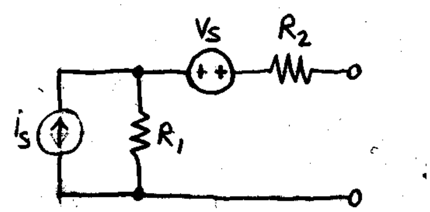
THÉVENIN'S THEOREM

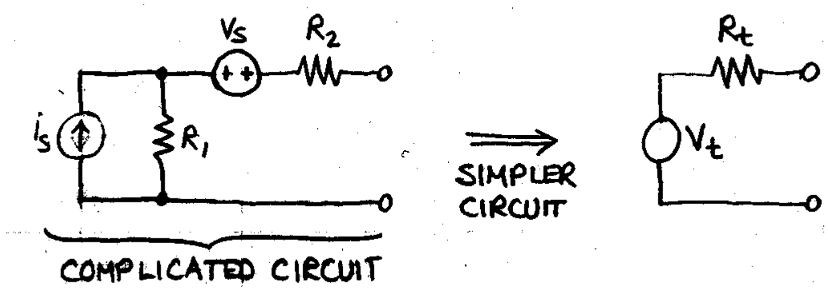
Often, we are interested in what is happening at a particular place in a circuit, and don't need all of the details about the rest of the circuit. Thévenin's Theorem says that we can replace the rest of the circuit by a very simple Equivalent Circuit as long as the circuit contains only resistances and sources. (Actually, it works for inductors and capacitors, too, but don't worry about that now.)

Say we have a circuit composed of R's and sources, and want to determine a simpler circuit which exhibits the same behavior at some pair of nodes in the circuit:



This is called a one-port network. The reason we are interested in a port consisting of two nodes is that we would like to know how the circuit will behave if we attach something to the nodes (such as a resistance.) For example, you might think of your stereo system as the circuit, with the wires to one of the speakers as the port of two nodes. Then, the speaker is the thing we would connect to the nodes. Obviously, the stereo is a complicated circuit, but maybe we could find a simple Equivalent Circuit which behaves the same as far as the speaker is concerned.

The Thévenin Equivalent Circuit consists of the following:



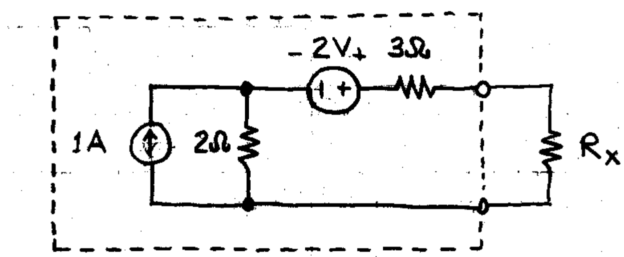
- Vt is the Thévenin Voltage;
- Rt is the Thévenin Resistance

Thevenin's theorem says that, if you choose Vt and Rt correctly, the simple circuit of one source and one resistor will have the exact same V-I characteristic as the more complicated circuit.

How do you choose Vt and Rt correctly? Usually the simplest way is to use the following procedure to find:

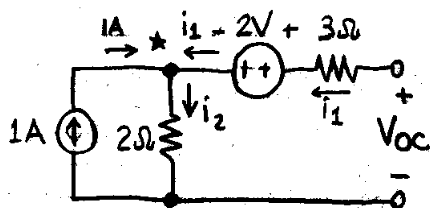
- Vt = Voc the open-circuit voltage measured at the terminals of the port;
- Rt = Req the equivalent resistance between the port terminals with all sources set to zero.
  (Recall: to set a voltage source to zero, replace it with a short circuit; to set a current source to zero, replace it with an open circuit.)

Example: Suppose we want to know what will happen when we connect a resistor of value Rx to the network in the dotted lines. With the full circuit, this becomes a tedious exercise in nodal analysis etc., which we would need to repeat for every different value of Rx. If all we care about is what Rx "sees", then we don't need all the information on the internal Vs and Is inside the dotted lines. We get a much simpler situation if we replace the circuit in the dotted lines by its Thévenin Equivalent.



To find the Thevenin equivalent:

- First, we remove Rx. Now, we want to
- Find Voc, the open circuit voltage.



Taking a KVL loop through the 2Ω resistor, the 2V source, and the 3Ω resistor:

$(i_2)(2\Omega) + 2V + (i_1)(3\Omega) - Voc = 0$ [1]

and KCL at the starred node gives

$1A + i_1 = i_2$ [2]

One nice thing about calculating the open-circuit voltage for the Thevenin procedure is that the condition of an open circuit at the output simplifies the analysis. Since the output is an open circuit, then we know current $i_1 = 0$ and [2] simplifies to

$1A = i_2$ [3]

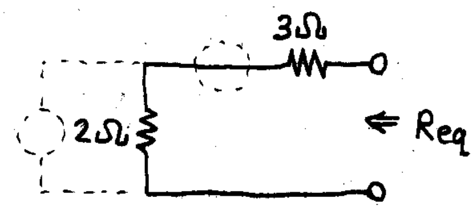
Substituting $i_1 = 0$ and $i_2 = 1A$ into [1] gives

$(1A)(2\Omega) + 2V + (0)(3\Omega) = V_{oc}$

$V_{oc} = +4V$

- Find Rt:

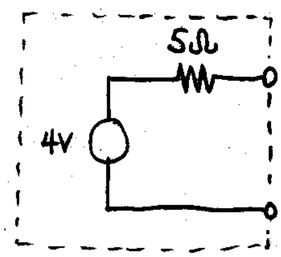
Turning off the current source results in an open circuit; turning off the voltage source results in a short circuit:



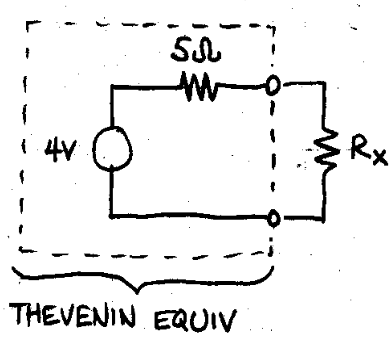
The equivalent resistance is just the series combination of

$R_{eq} = 2\Omega + 3\Omega = 5\Omega$

So, the Thévenin Equivalent with Vt = Voc and Rt = Req is:



Now we can hook Rx back up, and the resulting circuit is easier to work with.



For example, if we want to know the voltage Vx across Rx, we can now use the voltage divider relationship to get an expression that gives Vx for any Rx:

$$V_X = 4V\left(\frac{R_X}{5\Omega + R_X}\right)$$

Which is the same as we would have gotten from grinding through the (much more complicated) math for the complete original (much more complicated) circuit.