Team Control Number

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Problem Chosen

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Summary Sheet

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Summary

Honeybees play a crucial role in sustaining delicate ecosystems around the world, helping plants reproduce through the process of pollination. Humans depend on these plants for food and other resources, so logically humans depend on bees. Although many understand their importance, fewer understand their recent decline in population. Several factors play a part in this worryingly dire situation, so modeling a hive’s population over time would unravel the mysteries regarding their mass disappearance and discover the severity of these threats.

In order to create a model for the bee population, we first modeled a perfect bee colony with an infinite size. We assumed a birth rate of 2000 bees per day and a death rate of 1000 bees per day. Using this initial model, we developed an equation to predict the population after a certain number of days.

Using this equation as a basis, we utilized a computer code in order to predict the population of the bee colony for Model Two. This model was implemented in Java SE8 using the Eclipse IDE. We added different starting conditions that varied depending on the seasons, for instance the birth rate in spring was 3,000 bees per day while the birth rate in fall was 1,000 bees per day. We also implemented a maximum of 80,000 bees per hive. If the population exceeded this limit, some number of bees would leave the hive. By changing the different starting conditions, we could simulate different scenarios. For instance, if a virus infected the colony, the birth rate would decrease. We also changed the different starting conditions in order to perform a sensitivity analysis, for example, halving the lifespan of the bees.

Assuming normal conditions, the bee population was steady throughout all four seasons, never dropping below 35,000 bees after the first year. For our sensitivity analysis, the population decreased as expected. We modeled what would occur if the lifespan and birth rate were cut in half, the male to female ratio doubled, three viruses being introduced, and two predator attacks, a bear and small animal. The three viruses and the bear attack completely destroyed the colony in our trials. The population was significantly reduced when the lifespan and birth rate were halved, while there was no major difference for increasing the male to female ratio and for the small animal attack.

The next part of the problem was to predict how many hives would be needed to support a 20-acre parcel of land containing crops that could be pollinated. To solve this, we found farmer-recommended spacing and hives for various crops, and modeled where the hives would be located for an acre of the crops, then scaled it up to the 20-acre of farmland.

Infographic

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Introduction

In the 2007 blockbuster *Bee Movie*, an anthropomorphic bee in the midst of collecting nectar for his hive describes pollination as "a little bit of magic" [1]. This can not be any closer to the truth; honeybees are incredibly important for maintaining the balance of delicate ecosystems and supporting human life, allowing various plants and crops to grow with the spreading of pollen. A sharp reduction in honeybee population could mean the total loss of entire crops, which would likely lead to famine and mass deaths in many areas. However, due to various factors including predators and disease, a noticeable decline has been observed in the 21st century. Colony Collapse Disorder (CCD) was a term created in 2007 to describe the decrease in honeybee populations all over the world. Environmental factors like climate change, deforestation, habitat destruction, natural disasters, increased predation, the use of pesticides, viruses, and parasites are all factors that can be directly connected to the decline in bee population.

Interpretation of Problem

We will develop a mathematical model to determine the population of a honeybee colony overtime. Then, multiple sensitivity analyses will be conducted on this model to determine which of the following factors causes the most damage to honeybee colony size:

- Changes in lifespan
- Changes in egg laying rates
- Ratio between males and females
- The impact of different predator attacks
- The introduction of the common Varroa Mite
- Spread of viruses within the bee population

After conducting multiple sensitivity analyses, we must model and predict how many hives will need to be placed in a 20-acre plot of farmland to support pollination. After, a one page infographic will be created to provide the public with the information and models developed.

Assumptions for all models presented

- The bees in this study are all western honey bees (*Apis mellifera*)
- The colony is in the northern United States
- The colony is wild and is not attended to by humans
- The initial population of the hive is 12,500 bees
- The minimum hive size is 10,000 bees (if the population goes below this, the hive will die)
- The maximum hive size is 80,000 bees
- The worker bees have a lifespan of 30 days in the spring and summer, and 140 days in the fall and winter
- The drone bees have a lifespan of 30 days

Model One: A Hive's Population Considering Birth and Death Rates

Description

We began our process by creating a model for an isolated colony of western honeybees that would have access to the typical plants for pollination and gathering food, but with no access to potentially harmful factors like predators, viruses, and environmental disasters. Additionally, we also assumed that the colony can support an infinite number of bees for the time being. To create this model, we assumed that the queen produces 2,000 eggs/day and about 1,000 worker bees die due to natural causes during foraging [2]. In making this model, we created a simple equation for a beehive population over time with no outside factors:

$$P = P_{\mathrm{i}} + (B - D)$$

Equation 1

This equation determines the total population after one day. This is later extrapolated by plugging in the total population from the previous day into the place of initial population of the next day and repeating this calculation of the days in a given season.

