ECE3204 Lecture 19

Filter Design

Exam 2 Preview

Hand in: HW 5

Handouts:
Sallen & Key Circuit
Butterworth Examples

Filter Type and Specification (S&S Ch. 16 [6th edition]) IDEAL "BRICK WALL"

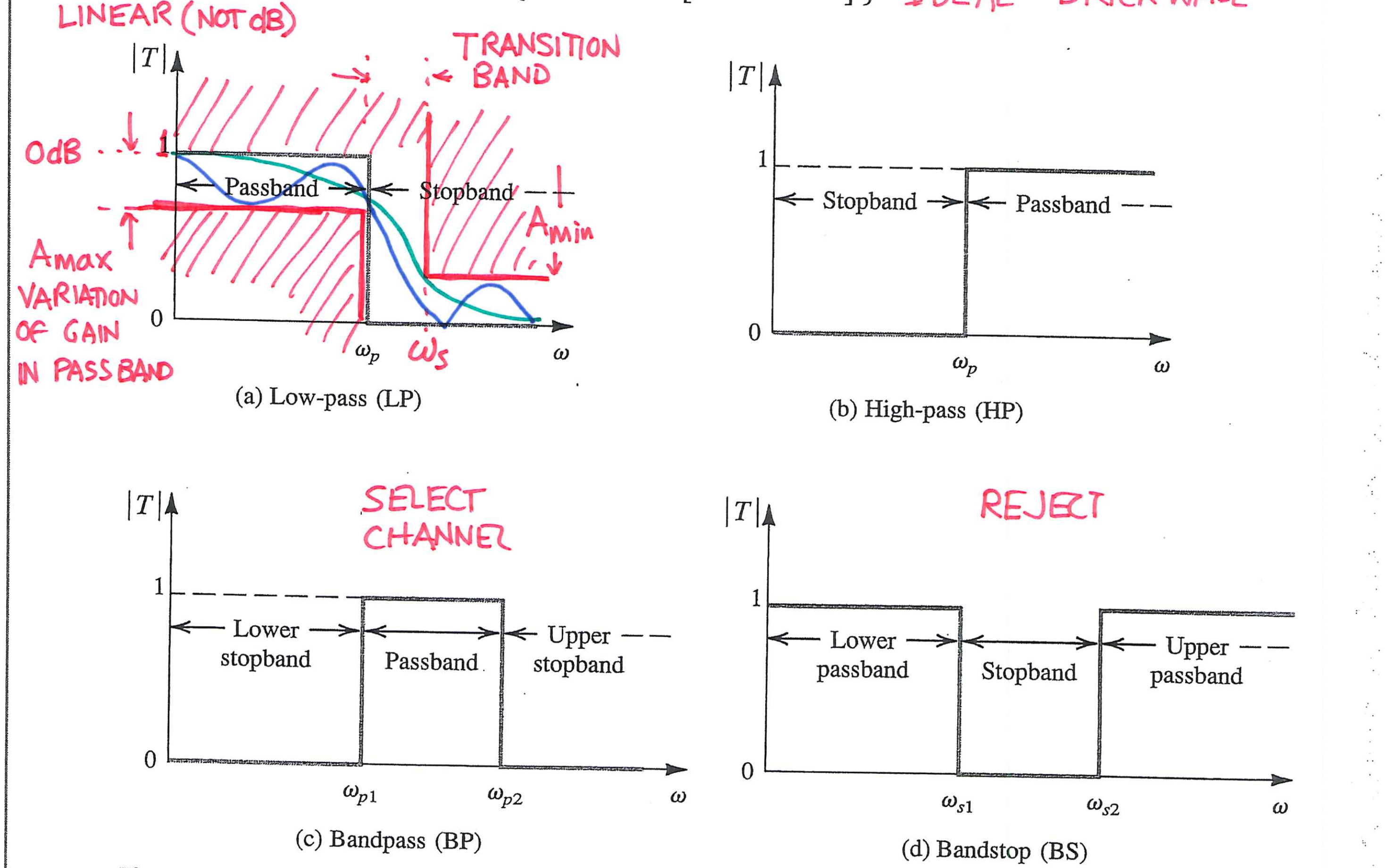


Figure 16.2 Ideal transmission characteristics of the four major filter types: **(a)** low-pass (LP), **(b)** high-pass (HP), **(c)** bandpass (BP), and **(d)** bandstop (BS).

