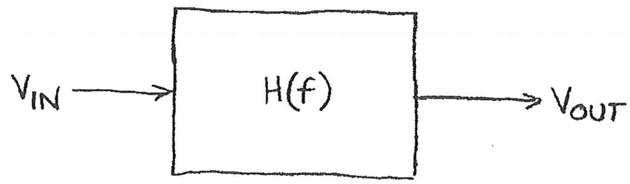
TRANSFER FUNCTION $H(f)$ $\big(H(s),\ H(\omega),\ H(j\omega),\ H(j2\pi f)\big)$

LTI SYSTEM

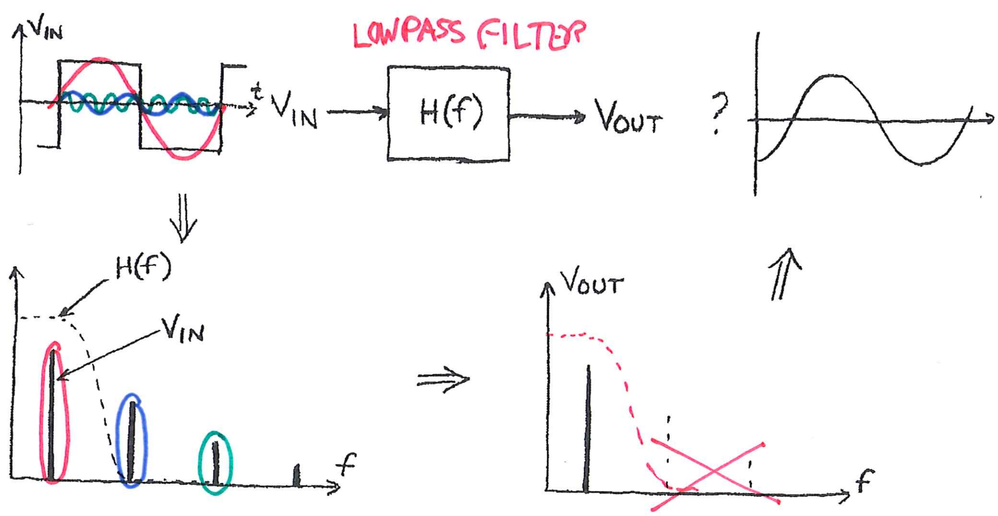


SAME f OUT AS IN

MAGNITUDE : SCALE
PHASE : SHIFT

$$\underbrace{V_{OUT}(f)}_{\text{OUTPUT SPECTRUM}} = \underbrace{V_{IN}(f)}_{\text{INPUT SPECTRUM}}\ \underbrace{H(f)}_{\text{TRANSFER FUNCTION}}$$

EXAMPLE



(DO THIS IN LAB 5!)

FOURIER TRANSFORM: REPRESENT V AS SUM OF COMPLEX EXPONENTIALS

$$V = V_0 e^{j\omega t} \qquad V = V_0 e^{st}$$

WHY? I-V FOR CAPACITOR

$$I = C\frac{dV}{dt}$$

CAN THIS BE AS EASY AS Ω'S LAW: $I = \frac{V}{R}$?

$$I = C\frac{d}{dt} V_0 e^{j\omega t}$$

$$I = j\omega C \underbrace{V_0 e^{j\omega t}}_{V}$$

$$I = (j\omega C)V$$

$$\frac{V}{I} = \frac{1}{j\omega C}$$

$\frac{V}{I}$ RATION

"IMPEDANCE": LIKE RESISTANCE, BUT COMPLEX

FREQUENCY DEPENDENT

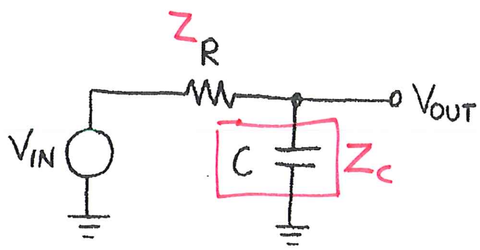
j: "ENCODES" 90° PHASE SHIFT $\qquad e^{st}$

