The Epsilon School

Distribution of Teachers

Megan Ashun, Gustavo Rodriguez, Jared Rosen, Abhinav K. Sharma

Problem Statement

Due to the increase in size of the student body of Epsilon School from 490 to 630, 7 more teachers will be hired. How can the new 7 faculty be distributed within each course in a way that is “fair”?

What defines fair? (I.E. Our Interpretation of the Problem)

"Fair" is the presence of as uniform of a student to teacher ratio as possible among all subjects. That is, there should be as low of a standard deviation from mean student-to-teacher ratio as possible:

Fair Example:

Class	Student-To-Teacher Ratio
Class A	10
Class B	11
Class C	8
Class D	9

$\bar{y}$ (mean ratio) = 9.5
s = 1
With a very small standard deviation, this set can be described as a rather fair assignment of teachers.

Unfair Example:

Class	Student-To-Teacher Ratio
Class A	4
Class B	17
Class C	8
Class D	11

$\bar{y}$ (mean ratio) = 10
s = 6
Comparatively, this set is less fair.

Assumption: Class Size

01

Since the total number of students in English equals 490, which is equal to the current size of the student body, the class sizes listed for English are the ‘23- ‘24 class sizes.

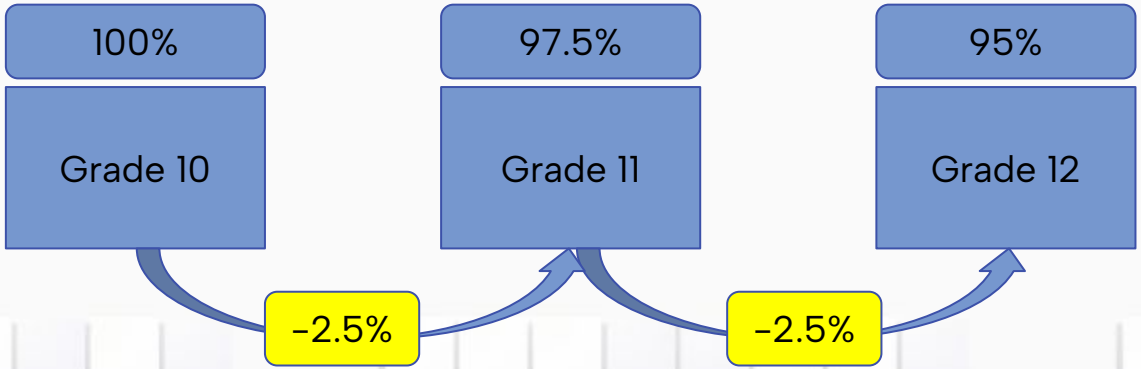
Course:	10th:	11th:	12th:	Total:
English	183	155	152	490

^^These are ‘23- ‘24 class sizes^^

Assumptions: Drop Out Rate

02

Given a 5% dropout rate, by the beginning of senior year, all students of that class have dropped out. Furthermore, 2.5% of people from sophomore to junior year drop out, aas do 2.5% of people from junior to senior year.



03 Assumptions: Number of Classes Per Student

There are six classes in a day and each student has a full schedule. This can be seen by dividing total number of students enrolled in all classes by class size, and doing the same thing at the school level, given 2023-2024 data:

	Incoming	10th	11th	12th	Total
Art		31	33	35	99
Biology		198	95	26	319
Chemistry		59	126	109	294
English	292	183	155	152	490
French		41	32	49	122
German		19	22	10	51
Spanish		51	26	33	110
Mathematics		184	201	262	647
Music		50	56	49	155
Physics		50	58	183	291
Social Studies		183	131	59	373
Class Size	300	183	155	152	490
		5.732240437	6.032258065	6.361842105	2951
					490
					6.02244898

Total number of students enrolled in courses

Class size (or school size)

Number of classes per student.

All round to six.

Assumptions: Class Interest

04 The percentage of people that take a given subject in each grade shall be consistent regardless of the changing qualities of the new student bodies.

