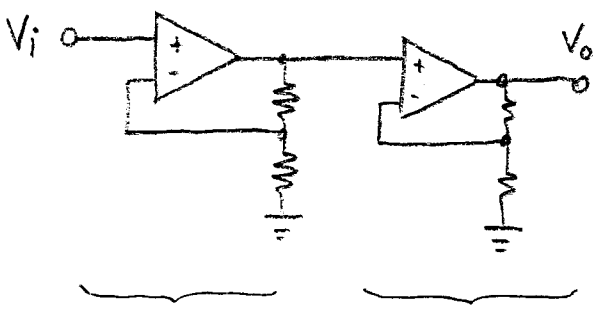
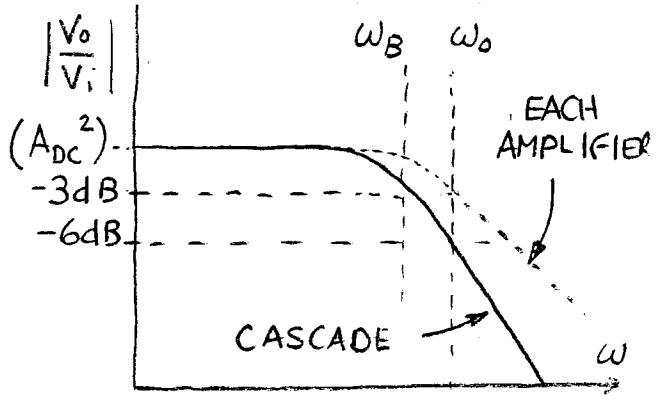
BANDWIDTH OF CASCADED STAGES:

Consider identical noninteracting single pole amplifiers in cascade





$$\frac{A_{DC}}{1+j\left(\frac{\omega}{\omega_o}\right)} \qquad \frac{A_{DC}}{1+j\left(\frac{\omega}{\omega_o}\right)}$$

Each individual amplifier's magnitude response is 3dB down at $\omega=\omega_o$

$\Rightarrow$ Cascade response will be 6dB down at ω_o

$\Rightarrow$ What is the -3dB frequency of the total response of the cascade?

Call this ω_B and solve for it

$$\frac{A_{DC}^2}{\sqrt{2}} = \frac{A_{DC}}{\sqrt{1+\left(\frac{\omega_B}{\omega_o}\right)^2}} \frac{A_{DC}}{\sqrt{1+\left(\frac{\omega_B}{\omega_o}\right)^2}} \Rightarrow \sqrt{2} = 1+\left(\frac{\omega_B}{\omega_o}\right)^2$$

$$\left(\frac{\omega_B}{\omega_o}\right)^2 = \sqrt{2}-1 \Rightarrow \omega_B = \sqrt{0.4142\ldots}\,\omega_o \Rightarrow \boxed{\omega_B = 0.64\,\omega_o}$$

IN GENERAL

$$\underbrace{\frac{A_1}{1+j\left(\frac{\omega}{\omega_1}\right)}}_{A_1} \quad \underbrace{\frac{A_2}{1+j\left(\frac{\omega}{\omega_2}\right)}}_{A_2}$$

Solve for ω_B of cascade

$$\frac{A_1 A_2}{\sqrt{2}} = \frac{A_1}{\sqrt{1+\left(\frac{\omega_B}{\omega_1}\right)^2}} \frac{A_2}{\sqrt{1+\left(\frac{\omega_B}{\omega_2}\right)^2}} \Rightarrow 2 = \left(1+\left(\frac{\omega_B}{\omega_1}\right)^2\right)\left(1+\left(\frac{\omega_B}{\omega_2}\right)^2\right)$$

Multiply out polynomial $\Rightarrow$ Can solve exactly using quadratic, but since we know $\omega_B < \omega_1$ and $\omega_B < \omega_2$

$$1 = \left(\frac{\omega_B}{\omega_1}\right)^2 + \left(\frac{\omega_B}{\omega_2}\right)^2 + \underbrace{\frac{\omega_B^4}{\omega_1^2\omega_2^2}}$$

assume this term negligible

$$\left(\frac{\omega_B}{\omega_1}\right)^2 + \left(\frac{\omega_B}{\omega_2}\right)^2 \approx 1 \Rightarrow \boxed{\omega_B^2 \approx \frac{1}{\frac{1}{\omega_1^2} + \frac{1}{\omega_2^2}}}$$

Compare approximation to exact:

ω_2/ω_1	ω_B EXACT	ω_B APPROX	APPROX OVERSTATES BANDWIDTH BY
1	0.64	0.71	10%
2	0.84	0.89	6%
3	0.91	0.95	4%
5	0.96	0.98	2%
10	0.989	0.999	1%