$$Z_R = R \qquad Z_R = R$$

$$Z_C = \frac{1}{j\omega C} \qquad Z_C = \frac{1}{sC}$$

$$Z_L = j\omega L \qquad Z_L = sL$$

ANALYSIS EXAMPLE: LOWPASS FILTER



IMPEDANCE DIVIDER

$$V_{OUT} = \left(\frac{Z_C}{Z_R + Z_C}\right) V_{IN}$$

$$V_{OUT} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} V_{IN} \qquad \frac{\frac{1}{sC}}{R + \frac{1}{sC}}$$

$$V_{OUT} = \underbrace{\frac{1}{1 + j\omega RC}}_{H(j\omega)} V_{IN}$$

TRANSFER FUNCTION: GENERALIZED GAIN V_{OUT}/V_{IN}

$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + j\omega RC} \qquad \frac{1}{1 + sRC}$$

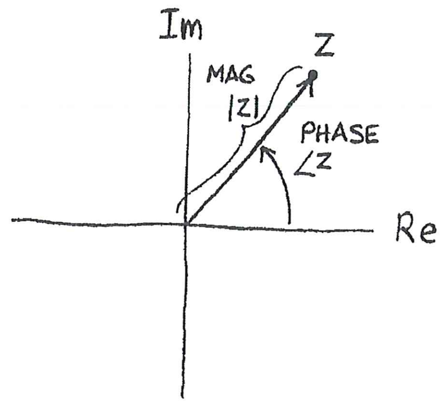
"FREQUENCY RESPONSE": FIND MAGNITUDE, PHASE $(s = j\omega)$

$$\left|\frac{V_{OUT}}{V_{IN}}\right| = \left|\frac{1}{1 + j\omega RC}\right|$$

$$\angle \frac{V_{OUT}}{V_{IN}} = \angle \frac{1}{1 + j\omega RC}$$

COMPLEX NUMBER REFRESHER

$$Z = \frac{a + jb}{x + jy} \quad \cancel{\frac{x - jy}{x - jy}}$$



MAGNITUDE

$$|Z| = \frac{\sqrt{a^2 + b^2}}{\sqrt{x^2 + y^2}}$$

$$|Z| = \frac{|NUM|}{|DEN|}$$

PHASE

$$\angle Z = \tan^{-1}(b/a) - \tan^{-1}(y/x)$$

$$\angle Z = \angle NUM - \angle DEN$$

TRANSFER FUNCTIONS (USUALLY) WILL BE RATIOS OF POLYNOMIALS IN COMPLEX VARIABLE s

WE WILL LET $s = j\omega$ TO GET FREQUENCY RESPONSE

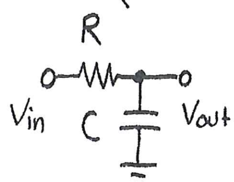
MAGNITUDE, PHASE OF STC LOWPASS

$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

$$|H(j\omega)| = \frac{\sqrt{1^2 + 0^2}}{\sqrt{1^2 + (\omega RC)^2}} = \frac{1}{\sqrt{1 + (\omega RC)^2}} = \frac{1}{\sqrt{1 + (2\pi f RC)^2}}$$

$$\angle H(j\omega) = \tan^{-1}(0/1) - \tan^{-1}(\omega RC/1) = -\tan^{-1}(\omega RC)$$
$$= -\tan^{-1}(2\pi f RC)$$

EXAMPLE: $R = 7.5\ k\Omega$
$C = 1000\ pF$



$RC = 7.5\ \mu sec$

f	MAGNITUDE $\lvert H(f) \rvert$	PHASE $\angle H(f)$ °	$20 \log(H(f))$ dB
1 Hz	1	−0.003°	0 dB UNITY GAIN
10 Hz	1	−0.03°	0
100	0.9999	−0.27°	0
1 kHz	0.9989	−2.7°	−0.01 dB
10 kHz	0.9046	−25.2°	−0.87 dB
21 kHz	0.707	−45.0°	−3.0 dB
100 kHz	0.2076	−78.0°	−13.7 dB
1 MHz	0.0212	−88.8°	−33.5 dB
10 MHz	0.0021	−89.9°	−53.5 dB