Courses:	10th	11th	12th
Art	0.1694	0.2129	0.2303
Biology	1.0820	0.6129	0.1711
Chemistry	0.3224	0.8129	0.7171
English	1.0000	1.0000	1.0000
French	0.2240	0.2065	0.3224
German	0.1038	0.1419	0.0658
Spanish	0.2787	0.1677	0.2171
Mathematics	1.0055	1.2968	1.7237
Music	0.2732	0.3613	0.3224
Physics	0.2732	0.3742	1.2039
Social Studies	1.0000	0.8452	0.3882

These proportions, which are the number of students per subject divided by total number of students in each grade for 2023 to 2024, are to remain the same across different school years. For example, 23.03% of rising seniors will take art, just as 23.03% of current seniors are doing.

Assumptions: Music and Art

05

Music and art classes meet only every other day. Meaning, students are split up into different groups that rotate every other day. Therefore, the student to teacher ratio for music and art classes can be divided by two.*

*Credit to Shaurya Patni's group for this idea

We represented this as double teachers for both subjects

Assumptions: Foreign Language

06

Foreign Language teachers can split their time unevenly between different languages

–E.g. A teacher who can teach French and German can spend two thirds of their time teaching French and one third of their time teaching German.

Additionally, the current (3) language teachers can teach: French and Spanish, Spanish and German, and French and German.

Hypotheses

- Given that students can/do double up in taking math courses (assuming class sizes are in fact what we have asserted them to be), it is likely that the mathematics department shall receive two additional teachers.

- Classes with a larger student-teacher ratio shall need more teachers

Steps (“Variables” solved for)

1. Find the class sizes for the 2024-2025 school year.
2. Project how many students will take each class.
3. Evaluate which classes need more students and add them accordingly.
    a. This is based off the class with the highest student/teacher ratio, based off calculations from step 2
4. Determine the distribution of language classes

01 Process/Model Development

Find the class sizes for the 2024–2025 school year.

x, y, & z = original class sizes, subscripts correspond to grade level

23-24:

Incoming 10th Graders = a

Current 10th Grades = x_{10} = x

Current 11th Grades = y_{11} = 0.975y → y = y_{11}/0.975

Current 12th Grades = z_{12} = 0.95z

24-25:

Current 10th Graders = a = a

Current 11th Graders = x_{11} = 0.975x

Current 12th Graders = y_{12} = 0.95y

As per the assumptions for dropout rate, 2.5% was subtracted per year until senior year.

The following are the class sizes for rising juniors and seniors (i.e. sophomores and juniors in '23-'24) as per assumption #1:

x_{10} = 183

y_{11} = 155

z_{12} = 152

2.5% ← Annual Assumed Dropout Rate*Original Sophomore Class Size

0.025*183 = 5 students to drop out per year

183 - 5 = 178 rising juniors for 2024-2025 (x_{11} = 178)

Annual Assumed Dropout Rate*Original Junior Class Size

0.025*(155/0.975) = 4 students drop out per year

155 - 4 = 151 rising seniors for 2024-2025 (y_{12} = 151)

"140 more than the graduating senior year class, plus any students who dropped out during that scholar year"

152 + 9 + 140 = 301 incoming sophomores (a = 301)

Process/Model Development

02 Project how many students will take each class

Number of enrollments current year				
	10th	11th	12th	Total
Biology	198	95	26	319
Number of enrollments per student				
Biology	1.081967213	0.613	0.171	1.743
Expected enrollments new year				Number of students expected next year
Biology	325.6721311	109.0967742	25.82894737	460.5978527

Finding trends of prior classes:

Ratio of students who take each class in nth grade in a given year =
(Number of students in **t** class in **nth grade**)/(Current number of students in **nth grade**)

Number of students expected to take **t** class in the new **nth grade** =
(Ratio of students who take class in nth grade in a given year)*(Number of students in the new year)

Process/Model Development

Evaluate which classes need more students and add them accordingly.

03

Rank the student-teacher ratio from greatest to least (expected students calculated in step 2 divided by number of teachers for the subject), and add one teacher starting at the top. Continue adding to the highest student teacher ratio present all seven new teachers are exhausted.

