

THÉVENIN'S THEOREM

Let's review the circuit analysis techniques we have developed so far.

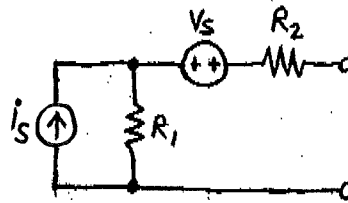
KVL (Kirchoff's Voltage Law) KCL (Kirchoff's Current Law)	Conservation laws that allow you to write equations about a circuit
Series Resistor Combination Parallel Resistor Combination	Equations that tell you how to calculate the value of one component that has the equivalent effect as a combination of two or more of the same kind of component
Voltage Divider Current Divider	Equations that allow you to (quickly) predict the behavior of voltage or current in a circuit
Nodal Analysis	A procedure for writing a system of equations that can always be solved to give the complete voltage and current behavior of a circuit
Thévenin's Theorem	A procedure that allows you to determine a different (but usually simpler) circuit that has the same V-I characteristic as the original circuit.

Thévenin's theorem is very powerful and useful, but since there are ways in which it is a fundamentally different technique, some students have difficulty with Thévenin. So, before getting into the details, some thoughts:

- Thévenin's theorem is not an equation, it is a procedure for finding a different circuit. Some people who are comfortable writing equations have difficulty with this additional layer of complexity.
- You will substitute one circuit for another; the Thévenin equivalent circuit will be simpler. For example, a circuit with multiple current sources, voltage sources, resistors will be replaced by a single voltage source and a single resistor. At first, some people find it hard to believe that the simpler circuit will really have the same behavior.
- You will lose visibility to most if not all voltages and currents internal to the original circuit. Some people don't like this, and indeed, if you need to know internal behavior, then Thévenin's theorem isn't what you want - then you need a technique like nodal analysis that provides complete information.

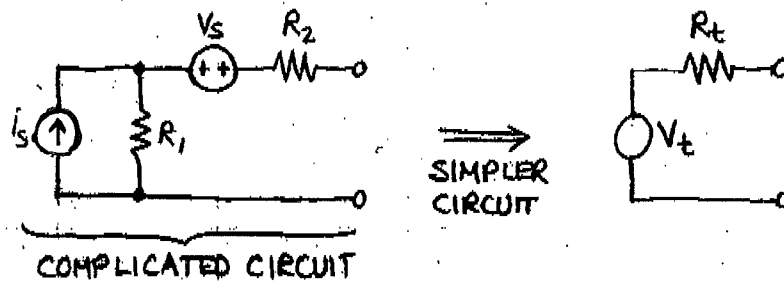
Thévenin's theorem is useful when we are interested in what is happening at a particular place in a circuit, and don't need all of the details about the rest of the circuit. Thévenin's Theorem says that we can replace the rest of the circuit by a very simple Equivalent Circuit as long as the circuit contains only resistances and sources. (Actually, it works for inductors and capacitors, too, but don't worry about that for now.)

Say we have a circuit composed of R's and sources, and want to determine a simpler circuit which exhibits the same behavior at some pair of nodes in the circuit:



This is called a one-port network. The reason we are interested in a port consisting of two nodes is that we would like to know how the circuit will behave if we attach something to the nodes (such as a resistance.) For example, you might think of an audio system as the circuit, with the wires to one of the speakers as the port of two nodes. Then, the speaker is the thing we would connect to the nodes. Even if the audio system is a complicated circuit, Thévenin's theorem says we can find a simple Equivalent Circuit which behaves the same as far as the speaker is concerned.

The Thévenin Equivalent Circuit consists of the following:



- V_t is the Thévenin Voltage;
- R_t is the Thévenin Resistance

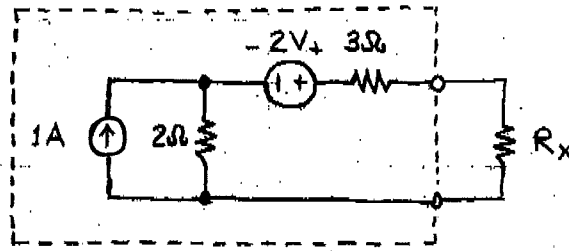
Thévenin's theorem says that, if you choose V_t and R_t correctly, the simple circuit of one source and one resistor will have the same V-I characteristic as the more complicated circuit.

