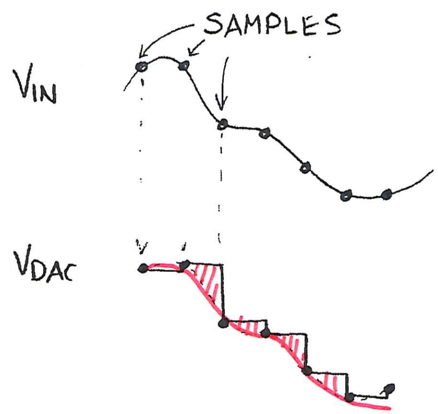
REST OF SLIDES

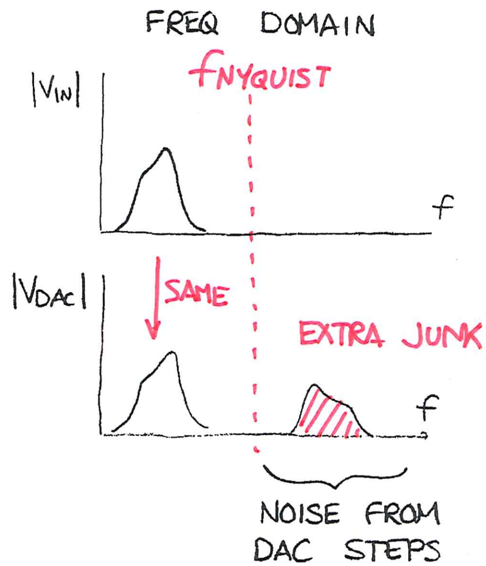
ECE3204 Lecture 18: FILTER DESIGN IN THE s-PLANE

Why filter? Look at reconstruction of signal with D/A converter:

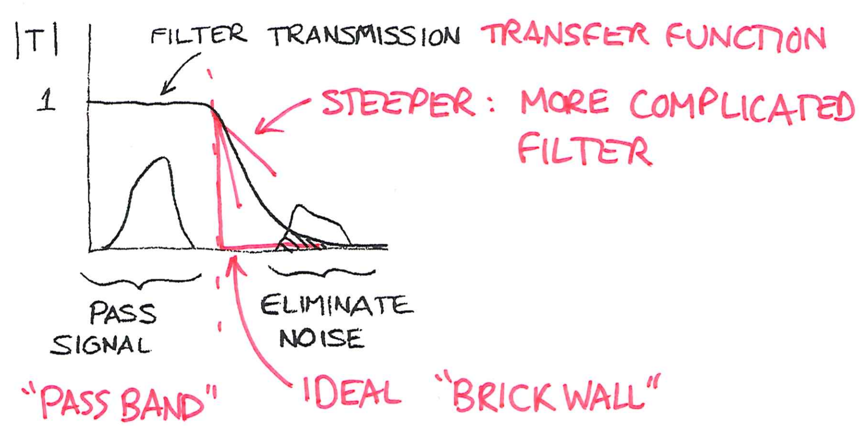


TIME DOMAIN INTERPRETATION

NEED TO "SMOOTH OUT" STEPS FROM DAC

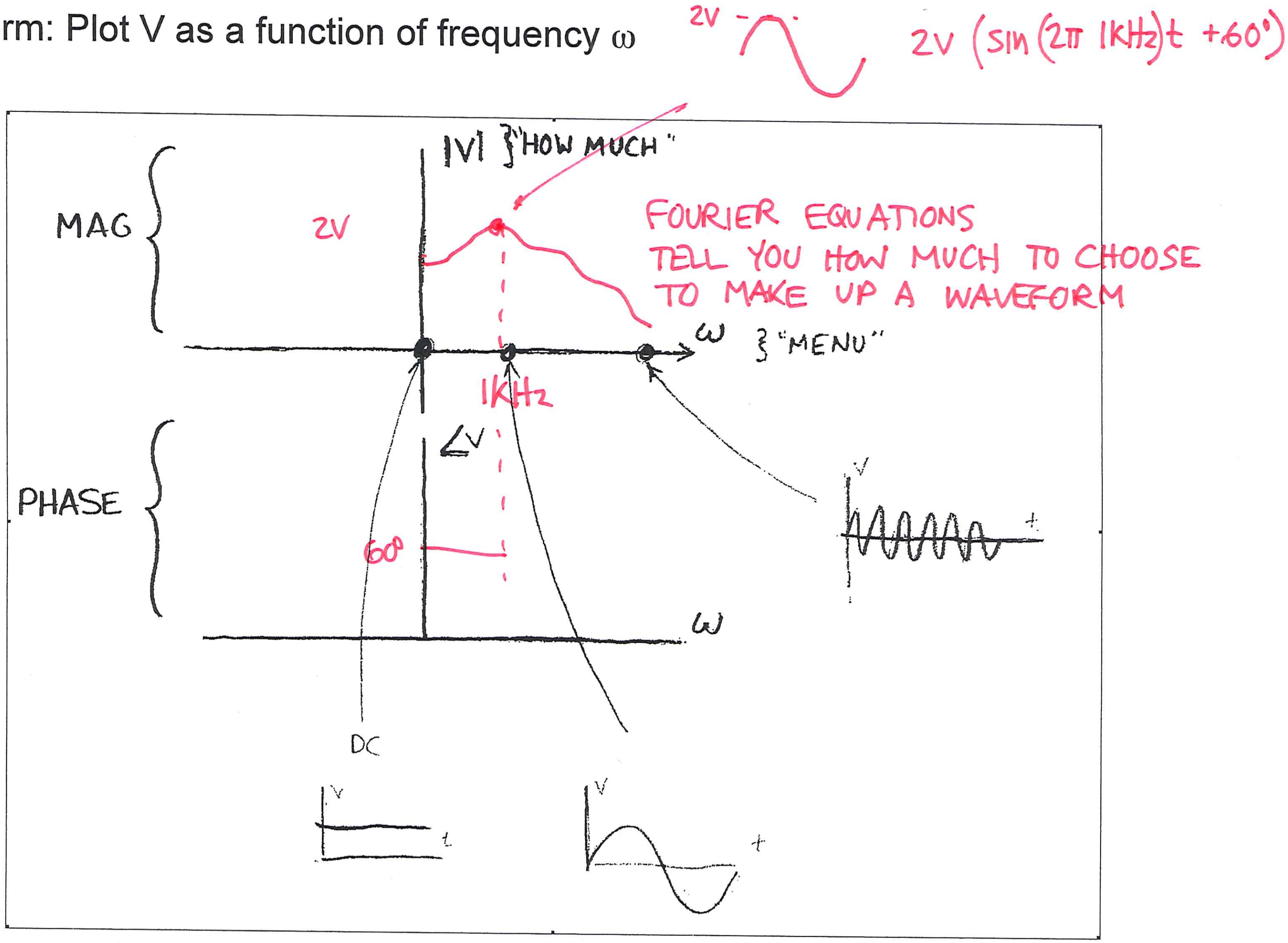


FREQ DOMAIN INTERPRETATION



Fourier transform: Plot V as a function of frequency ω

2V $\left(\sin\left(2\pi\ 1\text{kHz}\right)t + 60°\right)$



Assumes linearity: Adding sinusoids of different frequencies

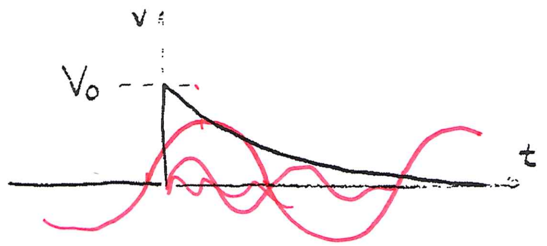
Any waveform can be expressed as sum of sinusoids: Why bother with s for Laplace?