FILTER SYNTHESIS: 1) SPECIFY { A_{max}, A_{min}, ω_p, ω_s, ...
SHAPE: LP, HP, ...
FAMILY BUTTERWORTH CHEBYSHEV

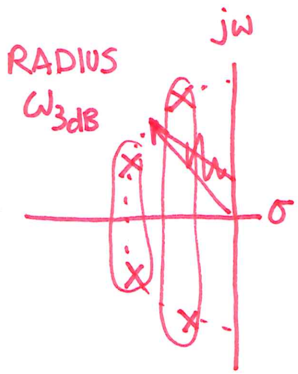
2) TRANSFER FUNCTION $T(s)$ IN S-PLANE

3) CIRCUIT WITH $\frac{V_{out}}{V_{in}} = T(s)$

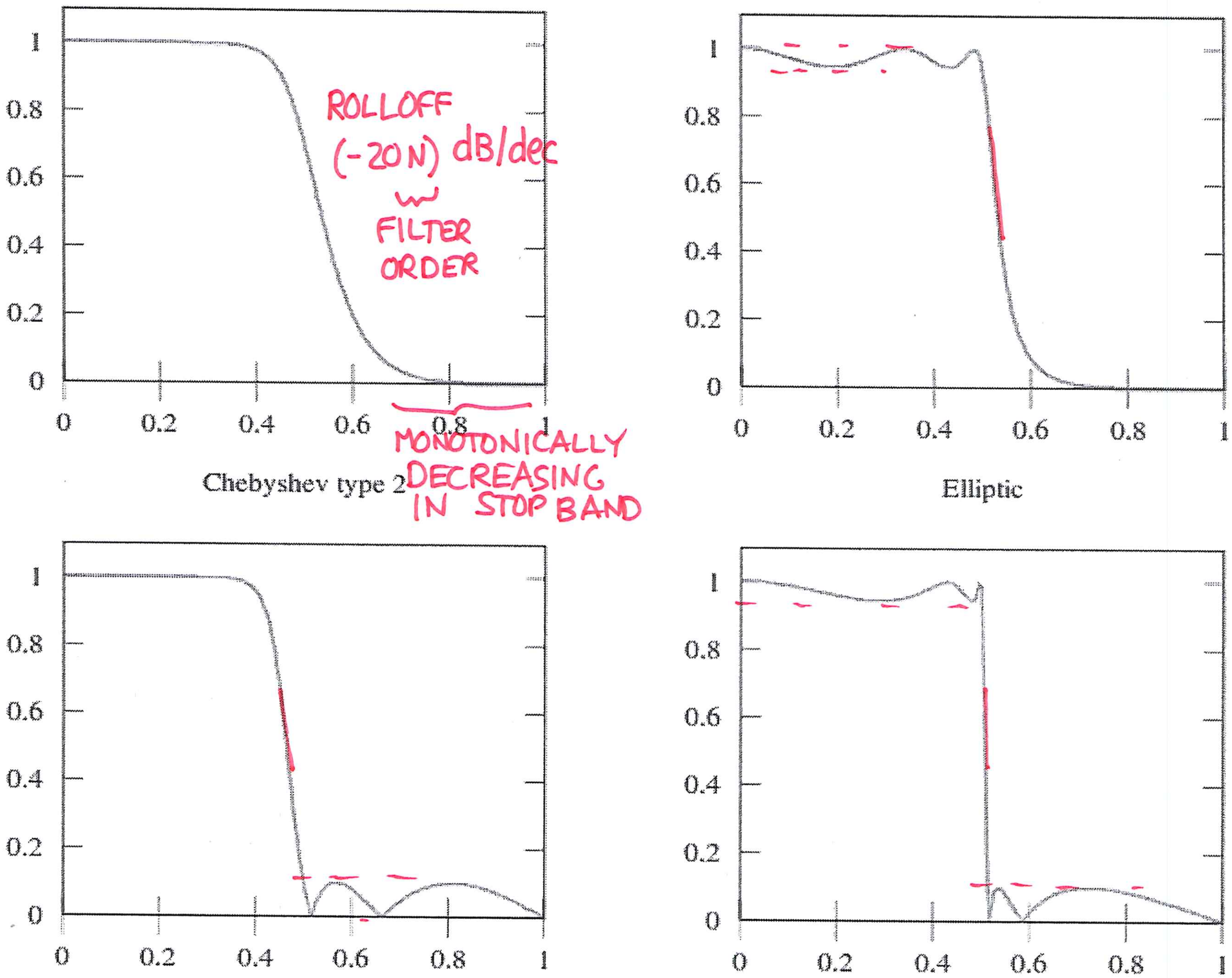
Comparison of Filter Family Transfer Functions:

"MAXIMALLY FLAT" IN PASSBAND

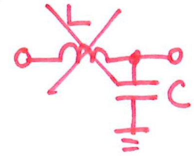
OTHER FAMILIES STEEPER TRANSITION



ALL POLE FILTER



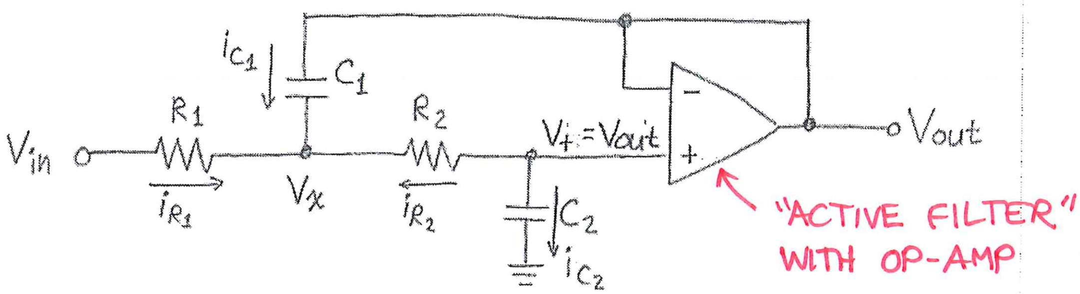
http://en.wikipedia.org/wiki/Elliptic_filter (accessed April 16, 2013)



SK1

"SALLEN AND KEY" CIRCUIT

For realizing a complex pole pair in the s-plane



Analysis: KCL using node voltages to describe currents.
At V_x (using $V_+ = V_{out}$, by ideal op-amp)

$$\underbrace{\frac{(V_{in} - V_x)}{R_1}}_{i_{R_1}} + \underbrace{(V_{out} - V_x)\,sC_1}_{i_{C_1}} + \underbrace{\frac{(V_{out} - V_x)}{R_2}}_{i_{C_2}} = 0 \qquad [1]$$

Collecting terms and multiplying through by R_1 gives

$$V_{in} + V_{out}\left(sR_1C_1 + \frac{R_1}{R_2}\right) = V_x\left(sR_1C_1 + \frac{R_1}{R_2} + 1\right) \qquad [2]$$

KCL at V_+ gives

$$\underbrace{\frac{V_{out} - V_x}{R_2}}_{i_{R_2}} + \underbrace{V_{out}\,sC_2}_{i_{C_2}} = 0 \qquad [3]$$

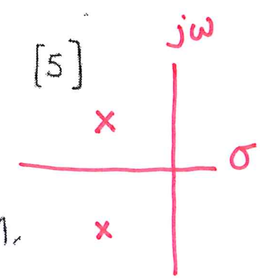
Collect and multiply by R_2, giving

$$V_x = V_{out}\left(sR_2C_2 + 1\right) \qquad [4]$$

Substituting [4] into [2] to eliminate V_x and solving for V_{out}/V_{in} gives

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{1}{s^2(R_1R_2C_1C_2) + s(R_1+R_2)C_2 + 1}} \qquad [5]$$

} NO ZEROS
} 2nd ORDER



Which is a second order, all pole transfer function.

