Nodal Analysis Procedure

1 Select reference node (usually one end of a voltage source) mark with ground 0V symbol

2 Label node voltages (implied with respect to 0V reference)

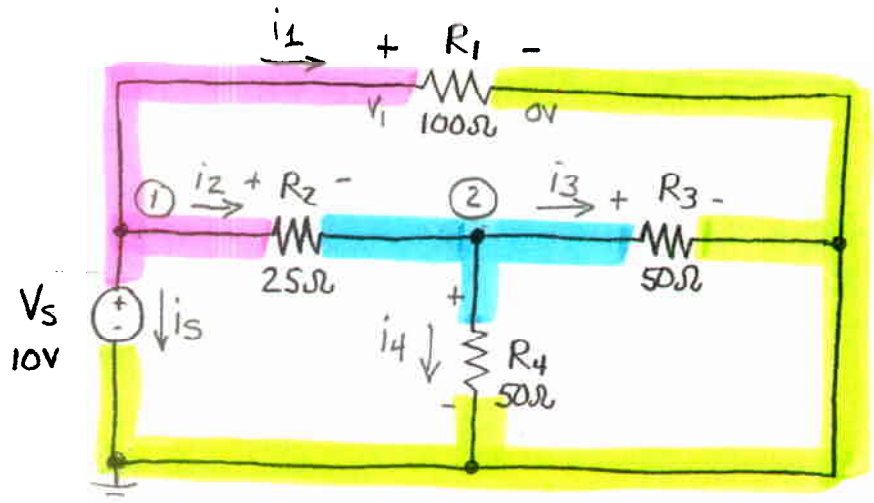
3 Label element currents

4 Write KCL equation at each node

5 Write element V-I constraints (may eliminate some “unknowns”)

6 Solve system of equations

If circuit is properly defined, procedure will always work



1 REF NODE: BOTTOM OF V_S

2 LABEL NODE VOLTAGES (ONLY 3 NODES: REF, ①, ②)

3 ELEMENT CURRENTS (PASSIVE REF CONFIG)

4 KCL AT EACH NODE

① $i_S + i_2 + i_1 = 0$

② $i_2 = i_3 + i_4$ OR $i_2 - i_3 - i_4 = 0$

5 ELEMENT V-I CONSTRAINTS

R_1-R_4: OHM'S LAW $V = iR \Rightarrow i = \frac{v}{R}$ } VOLTAGE DROP ACROSS R

$$i_1 = \frac{V_1 - 0}{R_1} = \left(\frac{1}{R_1}\right) V_1$$

$$i_2 = \frac{V_1 - V_2}{R_2} = \left(\frac{1}{R_2}\right) V_1 - \left(\frac{1}{R_2}\right) V_2$$

$$i_3 = \left(\frac{1}{R_3}\right) V_2$$

$$i_4 = \left(\frac{1}{R_4}\right) V_2$$

$i_S = ?$ VOLTAGE CONSTRAINT: $V_1 = V_S$ V_1 KNOWN

UNKNOWNS ARE i_S, V_2

6 SOLVE

6 SOLVE!

SUB INTO (1)

$$i_S + \underbrace{\left(\frac{1}{R_2}\right)\overbrace{V_1}^{V_S} - \left(\frac{1}{R_2}\right)V_2}_{i_2} + \underbrace{\left(\frac{1}{R_1}\right)\overbrace{V_1}^{V_S}}_{i_1} = 0 \Rightarrow -i_S + \left(\frac{1}{R_2}\right)V_2 = \left(\frac{1}{R_2}+\frac{1}{R_1}\right)V_S$$

SUB INTO (2)

$$\left(\frac{1}{R_2}\right)\overbrace{V_1}^{V_S} - \left(\frac{1}{R_2}\right)V_2 - \left(\frac{1}{R_3}\right)V_2 - \left(\frac{1}{R_4}\right)V_2 = 0 \Rightarrow \left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)V_2 = \left(\frac{1}{R_2}\right)V_S$$