Why s ? Look at STC exponential response

STC EXPONENTIAL RESPONSE $V = \underbrace{V_0 e^{-t/\tau}}$

VERY EASY: JUST ONE NUMBER τ

IF I ALLOW e^{t} WAVEFORMS



NEED LOTS OF SINES TO MAKE THIS WAVEFORM!

EXPAND "MENU"

$$e^{j\omega t} \rightarrow e^{st}$$

$$s = \sigma + j\omega$$

COMPLEX FREQUENCY s

ALLOWS BOTH REAL, IMAGINARY PARTS IN FREQUENCY

Transforms: Fourier vs. Laplace

FOURIER: ω $(2\pi f)$ LAPLACE s $(\sigma + j\omega)$

WAVEFORM: SUM OF COMPLEX EXPONENTIALS

$V = V_o e^{j\omega t}$ $V = V_o e^{st}$

WHY? I-V FOR CAPACITOR EXAMPLE

$$I = C\frac{dV}{dt}$$

CAN WE MAKE THIS AS EASY AS OHM'S LAW $R = \frac{V}{I}$?

$$I = C\frac{d}{dt}\left[V_o e^{j\omega t}\right] \qquad I = C\frac{d}{dt}\left[V_o e^{st}\right]$$

$$I = Cj\omega \underbrace{V_o e^{j\omega t}}_{V} \qquad I = Cs\underbrace{V_o e^{st}}_{V}$$

$$I = j\omega C\, V \qquad I = sC\, V$$

$$\frac{V}{I} = \frac{1}{j\omega C} \qquad \frac{V}{I} = \frac{1}{sC}$$

Fourier is "subset" of Laplace: Let s = jω

Impedances:

$$\left.Z_C = \frac{1}{j\omega C} \qquad Z_C = \frac{1}{sC}\right\} C$$

$$\left.Z_L = j\omega L \qquad Z_L = sL\right\} L$$

$Z_R = R$

s-Plane as "map" of waveforms.