We then determined the D or deaths per day through the following equations:

$$\frac{1}{l} = \%\, population\ dying\ daily$$

Equation 1a

$$P_{\mathrm{i}} * \%pdd = D$$

Equation 1b

The variables used in these equations are defined in Table 1.

Table 1: Variables Used in Equations 1, 1a, & 1b		
Variable	Meaning	Explanation
P	Total population	Total population during a single season
P_{i}	Initial population	Equal to the total population from the previous season
d	Number of days in a particular season	Spring: 92.8 Summer: 93.6 Fall: 89.8 Winter: 89 [3]
B	Births per day	Equal to the egg laying rate of the queen
D	Deaths per day	Changes based on the lifespan of the bees during a particular season

1	Lifespan of a bee during a particular season in days	Spring: 4-5 weeks Summer: 4-6 weeks Fall: 3-5 months Winter: 4-6 months [4]

Results from Model One

The results of Model One can be seen below in Figure 1:

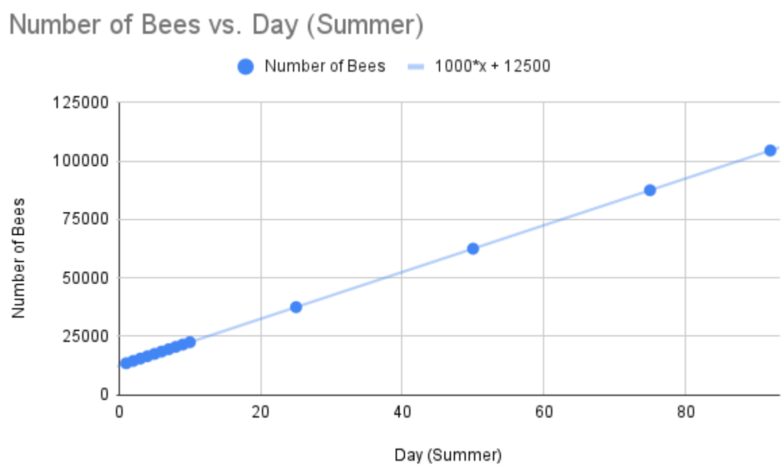

Figure 1: A starting model of an isolated colony of bees during the summertime

Analysis of Model One

In this model without outside factors or crowd control done by the hive, the “perfect summer hive” reaches a population of approximately 104,500 bees in total. This shows that, if neither external factors nor the birth rate changes, the population growth of the hive would be linear. However, the fact that this model does not have external factors nor a change in birth or death rates means that it is flawed. Additionally, an upper limit for the number of bees in a hive was not considered. The largest a typical beehive can get is about 80,000 bees, as any more would be overcrowded.

Although this model is overall unrealistic, it offers a good starting point to find the highest possible number of bees in a colony.

Model Two: A Hive’s Population Considering Internal Effects

Description

Our next step was to create a yearly model of the colony assuming there were still no outside influences like predators or diseases. We used the Eclipse IDE software in order to create the model, as this allowed us to conduct trials efficiently over multiple years. We used Equation 1 to model the population after every day. The code created is in Appendix 1. The parameters used are listed in Table 2:

Table 2: Parameters used for Model Two				
Parameters	Spring	Summer	Fall	Winter
B (bees/day)	3000	2000	1000	0
D (% of the population)	$\frac{1}{28} = 3.57\%$	$\frac{1}{28} = 3.57\%$	$\frac{1}{140} = 0.714\%$	$\frac{1}{140} = 0.714\%$
D_s (days)	93	93	90	89 (90 if leap year)
P_i (bees)	12,500 (This number changes in year 2)	*Calculated by the model*		

The model starts in the spring with an initial population of 12,500 bees. It calculates the population for the first day, then moves to the second day. This process continues until it reaches the end of spring, then it switches to using the parameters for summer. After going through the four seasons, the model loops back to the spring. Every four years, one day is added to winter because of the leap years, so the winter goes from 89 days to 90 days.

