ECE3204 D2015
HW Set 6

Due in class Friday May 1.

To make life easier on the graders:

- Be sure your NAME and ECE MAILBOX NUMBER are prominently displayed on the upper right of what you hand in.
- When appropriate, indicate answers with a box or underline
- Work as neatly as possible

All text problems from Sedra and Smith 6th edition

1) Text 16.1 (p. 1328) [Practice with lowpass transfer function magnitude]

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2) Text 16.6 (p. 1329) [Graphical representation of magnitude response]

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3) Text D16.15 (p. 1329) [Butterworth filter design]

"Design" for this problem does not mean to sketch a hardware realization; just specify the transfer function.

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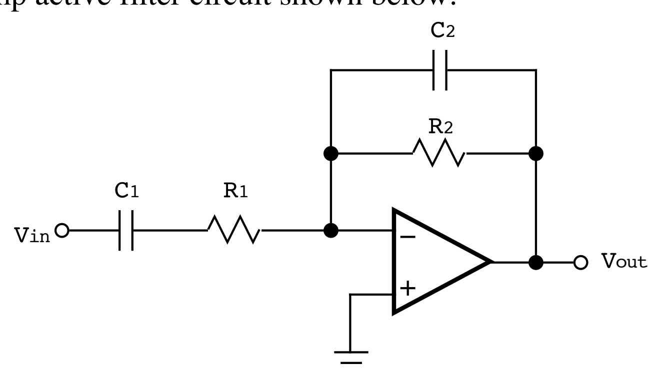
4) [Op-amp active filter]

Consider the op-amp active filter circuit shown below:



You may find it helpful to use the following substitutions:

$$\tau_1 = R_1C_1 \qquad \tau_2 = R_2C_2 \qquad \tau_N = R_2C_1$$

Assuming $\tau_1 >> \tau_2$,

a) Find the transfer function $T(s) = \frac{V_{out}}{V_{in}}$.

b) Sketch the pole-zero constellation in the s-plane.

c) Sketch the magnitude and phase Bode plots

d) What kind of filter is this?

5) [Filter design]

Design an active bandpass filter for an FM radio audio application. The lower and upper 3-dB frequencies should be 30Hz and 15kHz, and the midband voltage gain should be +32dB. The largest capacitor value should be no more than 2.2μF. A single pole (20dB/decade) rolloff in the stopbands is sufficient.

Be sure to specify the dynamic performance of any op-amp(s) used (both slew rate and gain-bandwidth product) so that a 10V peak output signal of any frequency in the passband can be processed without distortion.

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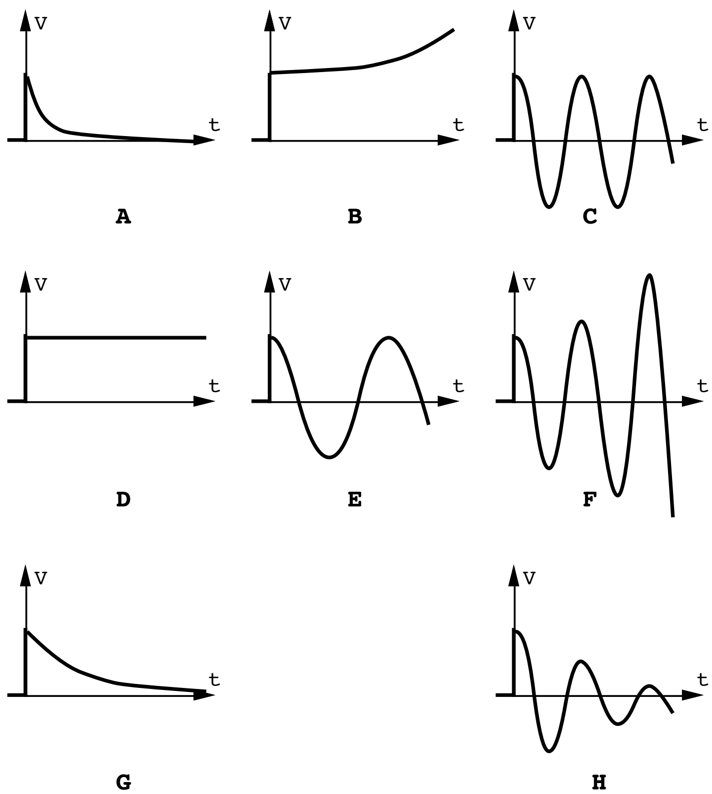
6) [Qualitative s-plane practice]

Sketch the pole locations in the s-plane that correspond to the following time-domain impulse responses. Superimpose the locations on the same plot, to show the relative locations corresponding to the various waveforms.