Plot real, imaginary parts of s

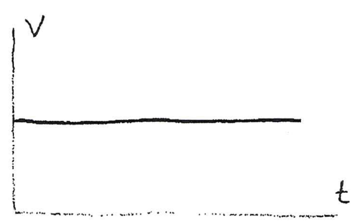
Waveform examples: DC

TIME DOMAIN

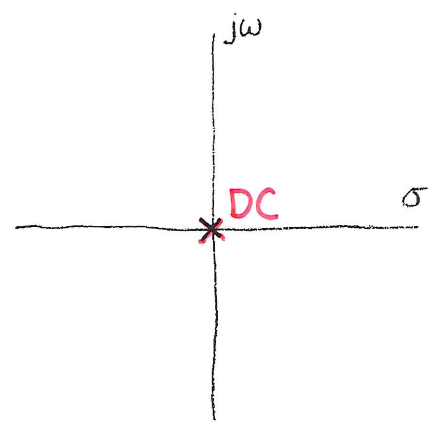
$s = 0$

$e^{st} = e^{0t} = 1$

TIME

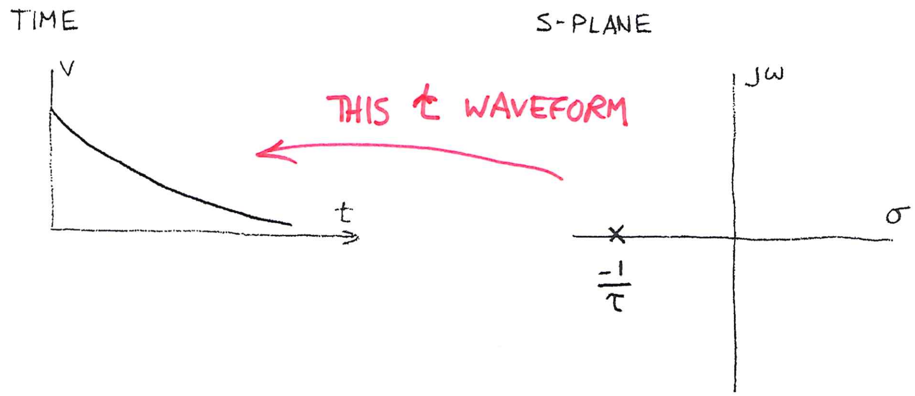


S-PLANE

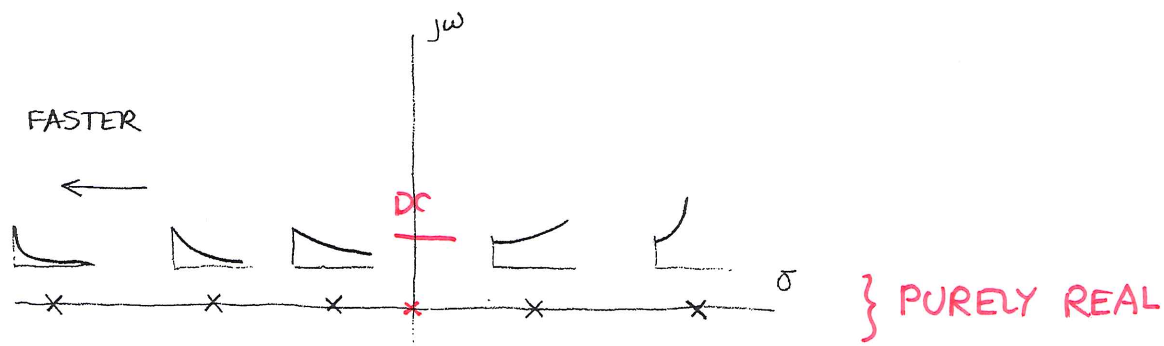


$$s = \frac{-1}{\tau}$$

$$e^{st} = e^{\left(\frac{-1}{\tau}\right)t} = e^{-t/\tau}$$



REAL AXIS: EXPONENTIALS



LHP: LEFT HALF PLANE
NEGATIVE REAL PART
DECAYING EXPONENTIAL

STABLE

RHP: RIGHT HALF PLANE
POSITIVE REAL PART
GROWING EXPONENTIAL

UNSTABLE

Pure sinusoids (complex conjugate pair)

$$s_1 = j(+\omega) \qquad s_2 = j(-\omega)$$

$$e^{s_1 t} = e^{j\omega t} \qquad e^{s_2 t} = e^{-j\omega t}$$

EULER RELATIONSHIP (COMPLEX NUMBERS)

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$e^{j\omega t} = \cos\omega t + j\sin\omega t \qquad e^{-j\omega t} = \cos\omega t - j\sin\omega t$$

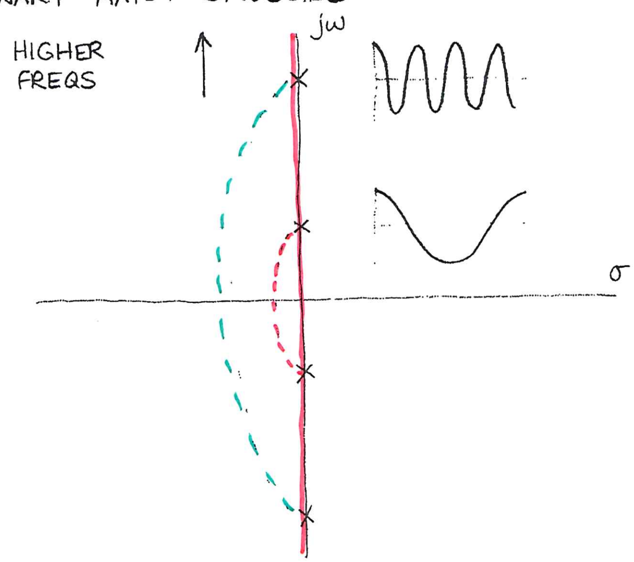
ADD BY SUPERPOSITION

$$e^{s_1 t} + e^{s_2 t} = \cos\omega t + \cancel{j\sin\omega t} + \cos\omega t - \cancel{j\sin\omega t}$$