Additional factors affect the change in population in a beehive over time. Starting in the late fall, all of the drones in a colony will be kicked out of the hive. Drones serve only two purposes in a population of bees: eating and mating [5]. They do not have a stinger or produce honey, and therefore drain vital resources that the hive needs during the wintertime. To prevent this, the queen makes the workers expel these bees from the colony. This causes the hive to lose about 15 percent of their total population over the course of the late fall season, as 15 percent of the population are drones [5].

Another natural factor affecting population changes is the practice of swarming. Swarming is a natural way bees reproduce after a hive has become overcrowded [6]. 50 percent of the hive's population leaves along with the queen, flying elsewhere to begin a new hive. This is modeled by using a check in the form of an "if" statement. The statement constantly checks if the population is over 80,000. If the check succeeds, it will divide the population by 2, simulating swarming.

Results from Model Two

The results of Model Two can be seen below in Figure 2:

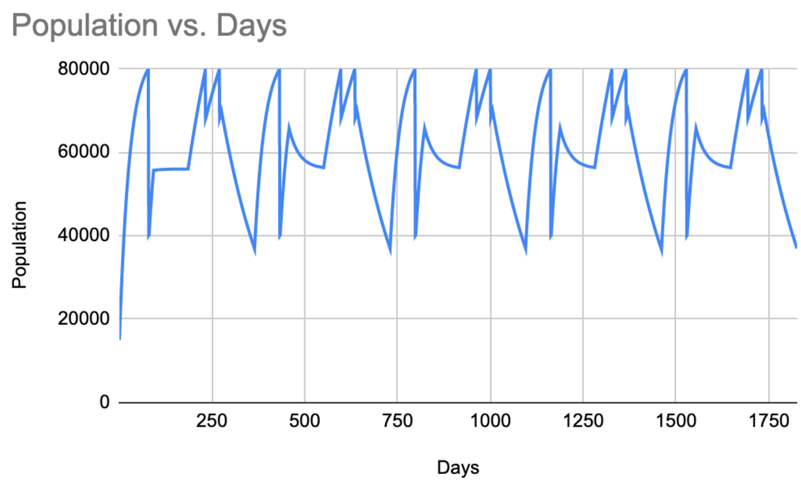

Figure 2: Model of how a honeybee hive population fluctuates due to seasonal behaviors such as drone eviction.

Analysis of Model Two

Model Two accounts for changes in birth rates and death rates, as well as swarming and drone eviction to see their effect on hive population. Swarming, which occurs when the bees sense that their hive is overcrowded, can be seen in sudden dips where the population suddenly drops from 80,000 to 40,000. These also happen regularly, and as seen in Appendix 1, the swarming is set to happen when the colony becomes overcrowded in the springtime, as this is when the birth rate surpasses the death rate quickly. This mirrors real life, with colonies swarming between the months of April and October [7]. Places in Figure 2 also show dips in the population before a massive drop occurs. These smaller dips occur during fall and show that the hive is getting too big and therefore starts to evict drones to save resources.

Some weaknesses include the abruptly changing birth rates per season and the absence of outside factors. The number of births and deaths per day instead of per season would give a more specific answer to the problem.