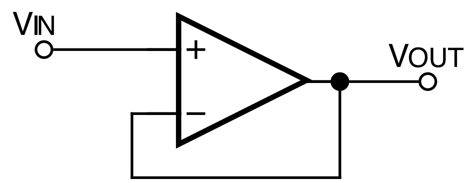


7) [Stability analysis practice]

An op-amp has a DC gain of $10^5 = 100{,}000$ and two poles: one at 100Hz, and one at 100 kHz.

a) Determine the expression for A(s), the op-amp open-loop gain transfer function. Sketch the magnitude and phase Bode plots $|A(f)|$ and $\angle A(f)$ for the op-amp open-loop gain. Or, if you prefer, $|A(\omega)|$ and $\angle A(\omega)$

The op-amp is to be used in the unity gain configuration:



b) Determine the feedback factor β, the fraction of the op-amp output that is fed back to the op-amp inverting input.

c) Sketch $|A|$, $|\beta|$, and $|A\beta|$, and use this plot to estimate the frequency at which the magnitude of the loop gain is equal to unity: when $|A\beta| = 1$.

d) Confirm your graphical estimate in (c) by using the expression in (a) to calculate a numerical value for the frequency at which $|A\beta| = 1$.

e) Sketch $\angle A\beta$ and use this plot to estimate the phase shift around the loop $\angle A\beta$ at the frequency at which $|A\beta| = 1$. What is the estimated phase margin? Indicate the phase margin on your plot.

f) Confirm your estimate in (e) with a calculation of the phase shift at the frequency from (d). What is the calculated phase margin?

g) Is this closed loop circuit stable?

8) [Stability analysis practice]

The same op-amp from the previous problem (DC gain of 10^5 = 100,000 and poles at 100Hz and 100kHz) is connected closed loop with a noninverting gain of 1000.

a) Sketch the circuit configuration.

b) Determine the feedback factor β, the fraction of the op-amp output that is fed back to the op-amp inverting input (both symbolic and numerical expressions).

c) Sketch $|A|$, $|\beta|$, and $|A\beta|$, and use this plot to estimate the frequency at which the magnitude of the loop gain is equal to unity: when $|A\beta| = 1$.

d) Confirm your graphical estimate in (c) by using the expression in (a) to calculate a numerical value for the frequency at which $|A\beta| = 1$.

e) Sketch $\angle A\beta$ and use this plot to estimate the phase shift around the loop $\angle A\beta$ at the frequency at which $|A\beta| = 1$. What is the estimated phase margin? Indicate the phase margin on your plot.

f) Confirm your estimate in (e) with a calculation of the phase shift at the frequency from (d). What is the calculated phase margin?

g) Is this closed loop circuit stable?

Hint: You should have found that the unity gain configuration for this op-amp in the previous problem was unstable, but the configuration with closed loop gain of 1000 was stable. The point of these two problems is that stability is not a property of only the op-amp – depending on the feedback, a particular op-amp may give stable or unstable performance with the feedback loop closed.

9) [Practice with stability, op-amp transfer functions]

A high performance op-amp has an open loop transfer function

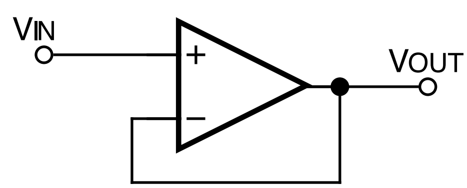
$$A(s) = \frac{(30{,}000)\left(1+\left[\frac{s}{2\pi(100kHz)}\right]\right)}{\left(1+\left[\frac{s}{2\pi(3kHz)}\right]\right)\left(1+\left[\frac{s}{2\pi(10kHz)}\right]\right)}$$

a) Determine numerical values for all pole(s) and/or zero(s) for A(s)

b) Plot the pole-zero constellation for the open-loop A(s) in the s-plane.

c) CAREFULLY sketch the magnitude and phase ($|A(\omega)|$ and $\angle A(\omega)$, or, if you prefer, $|A(f)|$ and $\angle A(f)$) Bode plot of the open-loop transfer function. Be sure to label and dimension your axes, and indicate any interesting points on your plot!

The op-amp is to be used in the unity gain configuration:



d) Determine the phase margin ϕ_M.

e) Indicate whether the closed-loop circuit will be stable or unstable.

10) Figure 6-10a shows a phase-shift oscillator: a cascade of three identical buffered RC lowpass stages, followed by an inverting amplifier. (For the purposes of this problem, the op-amps use ±10V supplies; assume the op-amps to be otherwise ideal.) A colleague is having trouble with the design and asks you for help.

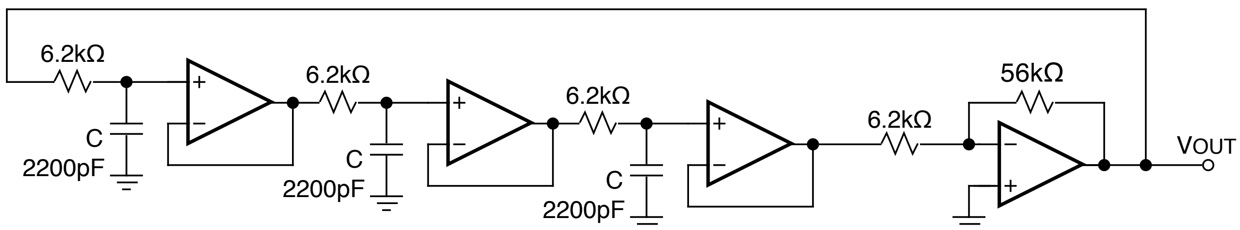
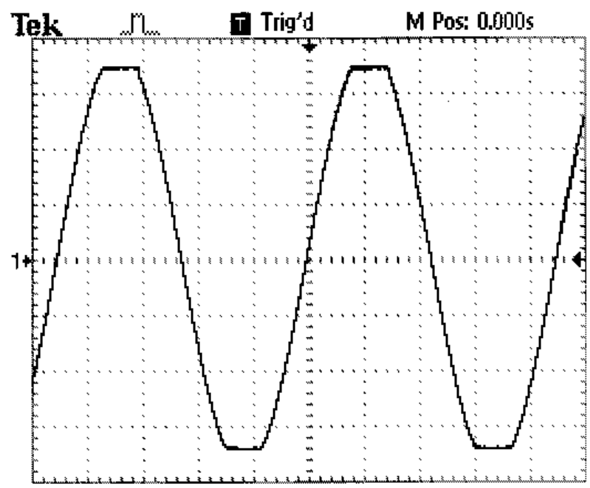