Design: How do we choose R_1, R_2, C_1, C_2 to place the complex pole pair where we want? We have 4 values to choose, yet the complex pole pair is completely determined by 2 numbers: the real and imaginary parts. So we have 2 free parameters among R_1, R_2, C_1, C_2.

It turns out design is simplified by letting

$$R_1 = R_2 = R \qquad [6]$$

Now 5 becomes

$$\frac{V_{out}}{V_{in}} = \frac{1}{s^2(R^2C_1C_2) + s(2RC_2) + 1}$$

The poles (roots of the denominator) are given by

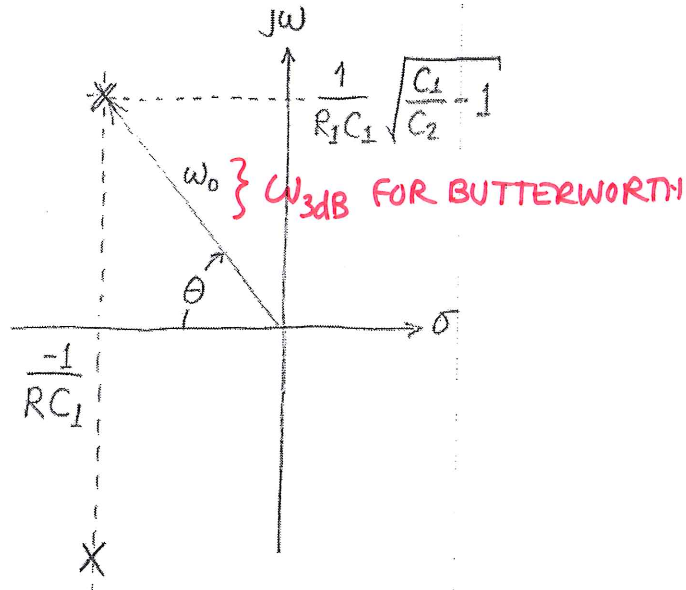
$$p_1, p_2 = \frac{-2RC_2 \pm \sqrt{4R^2C_2^2 - 4R^2C_1C_2}}{2R^2C_1C_2}$$

which reduces to

$$\boxed{p_1, p_2 = \frac{-1}{RC_1}\left(1 \pm j\sqrt{\frac{C_1}{C_2} - 1}\right)}$$

This can also be expressed in radial form. Some trigonometry gives

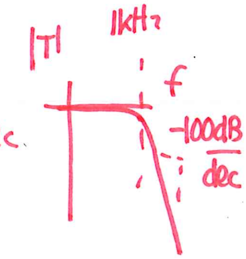
$$\boxed{\begin{aligned} \omega_0 &= \frac{1}{R\sqrt{C_1C_2}} \\ \tan\theta &= \sqrt{\frac{C_1}{C_2} - 1} \end{aligned}}$$



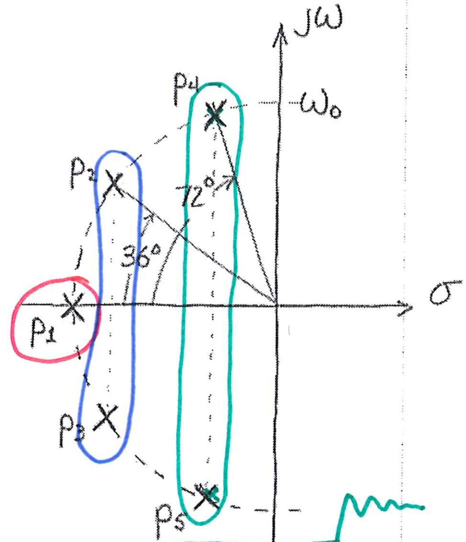
We still have R and C_1 or C_2 as free parameters, with the constraint that $C_1 > C_2$.