10X

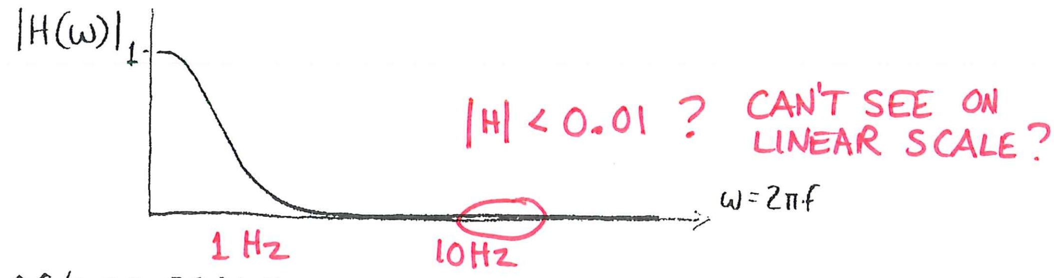
BANDWIDTH f_{-3d} "3dB FREQ"

3dB down from max value

−20dB

−20 dB PER DECADE f "ROLLOFF"

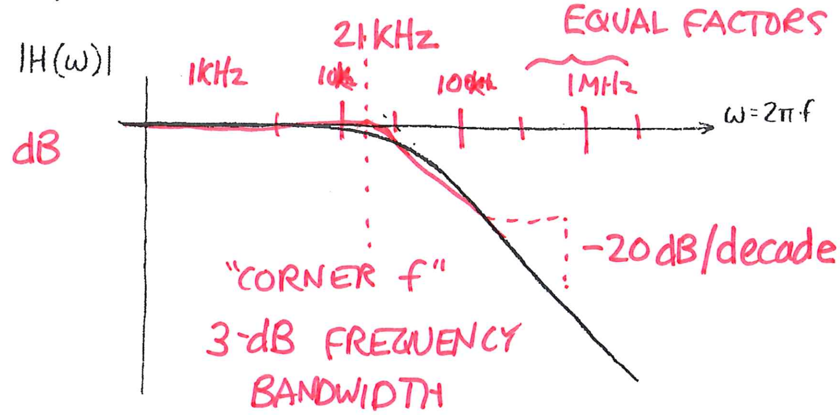
PLOT (LINEAR SCALE)



10 MHz

~~INSTITUTE PARK~~

LOG/LOG SCALE



"BODE PLOT": USE <u>dB</u>

$$\underbrace{\left(\frac{V_2}{V_1}\right)}_{\text{VOLTAGE RATIO}} \text{dB} = 20 \log\left(\frac{V_2}{V_1}\right)$$

EXAMPLE: $A = 50{,}000$

$$20 \log(A) = 20 \times 4.7 = +94 \text{ dB}$$

"3dB FREQUENCY"

$$20 \log\left(\frac{1}{\sqrt{2}}\right) = -3\text{ dB}$$

$$\frac{1}{\sqrt{2}} = 0.707$$

$|H(\omega)|$ HAS DROPPED TO $0.707 \times$ MAXIMUM VALUE

MAXIMUM VALUE = 1

$$|H(\omega_{3dB})| = \frac{1}{\sqrt{2}}$$

SOLVE

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+(\omega_{3dB}RC)^2}}$$

$$2 = 1 + (\omega_{3dB}RC)^2$$

$$1 = (\omega_{3dB}RC)^2$$

$$1 = \omega_{3dB}RC$$

$$\boxed{\omega_{3dB} = \frac{1}{RC}}$$

GENERAL RESULT $\omega_{3dB} = \frac{1}{\tau}$ } TIME CONSTANT

SINCE $\omega = 2\pi f$

$$2\pi f_{3dB} = \frac{1}{RC}$$

$$\boxed{f_{3dB} = \frac{1}{2\pi RC}}$$

FOR THIS EXAMPLE: $\frac{1}{2\pi(7.5\text{k}\Omega)(1000\text{pF})} = 21.2\text{ kHz}$

$$\frac{1}{2\pi\tau}$$