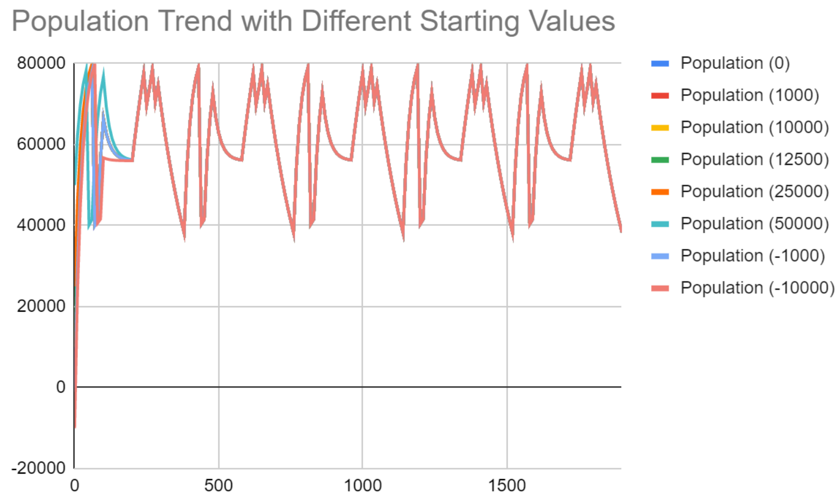

Figure 3: The results of Model Two with different starting populations

Another weakness can be found when changing the initial populations. As seen in Figure 3, even starting with initial populations of 0, -1000, and -10000 result in the same trend as starting populations of 1000, 10000, 12500, 25000, and 50000. The only difference is during the first year. This is likely due to two reasons, the first is how the birth rates work. The model assumes that there is the same birth rate even without a queen or any bees. The other reason is how the death rate is calculated. In equation 1b, the death rate is calculated by multiplying the percent of the population dying by the total population. If the population is negative, the death rate is negative as well. Therefore in equation 1, a negative death rate would be added to the population, causing it to increase.

This model still does not include outside influences, like viruses, parasites, and predators. Therefore, the next step is to find the damage caused by outside factors and the effect they have on bee population.

Model Three: A Hive's Population Considering External Effects

External Effects Considered For Model Three

Varroa Mites

Varroa mites are the most common honeybee parasite in the United States. Varroa mites live in almost every climate and area that western honey bees do, meaning they occur in almost every hive in the US [8]. The red-brown mites that attach themselves to the backs of western honeybees and feed on the hemolymph (the circulatory fluid or "blood" in honeybees) [citation]. This reduces the honeybee's lifespan significantly by harming brood size, flight ability, feeding patterns, and decreasing the weight of worker bees [9].

This infestation will always reduce the bee population to 0 assuming the hive is left unattended. Varroa mites also introduce new problems to the hive due to their feeding habits. Mites have the potential to transmit diseases between bees, so if one bee is infected with a disease a mite can quickly spread this disease to many other parts of the hive.

Deformed Wing Virus (DWV)

Deformed Wing Virus (DWV) is a disease that affects 85% of varroa mite infected bee colonies [10]. It is transmitted during the pupa stage of development [11] and causes shriveled wings, a bloated abdomen, decreased body size, discoloration, and negative impacts on learning and bee memory [12]. DWV reduces the lifespan of a bee from 4-6 weeks in summer and 4-6 months in winter to only 4-6 days. Since DWV directly harms the lifespan of bees, an equation can be created to predict the new death rate of bees infected and then the total population as well. These equations are expressed below in our model:

$$P_{v1} = P_i + (B - D_{v1})$$

Equation 3

$$D_{v1} = \left(\frac{1}{5}\right)P$$

Equation 3a

The new variables used in these equations are listed below in Table 3.

Table 3: Variables used in Equations 3 & 3a	
Variable	Meaning
P_{v1}	Total population of a hive infected with DWV
D_{v1}	Death rate of a hive with virus 1 (DWV)

Acute Bee Paralysis Virus (ABPV)

Acute Bee Paralysis Virus (ABPV) is a serious risk for beehives globally. It affects 20% of varroa mite infected hives [10]. It infects the jelly that nurse bees feed to developing larvae [13]. The infected larvae die before they are able to properly hatch, significantly affecting the birth rate. If untreated, a hive with ABPV will typically die within a single season. In our model, the equation expressing the updated birth rate is below:

$$P_{v2} = P_i + (B_{v2} - D)$$

Equation 4

$$B_{v2} = 0.25B_i$$

Equation 4a

Table 4: Variables for Equations 4 & 4a	
Variable	Meaning
P_{v2}	Total population of a hive infected with ABPV

B_{i}	Initial birth rate during a given season
B_{v2}	Birth rate of a hive infected with virus 2

Slow Bee Paralysis Virus (SBPV)