Example: DESIGN OF A 5th ORDER BUTTERWORTH LOWPASS FILTER USING SALLEN AND KEY CIRCUITS

Nth order Butterworth has N poles in the left half plane, on a semicircle of radius ω_o, equally spaced 180°/N apart. For N=5: $\times$ 20 dB/dec = −100 dB/dec.

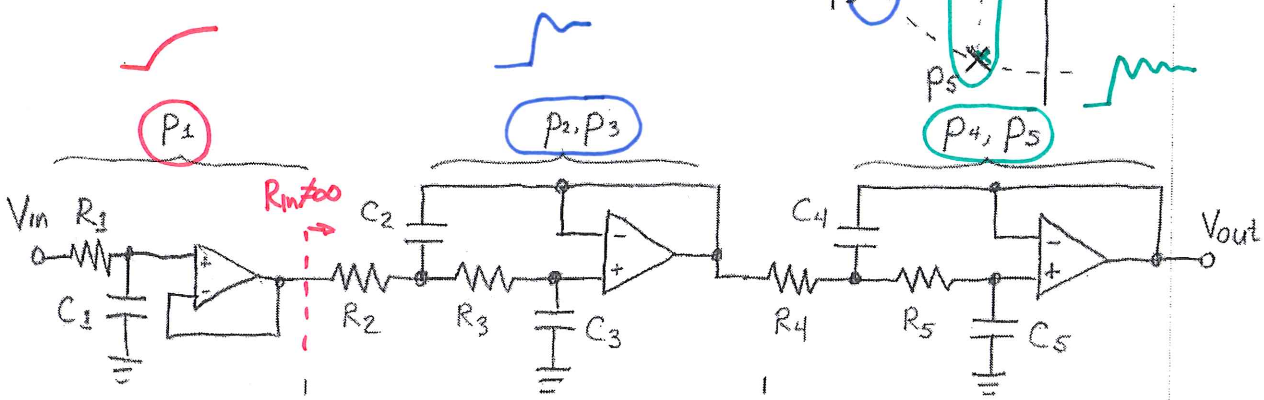


Pole p_1 can be realized with a simple RC circuit. Pole pairs p_2/p_3 and p_4/p_5 can each be realized with Sallen and Key circuits



Filter topology, for $f_{3dB} = 1\,kHz$

$\omega_o = 2\pi(1\,kHz)$



$$\frac{1}{2\pi R_1 C_1} = 1\,kHz$$

$R_1 = 16\,k\Omega$

$C_1 = 0.01\,\mu F$

$\omega_o = 995\,Hz$

$$\tan(36°) = \sqrt{\frac{C_2}{C_3} - 1} = 0.7265$$

$$\frac{C_2}{C_3} = 1.528$$

$C_2 = 0.015\,\mu F$, $C_3 = 0.01\,\mu F$ } $\theta = 35.3°$

$$2\pi(1\,kHz) = \frac{1}{R\sqrt{(0.015\,\mu F)(0.01\,\mu F)}}$$

$R = 12.99\,k\Omega \approx 13\,k\Omega$

$R_2 = R_3 = 13.0\,k\Omega$

$\omega_o = 999\,Hz$

$$\tan(72°) = \sqrt{\frac{C_4}{C_5} - 1} = 3.078$$

$$\frac{C_4}{C_5} = 10.47$$

$C_4 = 0.1\,\mu F$, $C_5 = 0.01\,\mu F$ } $\theta = 71.6°$

$$2\pi(1\,kHz) = \frac{1}{R\sqrt{(0.01\,\mu F)(0.1\,\mu F)}}$$

$R = 5.033\,k\Omega \approx 5.1\,k\Omega$

$R_4 = R_5 = 5.1\,k\Omega$

$\omega_o = 987\,Hz$

IDENTIFY FEEDBACK!

