ECE3204 Lecture 20

Differentiator

Stability analysis (Ch. 10)

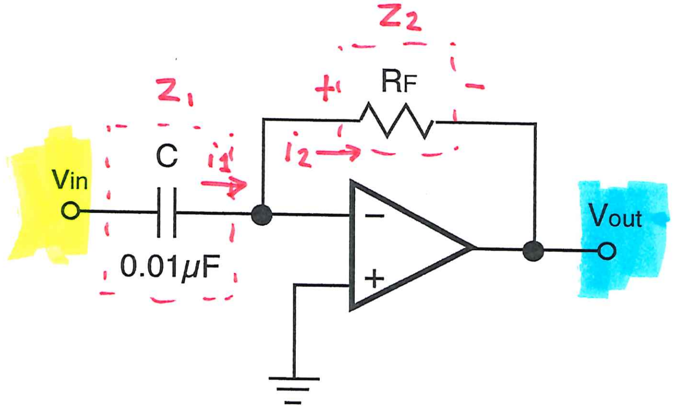
Handouts:

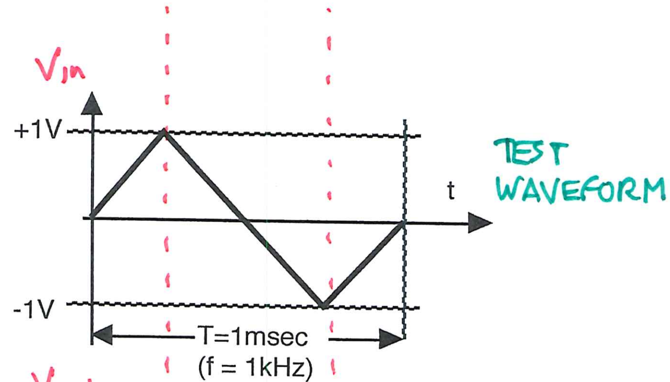
Ch. 10 Reading Guide

Differentiator

P1. Differentiator

The basic differentiator topology is shown in Figure 6.1. This topology is rarely (if ever) used in practice - you will see why when you build this circuit in the lab.

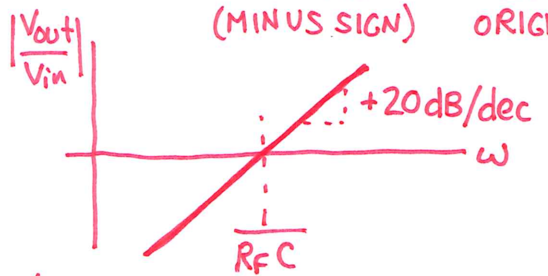


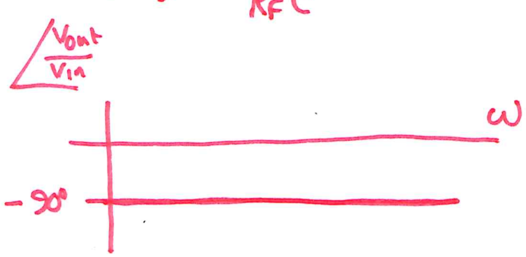


FREQ DOMAIN:

$$\frac{V_{out}}{V_{in}} = \frac{-Z_2}{Z_1} = \frac{-R_F}{1/sC} = -\underbrace{sR_FC}_{\text{ZERO AT ORIGIN}}$$

180° PHASE (MINUS SIGN)





TIME DOMAIN:

KCL AT − INPUT:

$i_1 = i_2$

VIRTUAL GROUND:

$V_{out} = -i_2 R_F$

CAPACITOR

$i_1 = C\dfrac{dV_{in}}{dt}$

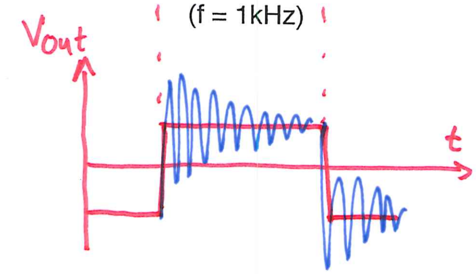
SUBSTITUTE

$$\boxed{V_{out} = -\underbrace{R_FC}_{sec}\underbrace{\frac{dV_{in}}{dt}}_{V/sec}}$$

EXPECTED

OUTPUT

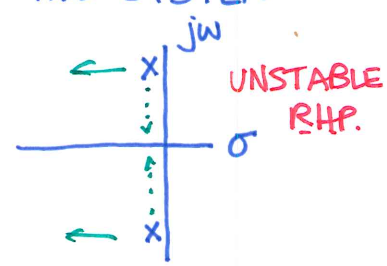
ACTUAL !?