Slow Bee Paralysis Virus (SBPV) is a more uncommon virus transmitted by varroa mites, only affecting about 10 percent of the mite infected hives. SBPV paralyzes the legs of honeybees within 12 days of infection [10] and causes them to lose their ability to walk and fly. Without the ability to fly and collect food, infected bees usually die about 1 day after their paralysis [14]. The death rate of a hive infected with SBPV as well as its effects on the total population are expressed below:

$$P_{\mathrm{v3}} = P_{\mathrm{i}} + (B - D_{\mathrm{v3}})$$

Equation 5

$$D_{\mathrm{v3}} = \left(\frac{1}{13}\right)P$$

Equation 5a

Table 5: Variables for Equations 5 & 5a	
Variable	Meaning
P_{v3}	Total population of a hive infected with SBPV (virus 3)
D_{v3}	Death rate of a hive infected with SBPV (virus 3)

The next step was to create a model that included these outside influences. We analyzed each influence separately, starting with just DWV, then just ABPV, and finishing with just SBPV. For these viruses, we used the equations given in the previous section, which would determine their effect on the bee population.

We also modeled the effects of a bear and a raccoon or other small animal attack on the beehive, which we determined to happen on certain days of the year to see the effect of the attack. For the bear attack, we arbitrarily assumed that the bear instantly kills 20,000 bees and destroys the hive. The rest of the bees migrate away because the hive is destroyed, therefore reducing the population of the hive to 0. For the small animal attack, we arbitrarily assumed that it happens at the end of every season and that the small animal would take 5,000 bees + 10% of the total population.

For all of these influences, we assumed that if the population went below 10,000, the bees would die out, as 10,000 bees is approaching the minimum required bees to sustain a hive, which is around 7,500 worker bees plus the queen [15].

Results of Model Three

Figures 3 and 4 show what would happen if we introduced the viruses and predator attacks to the population.

If the hive had DWV, the lifespan would shorten to 5 days, for ABPV the birth rate was decreased by 75%, and for SBPV, the lifespan diminished to 13 days.

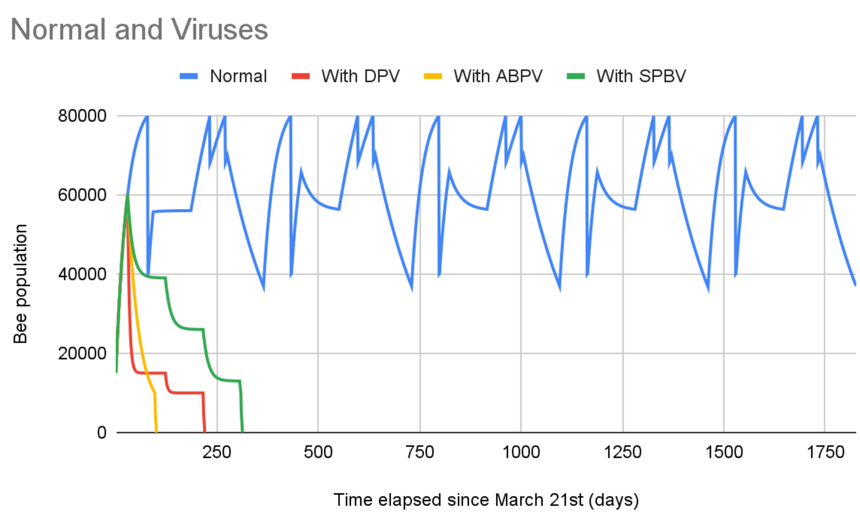

Figure 4: Line chart showing effect of viruses on bee population, with the blue line showing normal population trends

From this final model, our group was able to come to the conclusion that a colony contracting ABPV, a somewhat common disease, would be most devastating for a hive of western honeybees. The hive would die out almost instantly as this virus targets the larvae, the most vulnerable part of the entire colony. ABPV causes significant decline of the colony's birth rate, which does not allow the bees to have a chance to restore their population. Once the population hits about 10,000 bees, the hive is too small to sustain itself and dies shortly thereafter. It only takes about 101 days for a colony infected with ABPV to perish. DWV and SPBV would also destroy the population within a year. The change is most notable at the end of the season, likely due to the change in birth rates.

