"3dB FREQUENCY"

$$20 \log\left(\frac{1}{\sqrt{2}}\right) = -3\ \text{dB}$$

$$\frac{1}{\sqrt{2}} = 0.707$$

$|H(\omega)|$ HAS DROPPED TO 0.707 × MAXIMUM VALUE

MAXIMUM VALUE = 1

$$|H(\omega_{3dB})| = \frac{1}{\sqrt{2}}$$

SOLVE

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+(\omega_{3dB} RC)^2}}$$

$$2 = 1 + (\omega_{3dB} RC)^2$$

$$1 = (\omega_{3dB} RC)^2$$

$$1 = \omega_{3dB} RC$$

$$\boxed{\omega_{3dB} = \frac{1}{RC}}$$

GENERAL RESULT $\omega_{3dB} = \frac{1}{\tau}$ } TIME CONSTANT

SINCE $\omega = 2\pi f$

$$2\pi f_{3dB} = \frac{1}{RC}$$

$$\boxed{f_{3dB} = \frac{1}{2\pi RC}}$$

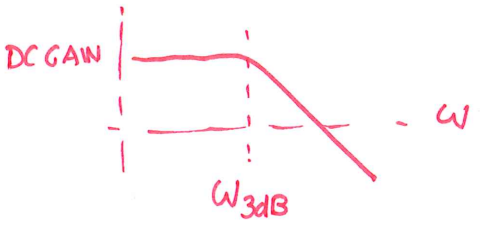
FOR THIS EXAMPLE: $\frac{1}{2\pi (7.5\,k\Omega)(1000\,pF)} = 21.2\ kHz$

$$\frac{1}{2\pi\tau}$$

1ST ORDER LOWPASS

GENERAL FORM OF TRANSFER FUNCTION

$$H(j\omega) = \frac{[\text{DC GAIN}]}{1 + j\dfrac{\omega}{[\text{3dB FREQUENCY } \omega]}}$$



$$H(j\omega) = \frac{1}{1 + RC} = \frac{1}{1 + j\dfrac{\omega}{1/RC}}$$

ω_{3dB}

$\omega_{3dB} = \frac{1}{\tau}$ } TIME CONSTANT OF 1ST ORDER SYSTEM

EXAMPLE

"A STC LOWPASS HAS A DC GAIN OF 20, AND A 3dB FREQUENCY OF 3 kHz. WHAT IS H(s)?"

