

AP Stats -Linear Regression Ttest, One-Way ANOVA, and Two-Way ANOVA

Math

Modeling

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Linear Regression t-test One-Way ANOVA











Linear Regression t-test

What is a Linear Regression t-test used for?

A linear regression t-test is used to identify if there is a significant linear relationship present between two variables.









What are Assumptions and Conditions for a Linear Regression t-

test?

- Independence between every (x, y) pair
- Linear Relationship between x and y
- Normal Distribution of y-values at every x-value
 - Or, Normal Distribution of the Residuals*
- No obvious patterns in the Residual Plots
- <u>(Homoscedasticity)</u>

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*Residual = predicted y - actual y
```







How is Linear Regression T-Test Calculated?

$\begin{array}{ll} p = presence \ of \\ linear \ relationship \\ between \ x \ and \ y \end{array} \begin{array}{ll} H_0: \ \beta_1 = 0 \\ H_A: \ \beta_1 >, <, =/= \ 0 \end{array} \begin{array}{ll} Parameter: \ \beta_1 \\ Statistic: \ b_1 \end{array}$

 $SE(b_{1}) = S_{r}/(S_{x}((\sqrt{(n-1)}))) T_{df}(0, SE(b_{1}))) S_{x}^{T} = \sqrt{((\sum(y_{act}-y_{pred})^{2})/(n-2))} T_{df}(0, SE(b_{1}))) S_{x}^{T} = n = n = 0$ $t = (\beta_{1} - 0)/SE(b_{1}) T_{df}(0, SE(b_{1})) T_{df}(0, SE(b_{1})) S_{x}^{T} = n = 0$ p = tcdf(t, UB, df) 0

 S_r = Standard Error of Residuals S_x = Standard Error of x-values n = sample size

TI-84: Stat>Calc>LinRegTTest (F) TI-Nspire: Menu>Statistics>StatTests>LinRegTTest (A)

Example (Taken from Abhinav's AP STATS HW Assignment)

A high school counselor was interested in finding out how well student grade point averages (GPA) predict ACT scores. A sample of the senior class data was reviewed to obtain GPA and ACT scores. The data are shown in the table to the right.



GPA	ACT	
3.25	24	
2.87	21	
2.66	18	
3.33	22	
2.87	22	1
3.21	22	
2.76	18	-
3.91	28	
3.55	29	- 1
2.55	18	
2.44	20	
3.22	24	
3.01	21	
3.44	24	
3.22	25	

Is there evidence of an association between GPA and ACT score?

Source Regression Residual	Sum of Squares 123.041 34.5589	df 1 13	Mean Square 123.041 2.65838	F-ratio 46.3
Variable	Coefficient	s.e. of Coef	f t-ratio	prob
GPA	7.39697	1.087	6.80	≤ 0.0001

Independence √

- <u>Linear Relationship</u> √
- Normal Distribution of the Residuals \checkmark
- <u>Homoscedasticity</u> √

Just state:

- Hypotheses ($\beta_1 = 0, \beta_1 = /= 0$)
- b₁(7.397)
- $df (n-2 \rightarrow 15-2 \rightarrow 13)$
- SE(b₁) (1.087)
- t (6.80)
- p (≤ 0.0001)
- Reject H₀





What is a One-Way ANOVA test used for?

In plain terms, given more than two population means, the One-Way ANOVA test determines if there is *any* significant difference between any of the population means. However, it not specify *which* means differ.



What are Assumptions and Conditions for One-Way ANOVA?

- **<u>Random</u>** Data is used (either through Random Assignment or Random Sampling)
- Data is **Independent** (both within and between groups)
- The distributions of each group is <u>Normal</u>; or if, not the sample size is large enough to ensure a normal sampling distribution (n > 20)
- **Equal Variance**: The standard deviations among the different groups compared is roughly equal
 - General rule of thumb: largest standard deviation should be no more than twice the smallest one. [according to <u>this</u> source]

