\omega) = \frac{1}{1 + \frac{\omega}{RC}} = \frac{1}{1 + j \frac{\omega}{1/RC}} \quad \omega_{3dB}$$

EXAMPLE

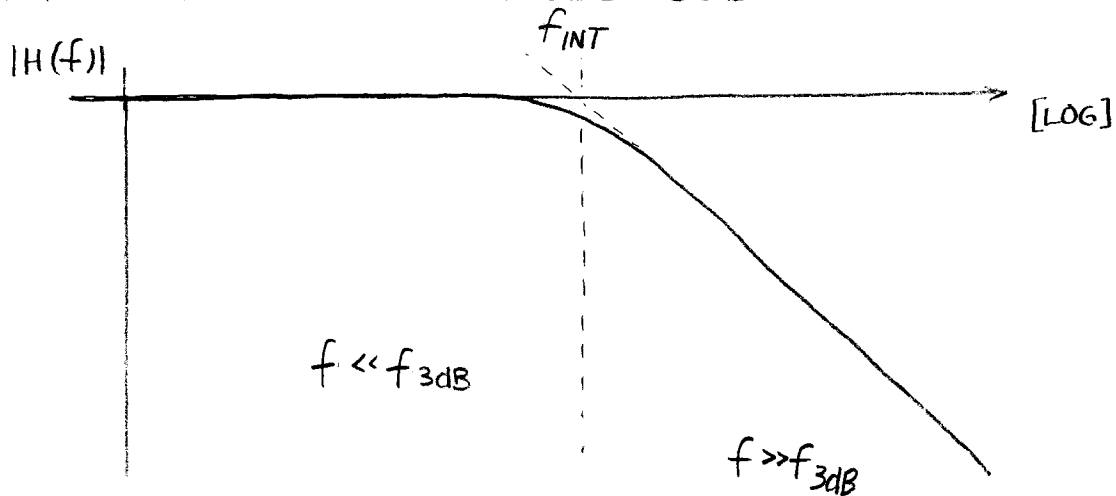
"A STC LOWPASS HAS A DC GAIN OF 20, AND A 3dB FREQUENCY OF 3 KHz. WHAT IS $H(s)$?"

$$H(j\omega) = \frac{20}{1 + j \frac{\omega}{2\pi(3\text{kHz})}}$$

ALTERNATIVE (SINCE $\omega = 2\pi f$)

$$H(jf) = \frac{20}{1 + j \frac{f}{(3\text{kHz})}}$$

ASYMPTOTES OF MAGNITUDE BODE PLOT



TRANSFER FUNCTION REWRITE

$$|H(f)| = \frac{1}{\sqrt{1 + (f/f_{3dB})^2}}$$

$$f \ll f_{3dB}$$

$$|H(f)| \approx 1$$

$$= 0 \text{ dB}$$

$$f \gg f_{3dB}$$

$$|H(f)| \approx \frac{1}{\sqrt{(f/f_{3dB})^2}}$$

$$\approx \frac{f_{3dB}}{f}$$

IN dB

$$20 \log \left(\frac{f_{3dB}}{f} \right)$$

f INCREASE BY X10 ("DECADE")

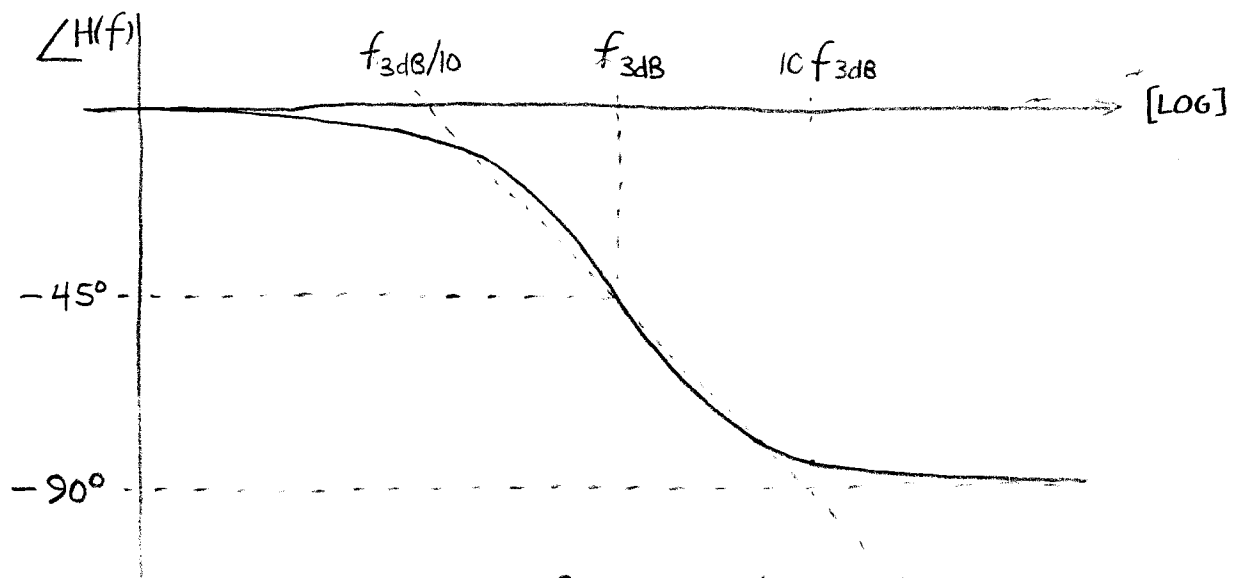
log DECREASE BY 1

dB "-20 dB PER DECADE"

INTERSECTION

$$1 = \frac{f_{3dB}}{f_{INT}} \Rightarrow \boxed{f_{INT} = f_{3dB}}$$

PHASE BODE PLOT



$$\angle H(f) = -\tan^{-1}(f/f_{3dB})$$

$$f \ll f_{3dB}$$

$$-\tan^{-1}(0)$$

$$= 0^\circ$$

$$f \gg f_{3dB}$$

$$-\tan^{-1}(\infty)$$

$$= -90^\circ$$

AT 3dB FREQUENCY

$$-\tan^{-1}(f_{3dB}/f_{3dB}) = -\tan^{-1}(1) = -45^\circ$$