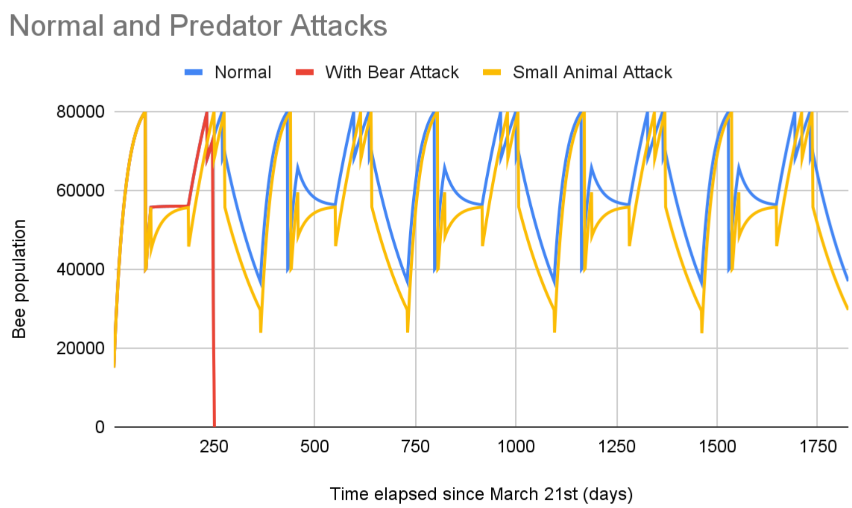

Figure 5: Line chart showing effect of various predator attacks on bee population, with the blue line showing normal population trends

As Figure 5 shows, the bear attack would have a devastating effect on a hive, reducing the population to 0 in a couple of days. However, the hive would be able to survive a small animal attack at the end of every season.

Analysis of Model Three

The results that we gained from the model for the viruses were similar to what we expected, which was that the hive would die within a year. Yet, similar to model 1, this model suffers from a lack of variety. The birth rate is constant during the season and abruptly drops at the end of the season, instead of a gradual change. Additionally, we estimated the chances and effects of the predator attacks on the hive. We assumed that the bear would destroy the beehive, which may not be true. We similarly assumed that the small animal would take a small portion and that it would come at the end of every season, which may also not be true. However, the data collected from Model Three allowed us to determine what was the largest factor that affected bee population size.

Sensitivity Analysis

We modified the parameters of the model to determine which factor would have the greatest effect on the population, shown in figure 6.

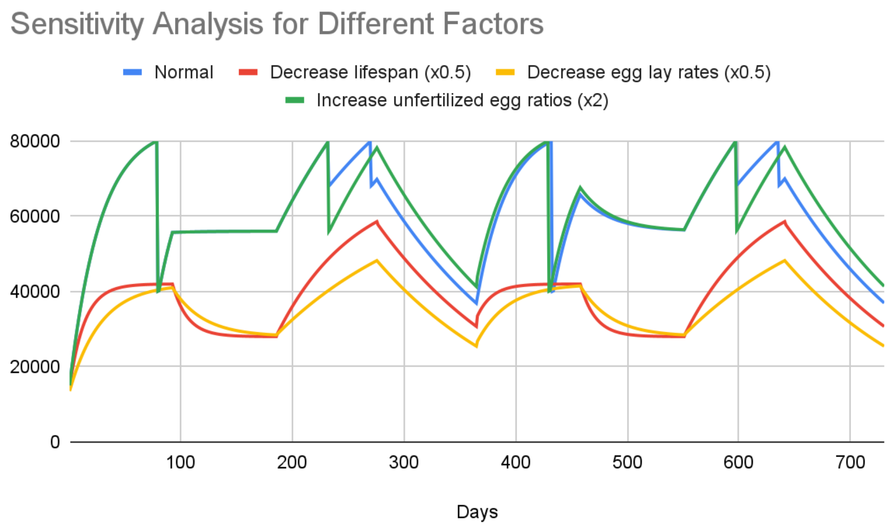

Figure 6: Different internal factors from the bee population compared to one another

Halving the lifespan and egg laying rates would decrease the population by a similar amount, although decreasing the egg laying rates would have a worse effect on the population than the lifespan. Increasing the fertilized to unfertilized egg ratios so that more unfertilized eggs are made (more males) would not have a great effect and is most notable in the winter, which is when the males are thrown out.

