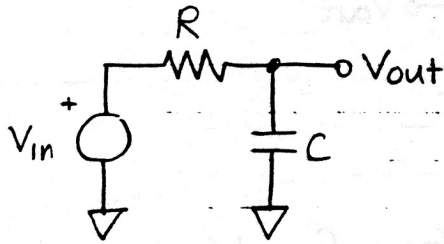


BANDWIDTH AND RISE TIME

Consider the single pole network



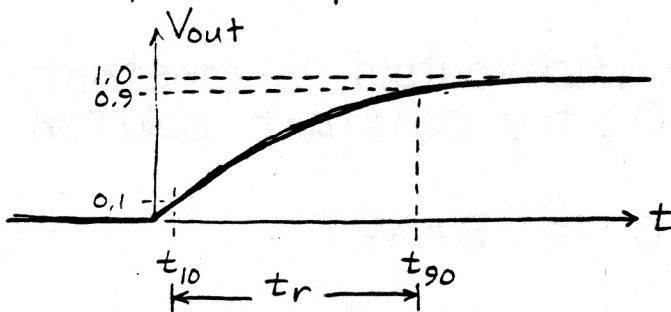
$$\frac{V_{out}}{V_{in}} = H(s) = \frac{1}{1+sRC} = \frac{1}{1+s\tau}$$

Letting $s=j\omega$, we have

$$H(j\omega) = \frac{1}{1+j\omega\tau} = \frac{1}{1+j\left(\frac{\omega}{\omega_0}\right)}$$

where $\omega_0 = 1/\tau$ is the 3dB bandwidth, in radians/sec

The "rise time" t_r is defined as the time required for the output to rise from 10% to 90% of the output step:



From general step response

$$\left. \begin{aligned} 0.1 &= 1.0 - e^{-t_{10}/\tau} \Rightarrow t_{10} = -\tau \ln(0.9) \\ 0.9 &= 1.0 - e^{-t_{90}/\tau} \Rightarrow t_{90} = -\tau \ln(0.1) \end{aligned} \right\} t_r = t_{90} - t_{10} = \tau \ln(9)$$

$$\text{Or, } \boxed{t_r = 2.2\tau}$$

Since $\tau = 1/\omega_0$, and $f_0 = \omega_0/2\pi = \text{BW}$ (3dB bandwidth in Hz)

$$t_r = 2.2 \frac{1}{\omega_0} = \frac{1}{\text{BW}} \frac{2.2}{2\pi} \Rightarrow \boxed{\text{BW} \cdot t_r = 0.35}$$

Approximately valid for any linear system with poles near real axis (little overshoot in step response)