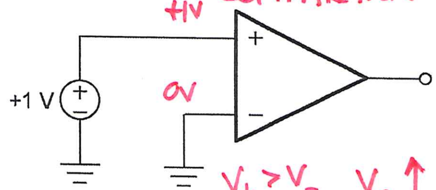
Review 1

NEG → V_-, V_+ =

NONE COMPARATOR SIGN $V_+ - V_-$

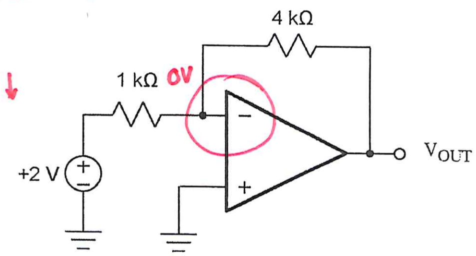
POS SIGN $V_+ - V_-$
OUTPUT DRIVEN ↑↓

NONE
COMPARATOR



+15V

$V_+ > V_-$ V_o ↑

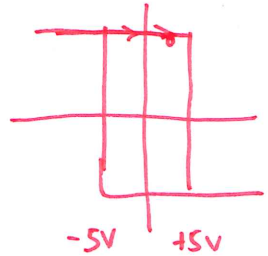


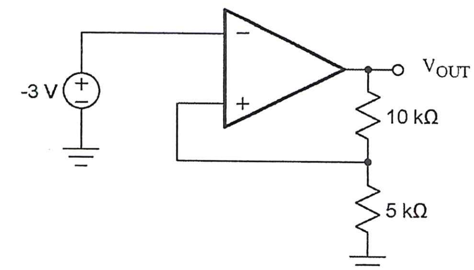
$V_{OUT} = \left(\frac{-4K}{1K}\right)(+2V) = -8V$

OK ±15V
NO CLIP ✓

NEGATIVE FB

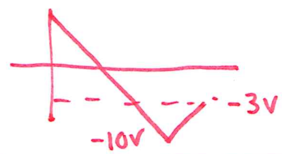
INVERTING S.T.