$$= 2\cos\omega t$$

NEED CONJUGATE TO MAKE IMAGINARY PART GO AWAY

IMAGINARY AXIS: SINUSOIDS



Both Real and Imaginary parts

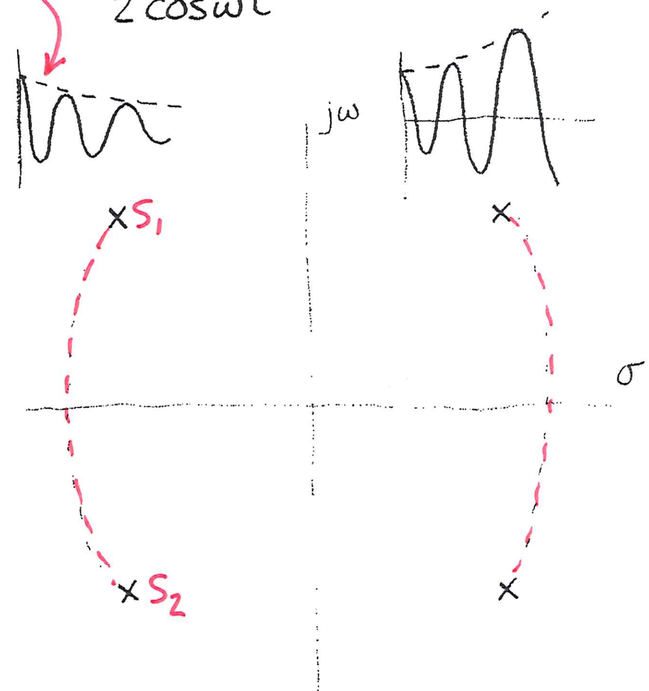
$S_1 = \sigma + j\omega$ $\qquad\qquad$ $S_2 = \sigma - j\omega$

$$e^{s_1 t} = e^{(\sigma + j\omega)t}$$
$$= e^{\sigma t} e^{j\omega t} \qquad\qquad = e^{\sigma t} e^{-j\omega t}$$