How is One-Way ANOVA Calculated?

Parameter: µ (population average of [])

 $H_0: \mu_1 = \mu_2 = \dots = \mu_n$

 H_A : $\mu_i = /= \mu_j$ is true for at least one pair of means within the set of groups.

1: Calculate Group Means

Group #	a_1	a ₂		a _n
	i_1	i_1	i_1	i ₁
Individual	i_2	i_2	i_2	i_2
Values				
	i _n	i _n	i _n	in
Average	\overline{a}_{1}	\overline{a}_2		\overline{a}_n



3: Calculate Between-Group...



What if sample sizes differ?

How is One-Way ANOVA Calculated (Cont.)?

TI-84: Stat>Calc>ANOVA (H) 2nd>Distr>Fcdf (0) TI-Nspire: Menu>Statistics>StatTests> ANOVA (C)

dfw

5: Calculate F-Critical Value

 $F_{crit}(df_{B}, df_{W})$

6: Calculate F-Statistic, P-Value, Judge H₀

$$F_{crit}(df_{B}, df_{W})$$
$$F = \frac{MS_{B}}{MS_{W}}$$

	∝ = (.05				1	1			
Tab	le A.6" C	ritical Valu	es of the F	² Distributi	on la la a	-	1		-	
	1	2	a	4	0.05(11,12		*			→
1	161.45	100.40	215 21	224.58	230.16	211.00	716.77	718 88	240.54	
2	18.51	19:00	19.16	19.75	19.30	19.33	10.15	19.17	19.18	
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	
5	6.61	5.79	5.11	5.19	5.05	4.95	4.88	4.82	4.77	
6	5.00	5.14	4.76	4.53	4 39	4.28	4.71	4.15	4.10	
7	5.59	4.74	4.35	4.12	3.97	3.87	3.70	3.73	3.68	
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.19	
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	
0	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	
1	4.84	1.98	3 50	1.16	3.20	3.09	3.01	2.05	2.90	
2	4.75	1.80	3.49	3.26	311	3.00	2.91	2.85	2.80	
3	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	
4	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	
5	4.54	3,68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	
6	4.89	3.63	3.24	3.01	2.85	2.74	266	2.50	2.54	
7	4.45	1.99	3.20	2.96	2.81	2.70	2.61	2.55	2.49	
8	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	
9	4.38	3.52	3.13	2.90	2.74	2.63	251	2.48	2.32	
0	4.35	3,49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	
1	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	
2	4.30	3.44	3.05	2.87	2.66	2.55	2.46	2.40	2.34	
3	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	
4	4.26	3,40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	
5	4.21	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	
6	4.23	3.37	2.98	2.74	2.59	2.47	2.19	2 32	2.27	
7	4.21	1,35	2.96	2.73	2.57	2.46	2 37	2.31	2.25	
8	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	
9	4.18	3,33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	
0	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	
0	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	
0	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	
0	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	
	3.8.5	3.00	3.60	3 37	2.24	2.10	2.01	1.04	1.69	

One-Way ANOVA Example

Ethan measures the length of three species of insects. Random sample is taken, and data area shown below:

	μ = Population Average of Length Among Different Insect Species (cm)
/	$H_0: \mu_1 = \mu_2 = \mu_3$
	$H_A: \mu_1 = /= \mu_2; \mu_2 = /= \mu_3; \mu_1 = /= \mu_3$
	(∝ = 0.05)

- <u>**Random</u> √ (Random Number Generator on TI-83 +)**</u>
- Independent √ (Reasonable to Assume)
- **<u>Normality</u>** \checkmark (see below- A (top), B (bottom) on left, C on right)
- <u>Equal Variance</u> √

A: SD= 2.378; B: SD = 2.300; C: SD = 2.658