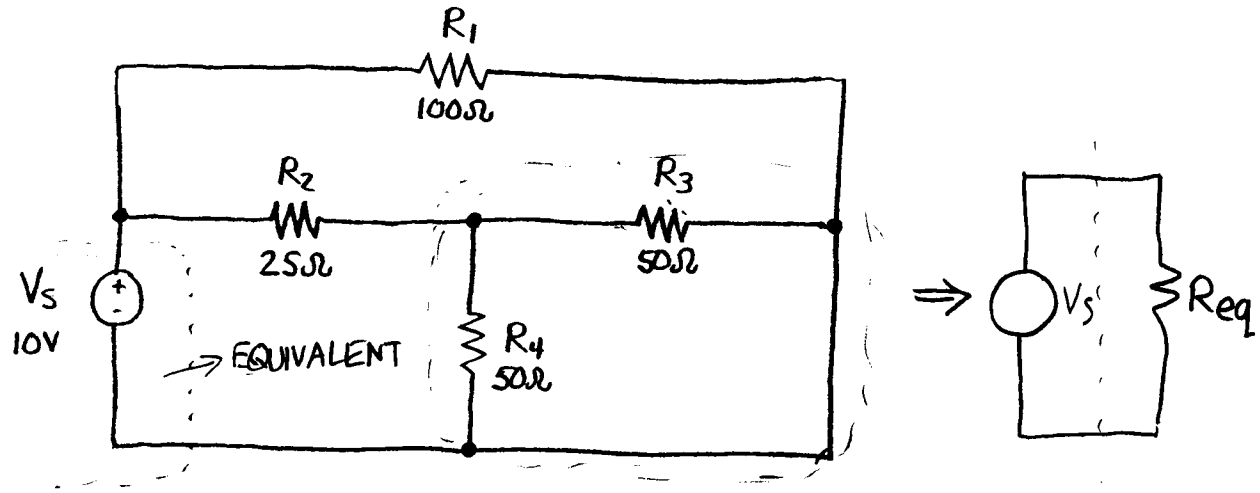
NUMERICALLY, IN MATRIX FORM

$$\begin{bmatrix} -1 & 0.04 \\ 0 & 0.08 \end{bmatrix}\begin{bmatrix} i_S \\ V_2 \end{bmatrix} = \begin{bmatrix} 1.4 \\ 0.4 \end{bmatrix}$$

SOLVING

$\Rightarrow V_2 = +5\,V$

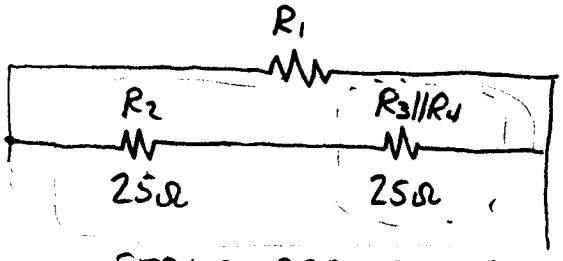
$i_S = 0.3\,A$



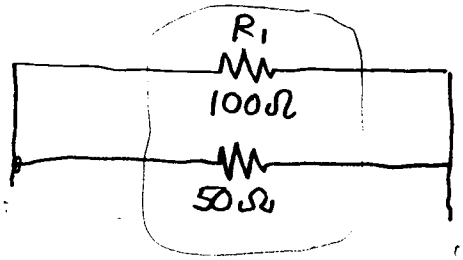
WHAT IF WE JUST NEEDED i_S?

USE SERIES, PARALLEL

$$\underbrace{R_3 \,||\, R_4}_{\text{"PARALLEL"}} = \frac{1}{\frac{1}{R_3}+\frac{1}{R_4}} = \frac{1}{\frac{1}{50\Omega}+\frac{1}{50\Omega}} = 25\Omega$$

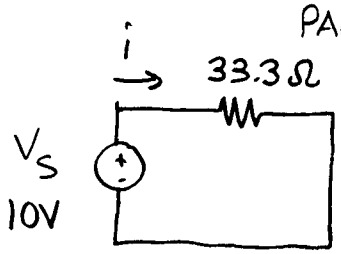


SERIES: $25\Omega + 25\Omega = 50\Omega$



$$\frac{1}{\frac{1}{100\Omega}+\frac{1}{50\Omega}} = 33.3\Omega$$

PARALLEL



OHM'S LAW: $i = \frac{10V}{33.3\Omega} = 0.3A$ ✓