Changing these factors do not have as great of an effect on the population than the viruses and bear attack, but the lifespan and egg laying rates would have a greater effect than the small animal attack.

Model Four: Positions of Hives for a 20-acre Plot

Our last task is to create a model that would provide insight on how many beehives to place in a 20-acre plot of land containing various crops that would benefit from pollination. We achieved this by creating a fourth model. Our first step was to research common plants that would benefit from pollination and determine their recommended spacing as well as hive recommendations. We chose apple trees, watermelon, cucumber, blueberries, and blackberries. These crops all have unique planting instructions and a recommended number of hives per acre. The crops' recommended spacings were placed into Table 6:

Table 6: Common Crops that Benefit from Pollination and their Recommended spacing

Crop	Spacing (between plants) [16]	Spacing (between rows of plants) [16]	Recommended hives/acre [17]	Plants per acre
Apple	30’	30’	1.5	36
Watermelon	2’	4’	1.8	5,408

Cucumber	1’	3’	2.2	10,868
Blueberry	1’	3’	3	14,352
Blackberry	3’	10’	2.7	1,499

While this table depicts some commonly grown fruits and vegetables, but does not cover everything. It is recommended for beekeepers to only put one hive per acre of tomatoes or other vine plants as over-pollination can cause complications like bees fighting for resources and dying as well as blossom drop [17]. As you can see there is no correlation between number of plants per acre and number of hives, as it relates more to the number of flowers on each of these plants than it does the plants themselves. For example, watermelons can be closely spaced together allowing for a higher number of plants, but they produce less flowers in need of pollination then the average apple tree. Therefore, they require a similar number of beehives to sustain life.

After this we created graphical models using Eclipse to model row and column spacing in various crops. Figure 7 is an example of this.

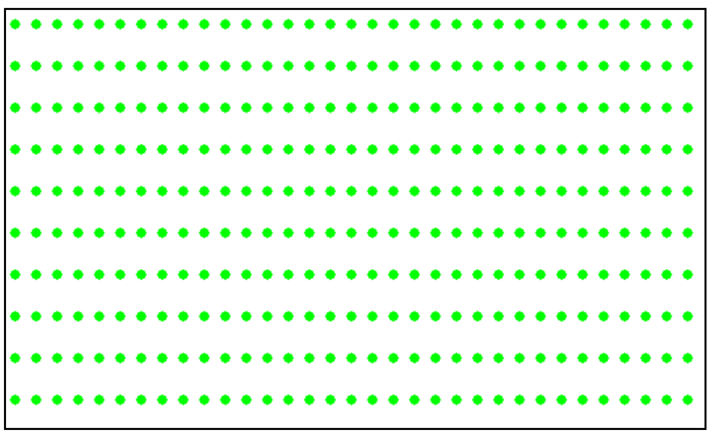

Figure 7: Watermelon crop recommended spacing

From these dot plots we created hive placement examples. For an apple orchard with trees spaced 30 ft apart in all directions, one or two hives should be placed in a horizontal orientation across the orchard. This allows for the best spacing when a 1 acre model is scaled up to 20 acres.

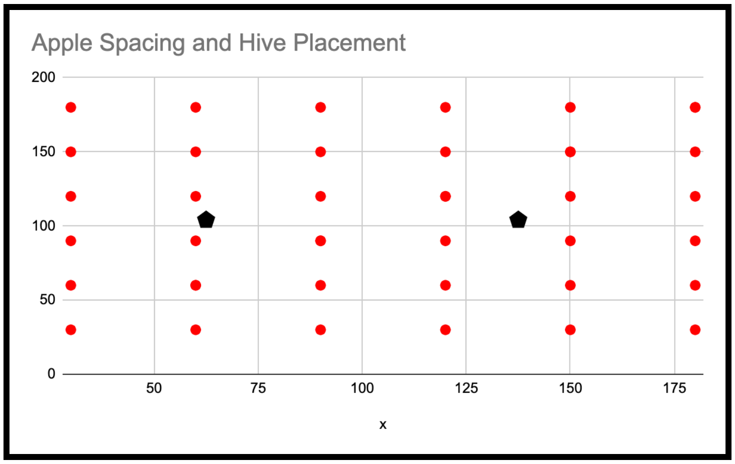

Figure 8: Ideal apple tree and honeybee hive placement. The black pentagons represent hives while the red dots represent apple trees.