Insect Species #	А	В	С
	9	6	4
	4	3	0
	4	4	4
	2	8	6
Length (cm)	6	8	3
	6	2	8
	8	7	7
	7	4	0
	4	4	3
	9	2	3
Average Length (cm)	5.9	4.8	3.8

One-Way ANOVA Example (Cont.)



One-Way ANOVA Example (Cont.)

Table A(> F-DistributionProbability Table



F = 3.35crit

F

(either p or f can be used... but result f either should confirm the other)

6: Calculate F-Statistic, P-Value, Judge H_n

dfw

$$p = Fcdf(F, UB, df_{B}, df_{w})$$

$$p = Fcdf(1.83, 999999, 2, 27)$$

$$p = 0.1785$$

According to this One-Way ANOVA test, p = 0.1785, which is above \propto = 0.05. Thus, the null hypothesis is not rejected. There is not statistically significant evidence to suggest that there is a difference in length among any two species of the insects measured.

Tab	le A.6" C	ritical Valu	es of the F	Distributi	0.0	-	1		_
				J	0.05 (12, 12)			
12	1	2	а	4	5	6	7	8	9
1	161,45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
2	18.51	19:00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8,81
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5	6.61	5.79	5,11	5.19	5.05	4.95	4.88	4.82	4.77
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8	5.32	4,46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2,70	2.65
15	4.54	3,68	3.29	3.06	2.90	2.79	2.71	2.64	2,59
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17	4,45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18	-4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19	4.38	3.52	3,13	2.90	2.74	2.63	251	2.48	2.32
20	4.35	3,49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21	4.32	3,47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23	4.28	3,42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24	4.26	3,40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25	4.21	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27	4.21	1.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29	4.18	3,33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	196
00	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88



What is a Two-Way ANOVA test used for?

Unlike a one-way ANOVA test which evaluates the impact of one variable across multiple groups, the two-way ANOVA test evaluates the impact of *two* variables across multiple groups, and whether or not there are significant relationships and/or interactions.



What are Assumptions and Conditions for Two-Way ANOVA?

Equal Variance: The standard deviations among the different groups compared is roughly equal

-

-

- General rule of thumb: largest standard deviation should be no more than twice the smallest one.
- The distributions of the data within all groups is **normal**; or, if not, they have a larger sampling size and/or the distribution of residuals (the distribution of error) is normal.
 - **Independence** among groups. The samples should not be matched or paired in any way.

How is Two-Way ANOVA Calculated (Using <u>Example</u>)?

1: Calculate Means

-Averages for all combinations of A and B.

- -Averages for i
- -Averages for B -Averages for A and B in total.

	Drug A	Drug B	
	6	4	
	4	5	
Males	7	6	
	9	7	
	3	5	Gender Averages
Male Drug Averages	5.8	5.4	5.6
	8	3	
	3	5	
Females	5	9	
	8	2	
	6	3	
Female Drug Averages	6	4.4	5.2
Drug Averages	5.9	4.9	5.4 (Total Average)

There are *three* pairs of null-alternative hypotheses. H_{A0} : There is no significant difference among results for factor A. H_{A1} : There is a significant difference among results for factor A.