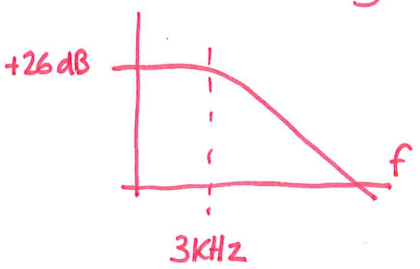
f $\omega = 2\pi f$

$$H(j\omega) = \frac{20}{1 + j\dfrac{\omega}{2\pi(3\text{kHz})}}$$

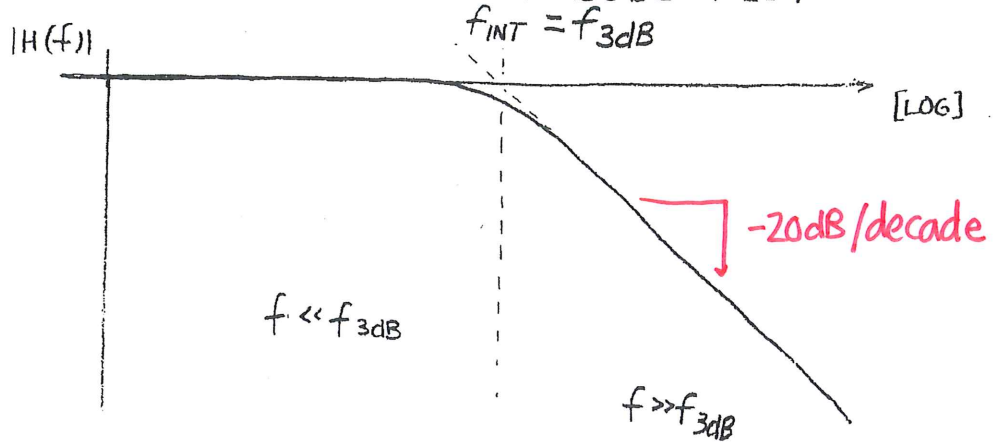
$|H(j\omega)|$ $20\log(20) = +26$ dB

ALTERNATIVE (SINCE $\omega = 2\pi f$)

$$H(jf) = \frac{20}{1 + j\dfrac{f}{(3\text{kHz})}}$$



FIRST ORDER LOWPASS TRANSFER FUNCTION
ASYMPTOTES OF MAGNITUDE BODE PLOT



TRANSFER FUNCTION REWRITE

$$|H(f)| = \frac{1}{\sqrt{1 + (f/f_{3dB})^2}}$$

$f \ll f_{3dB}$

$|H(f)| \approx 1$

$= 0$ dB

$f \gg f_{3dB}$

$$|H(f)| \approx \frac{1}{\sqrt{(f/f_{3dB})^2}}$$

$$\approx \frac{f_{3dB}}{f}$$

IN dB

$$20 \log\left(\frac{f_{3dB}}{f}\right)$$

f INCREASE BY X10 ("DECADE")

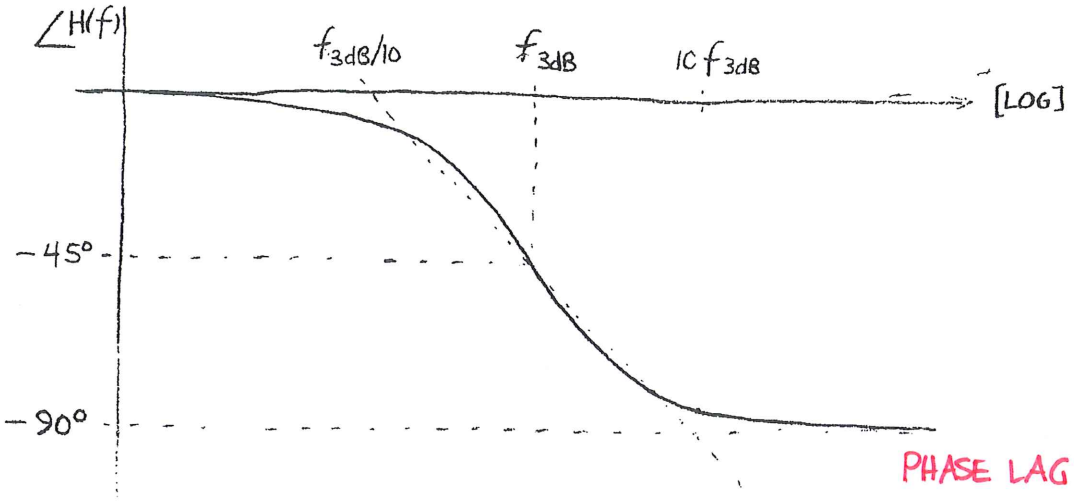
log DECREASE BY 1

dB "-20 dB PER DECADE"

INTERSECTION

$$1 = \frac{f_{3dB}}{f_{INT}} \Rightarrow \boxed{f_{INT} = f_{3dB}}$$

PHASE BODE PLOT



$$\angle H(f) = -\tan^{-1}(f/f_{3dB})$$

$f << f_{3dB}$

$-\tan^{-1}(0)$

$= 0°$

$f >> f_{3dB}$

$-\tan^{-1}(\infty)$

$= -90°$

AT 3dB FREQUENCY

$$-\tan^{-1}(f_{3dB}/f_{3dB}) = -\tan^{-1}(1) = -45°$$