+15V	V_+
+15V	+5V
−15V	−5V

← −3V



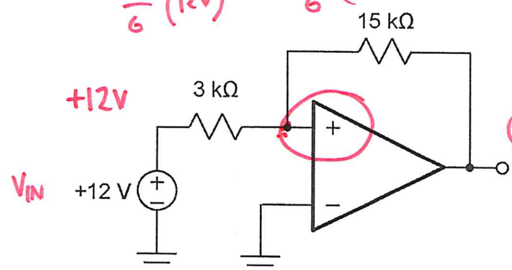
DON'T KNOW IF $V_+ > V_-$?! NEED PAST

+15 −15

VALUES OF V_+

$V_+ = \left(\frac{15}{18}\right) 12V + \left(\frac{3}{18}\right) V_{OUT}$

$\frac{5}{6}(12V)$ $\frac{1}{6}(-15V)$ $V_+ > 0$



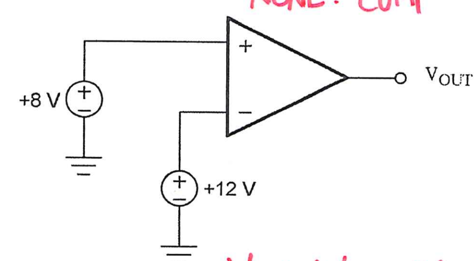
+15V
OR
−15V

NONE: COMP

$V_+ - V_-$

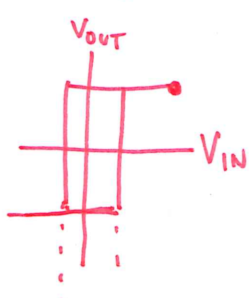
$8V - 12V = -4V$

$V_{OUT} = -15V$



$V_- > V_+$: V_{OUT} ↓

POSITIVE FB
SCHMITT TRIGGER

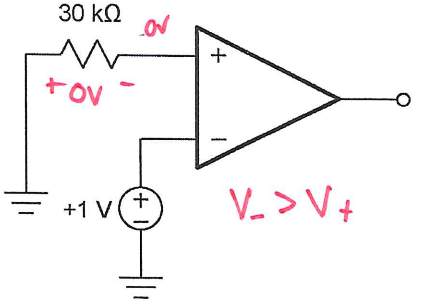


V_{TL}, V_{TH} $\frac{3k\Omega}{15k\Omega} \pm 15V = \pm 3V$

INSIDE HYSTERESIS BOX?

Review 2

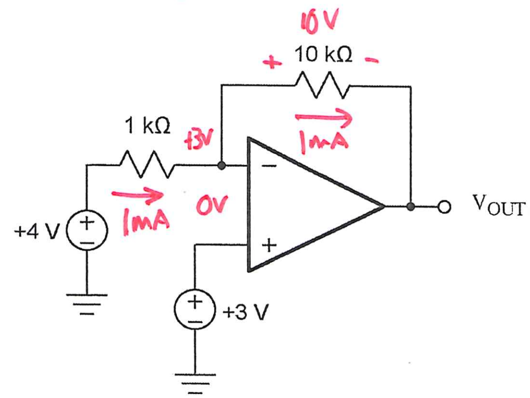
COMP



−15V

$V_- > V_+$

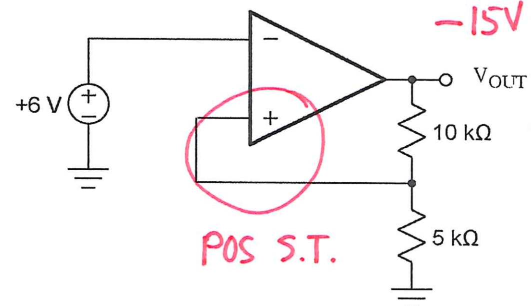
NEG FEEDBACK

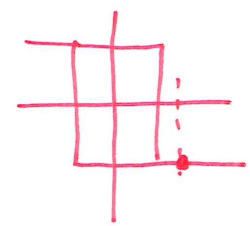


$V_{OUT} = 3V - 10V = -7V$

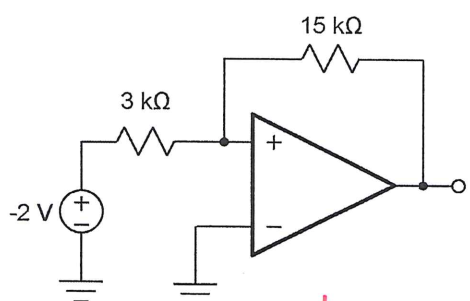
OK CLIP

−15V



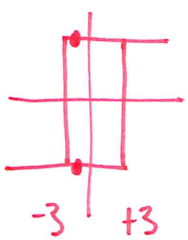


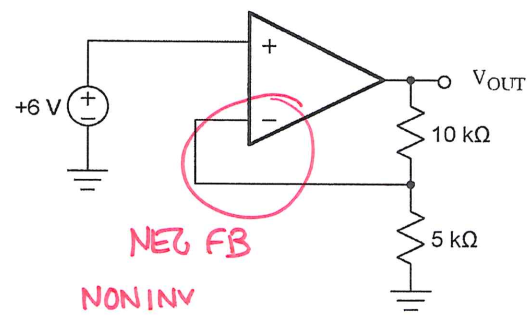
POS S.T.



±15V

NEED PAST





$V_{OUT} = \underbrace{\left(\frac{10+5}{5}\right)}_{3}(+6V) = \cancel{+18V}$

CLIP

+15V

NEG FB

NONINV