Figure 6-10a.

Your colleague expects a pure sine wave at the output but instead sees the clipped sine wave in Figure 6-10b:



To analyze the oscillator, you "break the loop" and inject a sinusoidal test signal as shown in Figure 6-10c.

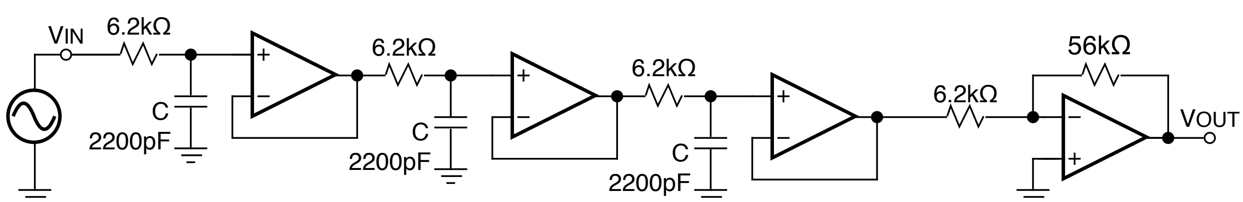


Figure 6-10c.

a) At what frequency $f_{\phi=0}$ will the total phase shift from V_{IN} to V_{OUT} equal zero? (Or, equivalently, at what frequency $f_{\phi=0}$ will V_{IN} to V_{OUT} be exactly in phase?)

b) With an input sine wave at $f_{\phi=0}$, what will be the gain magnitude from V_{IN} to V_{OUT}? (Or, equivalently, with an input sine wave at $f_{\phi=0}$, what is the magnitude of the transfer function v_{out}/v_{in}?)

c) Explain what is wrong: why is the output waveform a clipped sine wave, rather than a pure sine wave with a stable amplitude?

11) [Analysis techniques]

To determine the oscillation frequency f_{OSC} of the Wien bridge oscillator used in Lab 6, our analysis takes place in the frequency domain and considers (among other things) transfer functions, the s-plane, and phase shift as a function of frequency.

To determine the oscillation frequency of the Schmitt trigger oscillator used in Lab 3 (and shown below in figure 7-6 to refresh your memory), our analysis took place in the time domain to find the oscillation period

$$T = \frac{4R_2RC}{R_1}$$

The oscillation frequency was then determined as

$$f_{OSC} = \frac{1}{T} = \frac{R_1}{4R_2RC}$$

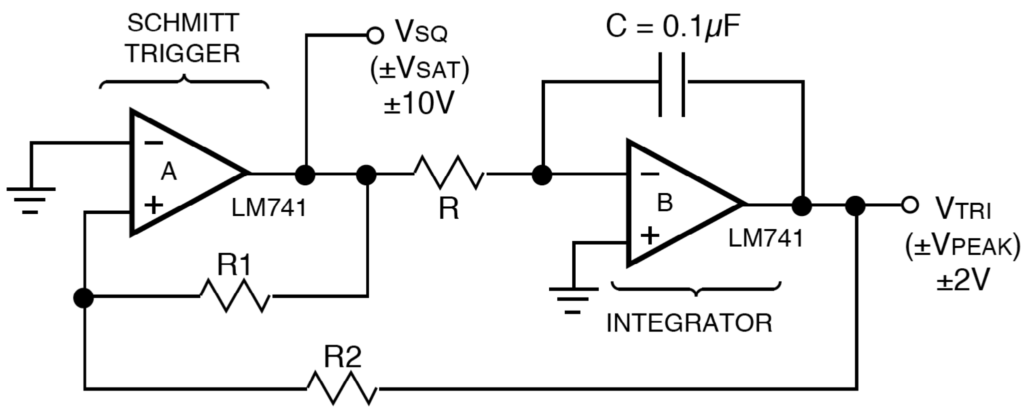


Figure 6-11.

The question is: Can the same frequency domain techniques used for the Wien bridge oscillator be used to determine the oscillation frequency for the circuit of Figure 6-11?

If so, use the frequency domain technique to perform the analysis and verify the expression for the oscillation frequency f_{OSC}. If not (frequency domain techniques cannot be used), explain why not.