How do you choose V_t and R_t correctly? Usually the simplest way is to use the following procedure to find:

- $V_t = V_{oc}$ the open-circuit voltage measured at the terminals of the port;
- $R_t = R_{eq}$ the equivalent resistance between the port terminals with all sources set to zero.

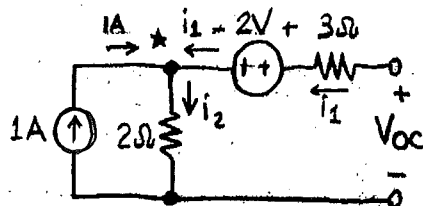
(Recall: to set a voltage source to zero, replace it with a short circuit; to set a current source to zero, replace it with an open circuit.)

Example: Suppose we want to know what will happen when we connect a resistor of value R_x to the network in the dotted lines. With the full circuit, this becomes a tedious exercise in nodal analysis etc., which we would need to repeat for every different value of R_x . If all we care about is what R_x "sees", then we don't need all the information on the internal V_s and I_s inside the dotted lines. We get a much simpler situation if we replace the circuit in the dotted lines by its Thévenin Equivalent.



To find the Thévenin equivalent:

- First, we remove R_x . Now, we want to
- Find V_{oc} , the open circuit voltage.



Taking a KVL loop through the 2Ω resistor, the $2V$ source, and the 3Ω resistor:

$$(i_2)(2\Omega) + 2V + (i_1)(3\Omega) - V_{oc} = 0 \quad [1]$$

and KCL at the starred node gives

$$1A + i_1 = i_2 \quad [2]$$

One nice thing about calculating the open-circuit voltage for the Thévenin procedure is that the condition of an open circuit at the output simplifies the analysis. Since the output is an open circuit, then we know current $i_1 = 0$ and [2] simplifies to

$$1A = i_2 \quad [3]$$

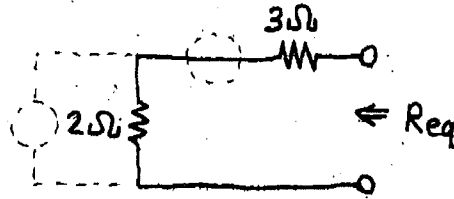
Substituting $i_1 = 0$ and $i_2 = 1A$ into [1] gives

$$(1A)(2\Omega) + 2V + (0)(3\Omega) = V_{oc}$$

$$V_{oc} = +4V$$

- Find R_t :

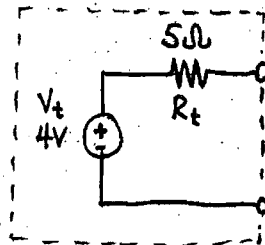
Turning off the current source results in an open circuit; turning off the voltage source results in a short circuit:



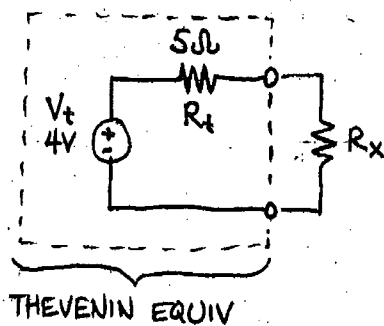
The equivalent resistance is just the series combination of

$$R_{eq} = 2\Omega + 3\Omega = 5\Omega$$

So, the Thévenin Equivalent with $V_t = V_{oc}$ and $R_t = R_{eq}$ is:



Now we can hook R_x back up, and the resulting circuit is easier to work with.



For example, if we want to know the voltage V_x across R_x , we can now use the voltage divider relationship to get an expression that gives V_x for any R_x :

$$V_x = 4V \left(\frac{R_x}{5\Omega + R_x} \right)$$

Which is the same as we would have gotten from grinding through the (much more complicated) math for the complete original (much more complicated) circuit.