FROM $Re\{s\}$

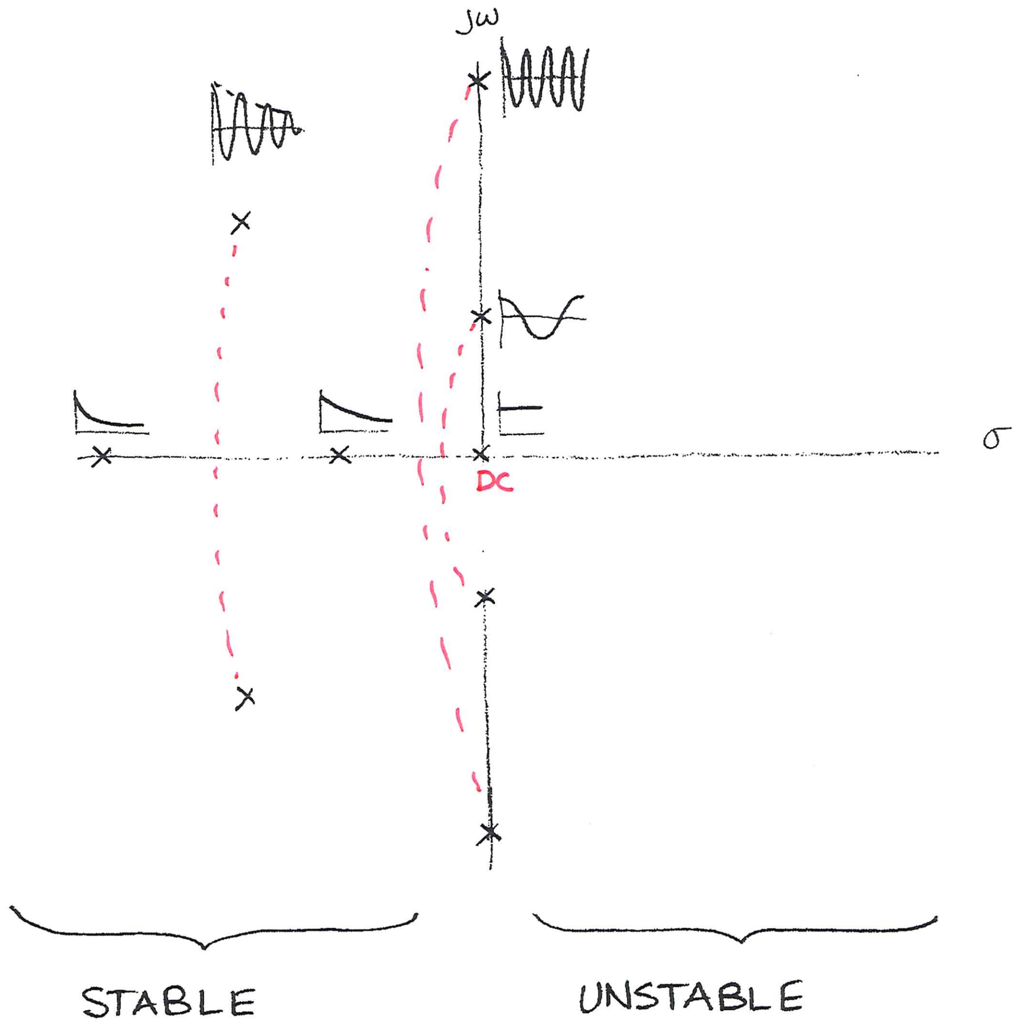
ADD

$$\underbrace{e^{\sigma t}}\underbrace{\left(e^{j\omega t} + e^{-j\omega t}\right)}_{2\cos\omega t}$$



STABLE $\qquad\qquad$ UNSTABLE

s-Plane summary



Forms of transfer function

$$\frac{V_{out}}{V_{in}} \quad T(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \cdots + a_0}{s^n + b_{n-1} s^{n-1} + \cdots + b_0}$$

POLYNOMIALS IN S
a_i, b_i REAL NUMBERS

$T(s)$ FILTER TRANSMISSION

$H(s)$ GENERAL TRANSFER FUNCTION

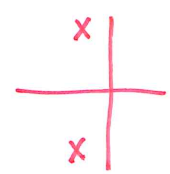
n ORDER OF THE NETWORK

$m \leq n$ FOR REAL SYSTEM

BETTER FORM FOR TRANSFER FUNCTION!
SHOW ROOTS EXPLICITLY

$$T(s) = \underbrace{K}_{\substack{\text{DC}\\\text{GAIN}\\(s=0)}} \frac{\left(1-\frac{s}{z_1}\right)\left(1-\frac{s}{z_2}\right)\cdots\left(1-\frac{s}{z_m}\right)}{\left(1-\frac{s}{p_1}\right)\left(1-\frac{s}{p_2}\right)\cdots\left(1-\frac{s}{p_n}\right)}$$

$p_1, p_2, \ldots, p_n$ POLES: VALUES OF S FOR WHICH
DENOMINATOR $\rightarrow 0$
$\Rightarrow |T| \rightarrow \infty$
OUTPUT FOR $\rightarrow 0$ INPUT
"NATURAL MODE OF A SYSTEM"



SWING

LOAD SYSTEM WITH INITIAL ENERGY

$z_1, z_2, \ldots, z_m$ ZEROS: VALUES OF S FOR WHICH
NUMERATOR $\rightarrow 0$ $\Rightarrow$ $|T| = 0$
NO OUTPUT FOR ANY INPUT

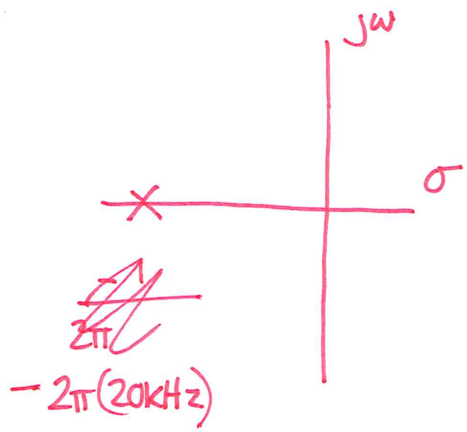
Poles, zeros: Plot these special values of s in the s-plane