WHY?

POLES OF THIS SYSTEM

HOW TO MAKE POLES MORE STABLE?



ASSUMED $V_- = V_+$ WITH $\underset{\sim}{A}$ LARGE

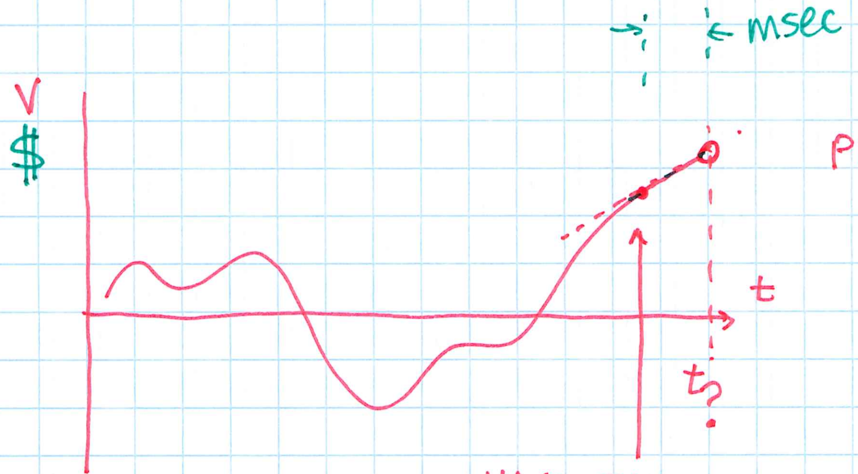
A IS REALLY A(ω)

NOT LARGE FOR ALL ω (GBWP)

∠A(ω) ≠ 0 FOR ALL ω!

WHY CARE ABOUT DERIVATIVE ANYWAY?

TIME PLOT OF WAVEFORM

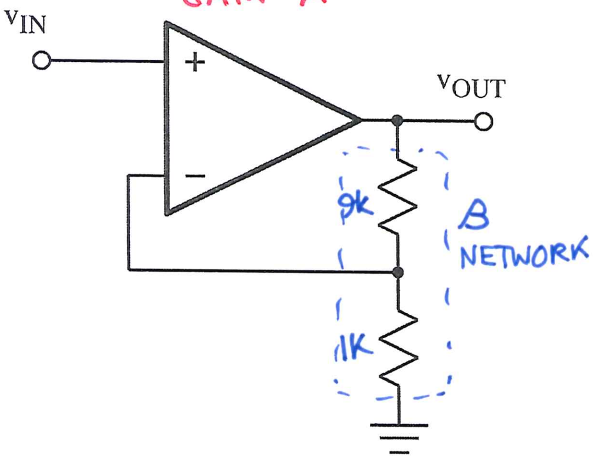


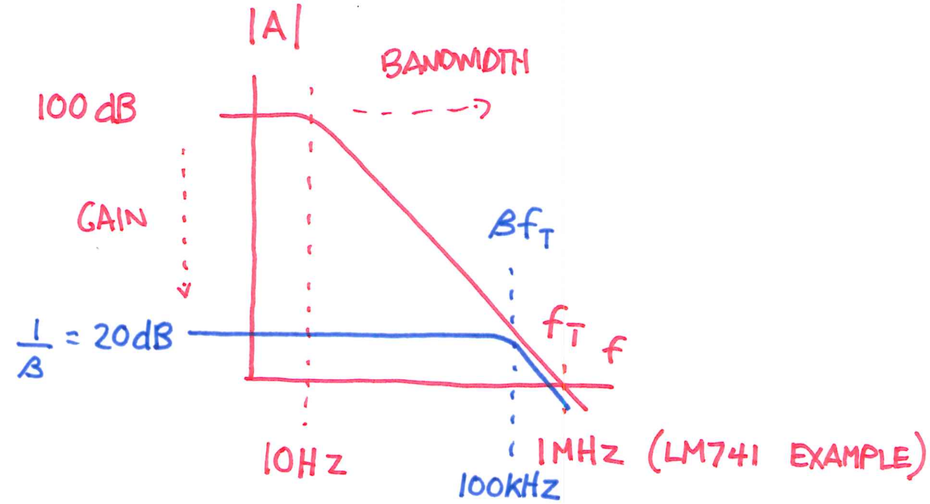
PREDICT: NEED $\frac{dv}{dt}$

HAVE THIS VALUE... WHERE WILL IT BE AT $t_?$ IN THE FUTURE?