Course	#Of Faculty	Av. # of students per teacher	
Mathematics	6	132	+1
English	5	126	+1
Language*	3	122	+1
Chemistry	3	117	+1
Biology	4	115	+1
Av. Ratio	33	114	
Physics	3	110	+1
Social Studies	5	102	
Music*	2	98	
Art*	2	62	
	Original		

Course	#Of Faculty	Av. # of students per teacher	
Mathematics	7	113	+1
English	6	105	
Social Studies	5	102	
Music*	2	98	
Av. Ratio	39	96	
Biology	5	92	
Language*	4	92	
Chemistry	4	88	
Physics	4	83	
Art*	2	62	
	Phase 1		

Course	#Of Faculty	Av. # of students per teacher	
English	6	105	
Social Studies	5	102	
Mathematics	8	99	
Music*	2	98	
Av. Ratio	40	94	
Biology	5	92	
Language*	4	91	
Chemistry	4	88	
Physics	4	83	
Art*	2	62	
	Phase 2		

Process/Model Development

04a.

Determine the distribution of language classes

New number of language teachers: 4
French students: 153
German students: 66
Spanish students: 147

Class	Total Students	# Faculty
French	153	4
German	66	
Spanish	147	

To get the fairest student to teacher ratio, The ratio of each class' teachers (to the nearest sixth) should be as close to the ratio of students per class.

Ratio: 147:153:66 → 147+153+66 = 366 = 4(r)
r = 91.5, so dividing the ratio by 91.5 will give you the optimal ratio of teachers:
(147:153:66)/91.5 ~ 1.61:1.67:0.72

The closest ratio, to the nearest sixth, is 1 ⅔ Spanish teachers, 1 ⅔ French teachers, and ⅔ German teachers.

Process/Model Development

Determine the distribution of language classes

04b.

To find which two languages we need to know the new demand of language classes in comparison to the original distribution

To find the original distribution, the same process can be used as in the previous calculation:

Class	Total Students	# Faculty
French	122	3
German	51	
Spanish	110	

122:51:110 → 122+51+110 = 283 = 3r
r = 94.3 → (122:51:110)/94.3 = 1.29:0.54:1.16 ~ 1 ⅓ : ½ : 1 ⅙

Difference: 1 ⅔ : 1 ⅔ : 1 ⅔ – 1 ⅓ : ½ : 1 ⅙ = ⅓ : ⅙ : ½ → By hiring a new teacher who can speak Spanish and French, one of the German teachers can teach ⅙ more German while the new teacher fills in for the remaining demand for French and Spanish classes. Therefore, the new Language teacher should be able to teach French and Spanish.

Solution

Based on the results of the previous calculations, we found that we should hire the following teachers:

- 2 Math Teachers
- 1 English Teacher
- 1 Chemistry Teacher
- 1 Biology Teacher
- 1 Physics Teacher
- 1 Language Teacher, who can teach Spanish and French

Why Is Our Model Effective (Strengths)?

- Even distribution of dropouts neatly accounts for unpredictability of students
- Takes into account the variability in course loads of various grades of students
- Low standard deviation of 13, 7.6 excluding art class
- All student-to-teacher ratios are within two standard deviations of the average ratio (using 7.6 and ignoring art)
- Allows uneven distribution of language teachers, allowing for more precise decision making for faculty

Course	#Of Faculty	Av. # of students per teacher
English	6	105
Social Studies	5	102
Mathematics	8	99
Music*	2	98
Av. Ratio	40	94
Biology	5	92
Language*	4	91
Chemistry	4	88
Physics	4	83
Art*	2	62
	Phase 2	

Evaluating Model (Weaknesses)

- Does not take into account how a new group of students may act differently from the previous class (i.e. percentages for each subject for each grade most likely wouldn't be the same year to year - assumption 4)
- Does not take into account how scheduling of students among six blocks will create different class sizes, not properly reflecting student-to-teacher ratios used in calculations.
- Art student-to-ratio is incredibly lower than all other subjects

Future Work

- Applying these methods to other schools with different class sizes, number of departments, student to teacher ratios
- Optimize model by taking into account different level classes (AP, Honors, CP)
  - Accounting for teacher education
- Construct a model that unites all variables considered, one where values can be plugged into each of the variables to produce output (i.e. develop a formula)

Acknowledgements

Mrs. Burns – Assistance with the 5% dropout rate

Shaurya Patni – Music and Art rotation assumption

Thank You!

Any Questions?