Another way to specify transfer function

LOWPASS FILTER : DC GAIN OF 10

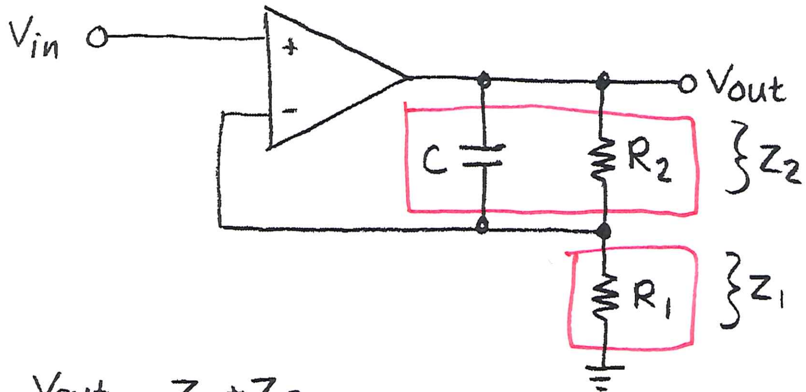
f_{3dB} OF 20 kHz

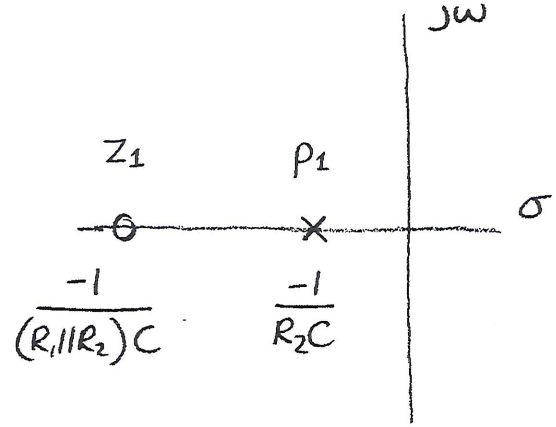
$$\frac{10}{1 + \frac{s}{2\pi\, 20\text{kHz}}}$$



S-PLANE DOESN'T TELL DC GAIN

Example:





$$T(s) = \frac{V_{out}}{V_{in}} = \frac{Z_1 + Z_2}{Z_1}$$

$$= \frac{R_1 + \overbrace{\frac{R_2}{1 + sR_2C}}^{Z_2}}{R_1}$$

MASSAGE INTO STANDARD FORM

1st ORDER SYSTEM

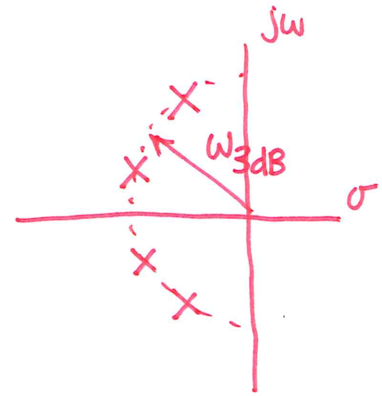
$$= \frac{R_1 + R_2 + sR_1R_2C}{R_1(1 + sR_2C)}$$

KNOW DC GAIN

$$T(s) = \underbrace{\left(\frac{R_1 + R_2}{R_1}\right)}_{\text{DC GAIN}} \frac{1 + s\overbrace{\left(\frac{R_1R_2}{R_1 + R_2}\right)}^{R_1 \| R_2}C}{1 + sR_2C}$$

ZERO: $z_1 = \frac{-1}{(R_1 \| R_2)C}$

POLE: $p_1 = \frac{-1}{R_2C}$



BUTTERWORTH (4TH ORDER)

Bode plot