Feedback Changes Pole Locations

OPEN LOOP
GAIN A





$$V_{out} = A(V_{in} - \beta V_{out})$$

$$V_{out}(1 + A\beta) = A(V_{in})$$

$$\frac{V_{out}}{V_{in}} = \frac{A}{1 + A\beta}$$ CLOSED LOOP TRANSFER FUNCTION

SIMPLIFY IF $A\beta \gg 1$

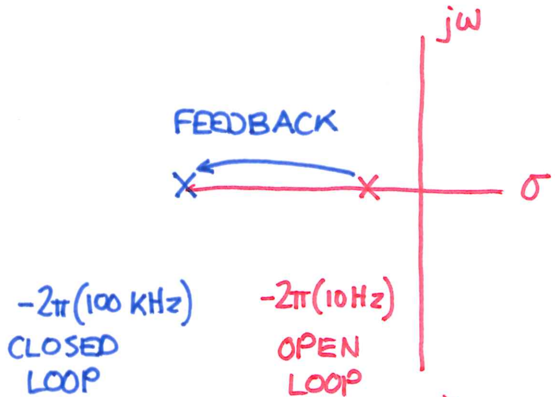
$$\frac{V_{out}}{V_{in}} = \frac{\cancel{A}}{\cancel{1} + \cancel{A}\beta} = \frac{1}{\beta}$$

$\underbrace{\beta = 0.1}_{\text{EXAMPLE}}$

CHOOSE $R_2 = 9K$
$R_1 = 1K$

WANTED CLOSED LOOP GAIN = 10

S-PLANE PLOT FOR $A(f)$



WHAT IF OUR FEEDBACK MOVES POLES INTO RHP!?

BAD!

SYSTEM IS UNSTABLE

Closed Loop Transfer Function

$$\underbrace{\frac{V_{out}}{V_{in}}}_{\text{CLOSED LOOP TRANSFER FUNCTION}} = \frac{A}{1+A\beta} \xRightarrow{\text{REALLY}} \frac{A(s)}{\underbrace{1 + A(s)\,\beta(s)}_{\text{BOTH } A,\beta \text{ ARE TRANSFER FUNCTIONS}}}$$

WHERE ARE THE POLES?

ROOTS OF DENOMINATOR POLYNOMIAL

$$1 + A(s)\,\beta(s) = 0$$

TO DO THIS

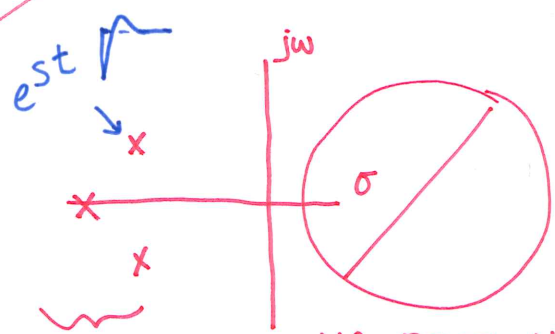
~~MULTIPLY OUT POLYNOMIAL~~

~~FIND ALL ROOTS $S_{p1}, S_{p2}, \ldots$~~

~~PLOT IN S - PLANE~~

TOO MUCH INFORMATION

MODE OF NATURAL RESPONSE



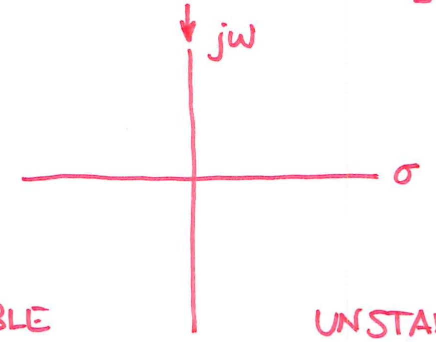
NO POLES HERE!

ONLY CARE THAT WE ARE IN L.H.P.

EXACT LOCATION NOT CRITICAL

Do I HAVE TO find all exact pole locations? NO!

JUST LOOK ON BOUNDARY ON jw AXIS



ANY POLES ON THE BOUNDARY?

$$1 + A(j\omega)\,\beta(j\omega) = 0$$

S = jw LOOK ON jw AXIS?

IS THIS REALLY EASIER?

RESTATE AS $\underbrace{A(j\omega)\beta(j\omega) = -1}_{\text{BODE PLOT!}}$

$$|A(\omega)|\,|\beta(\omega)| = 1$$

$$\underbrace{\angle A(\omega) + \angle\beta(\omega) = -180^\circ}_{\text{CAN WORK WITH BODE PLOTS OF } A,\ \beta}$$