After analyzing other types of crops, we determined that this table represented the best hive amount for a 20 acre plot of land. Using this model, crops should not be over pollinated, bees will have adequate food, and crops will thrive.

Table 7: Hives per 20-acre

Crop	Recommended hives/acre [17]	Our model hives/acre	Ideal hives/20 acre placement
Apple	1.5	2	40
Watermelon	1.8	2	40
Cucumber	2.2	2	40
Blueberry	3	3	60
Blackberry	2.7	3	60

Results and Conclusion

Using data from various sources we were able to develop a somewhat accurate model of the perfect beehive's fluctuations in population as well as how quickly a hive would die if attacked by viruses, a parasite, and a bear. This model was developed using a mixture of simple algebraic equations as well as computer science elements.

This model may be able to be used in a future app for beekeepers, environmental scientists, and wildlife protection officials to recognize when their hive population is dipping below normal. Ideally, beekeepers and researchers would be able to input their observed average lifespan during the four seasons, egg laying rates, and starting population to get a quick and accurate model of what their hive's fluctuations should look like. This model can be used as a reference and will allow beekeepers to notice when populations dip significantly and quickly assess the hive for disease before it becomes untreatable.

The results for our third model on beehive placement in fields aligns with national agricultural standards and will allow for specific guidance for farmers.

Some flaws of our model include the fact that habits like swarming and drone eviction before wintertime both happen instantaneously instead of the more realistic gradual change. If we had more time, we may be able to develop an algorithm for a start and end point of these natural habits and more realistically represent population changes through doing so. Another flaw is the fact this model is meant to have a starting population of 12,500 bees. Populations that start lower will grow incredibly quickly, which is unrealistic to what happens in nature.

Additionally, this model is best suited for the behavioral patterns and lifespans of the western honeybee, and may not be able to accurately portray the patterns of other species like the eastern honeybee. However, we do not believe that it would be too difficult to develop models for those species as well given what we know now. In fact, it may be easier as a few of these species are completely resistant to parasites and some diseases [10].

In conclusion, our group was able to develop a model that could be utilized by scientists to ensure the bee population globally remains at a high enough level to sustain agricultural practices and prevent mass starvation.

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Appendices

Appendix 1: Java code for yearly bee population assuming no outside influences

```
public class Bees {

	public static void main(String[] args) {

		int y = 1; // Starts at the first year
		double P = 12500; // Population
		int day = 0; // Days
		int b = 3000; // Birth rate
		double d = ((double)1/28)*P; //Death rate
		int winterDays = 89; // Days in winter

		while(y<=5) { // Goes through the first 5 years

		// Spring
		b = 3000; // Birth rate in spring is 3000 bees/day
		d = ((double)1/28)*P; //Death rate starts at 3.57% (1/28) of the population

		while (day<93) { // 93 days in spring
			P = P + (b-d); // Finds the new population after every day
			d = ((double)1/28)*P; // Changes the death rate to match the new population
			day++;

			if (P>80000) { // Swarming
				P = P/2;
			}
			System.out.println(P); // Prints the population every day
		}

		// Summer
		b=2000;
		while(day<186) { //93 days in spring + 93 days in summer
			P = P + (b-d);
			d = ((double)1/28)*P;
			day++;

			System.out.println(P);
		}

		// Fall
		b=1000;
		d=((double)1/140)*P; //New lifespan is 140 days
		while(day<276) { //186 days in spring and summer + 90 days in fall
			P = P + (b-d);
			d = ((double)1/140)*P;
			day++;

			if(P>80000) {
				P = 0.85 * P; // Males thrown out
			}
			System.out.println(P);
		}

		// Winter
		b=0;
		while(day<winterDays+276) { //276 days in fall, spring, and summer + 89 or 90 days in winter
```

```
                P = P + (b-d);
                d = ((double)1/140)*P;
                day++;
                System.out.println(P);
            }

            y++; // New year
            if(y%4==0)  // Leap year is every four years
                winterDays=90;
            else
                winterDays=89;
            }
        }
}
```