 H_{B0} : There is no significant difference among results for factor B. H_{B1} : There is a significant difference among results for factor B.

 H_{AB0} : The two factors in question are independent. H_{AB1} : The two factors in question are not independent.

- **Equal Variance/SD** \checkmark
 - When SD is calculated out for all groups, all values are in a similar range.
- <u>Normality</u> √
 - Normal shape of distribution displayed for all groups
- **Independence** ✓ (Reasonable to assume)

<u>Ex.</u> These data are of the reduced blood pressure among patients, stratifying for both drug type and gender.

Calculating Two-Way ANOVA (con.)

2: Calculate Sum of Squares/Degrees of Freedom

	Drug A	Drug B	
	6	4	
	4	5	
Males	7	6	
	9	7	
	3	5	Gender Averages
Male Drug Averages	5.8	5.4	5.6
	8	3	
	3	5	
Females	5	9	
	8	2	
	6	3	
Female Drug Averages	6	4.4	5.2
Drug Averages	5.9	4.9	5.4 (Total Average)

 $SS_{tot} = SS_A + SS_B + SS_{AB} + SS_{err}$ A $SS_{AB} = SS_{btw} - (SS_A + SS_B)$ $SS_{tot} = \Sigma(6 - 5.4)^2 + (4 - 5.4)^2 + ... + (3 - 5.4)^2$

Square of Difference of Each Individual Data Point from Total Average. $SS_{tot} = 84.8$

$$df_{tot} = npq - 1$$

N = number of people per group p/q = number of levels for factors A/B, respectively.

$$df_{tot} = (5)(2)(2) - 1$$

 $df_{tot} = 19$

 $o_{btw} =$

 $(5.4)^2$

$$df_{btw} = p \cdot q - 1$$

= 2 \cdot 2 - 1
= 3
$$\sigma_{btw}^2 = \frac{SS_{btw}}{df_{btw}} = \frac{7.6}{3} = 2.53$$

 $\sigma_{tot}^2 = \frac{SS_{tot}}{df_{tot}}$ σ_{tot}^2 $=\frac{84.8}{19}$ $\sigma^2_{tot} = 4.46$

$$SS_{btw} = 5 * \Sigma(5.8 - 5.4)^{2} + (5.4 - 5.4)^{2} + (6 - 5.4)^{2} + (4.4 - 5.6)^{2} + (4.4 - 5.6)^{2} + (4.4 - 5.6)^{2} + (5.4 - 5.6)^{2$$

B

Calculating Two-Way ANOVA (con.)

$$SS_{A} = 5 * 2((5.9 - 5.4)^{2} + (4.9 - 5.4)^{2})$$
$$SS_{A} = 5$$

Here, n, number of samples per groups, is n = 5. Multiply the sum of the squares of the differences by 2, as there are 2 levels (both for Factors A and B)

$$df_{A} = 2 - 1$$
$$= 1$$
$$\sigma_{A}^{2} = \frac{SS_{A}}{df_{A}} = \frac{5}{1} = 5$$

D
$$SS_B = 5 * 2((5.6 - 5.4)^2 + (5.2 - 5.4)^2)$$

 $SS_B = 0.8$

$$\sigma_B^2 = \frac{SS_B}{df_B} = \frac{0.8}{1} = 0.8$$

 $df_{\rm B} = q - 1$ $df_{\rm B} = 2 - 1$

$$SS_{AB} = SS_{btw} - (SS_{A} + SS_{B})$$

$$SS_{AB} = 7.6 - (5 + 0.8)$$

$$SS_{AB} = 1.8$$

$$df_{AB} = (p - 1) \cdot (q - 1)$$

$$df_{AB} = 1 \cdot 1$$

$$= 1$$

$$\sigma_{AB}^{2} = \frac{SS_{AB}}{df_{AB}} = \frac{1.8}{1} \text{ [1.8]}$$
Here, the sum of the squared differences for each combination of treatments A and B is calculated.
$$SS_{err} = \Sigma(6 - 5.8)^{2} + (4 - 5.8)^{2} + ... \Sigma(4 - 5.4)^{2} + (5 - 5.4)^{2} + ... + \Sigma(8 - 6)^{2} + (3 - 6)^{2} + ... \Sigma(3 - 4.4)^{2} + (5 - 4.4)^{2} + ...$$

$$SS_{err} = 77.2$$

 $df_{err} = (n-1) \cdot p \cdot q$ = 4 \cdot 2 \cdot 2 = 16 $\sigma_{err}^2 = \frac{SS_{err}}{df_{err}} = \frac{77.2}{16} = 4.83$

Calculating Two-Way ANOVA (con.)

3: Calculate F-Statistic, P-Value, Judge H₀'s

$F_A = \frac{\sigma_A^2}{\sigma_{err}^2} = \frac{5}{4.83}$	$rac{\sigma_B^2}{\sigma_{err}^2} = rac{\sigma_B^2}{\sigma_{err}^2} = rac{0.8}{4.83} = 0.17$	$F_{AB} = \frac{\sigma_{AB}^2}{\sigma_{err}^2} = \frac{1.8}{4.83} = 0.373$	TI-Nspire:
$df_{A} = 1$	$df_{\rm B} = 1$	$df_{AB} = 1$	Menu>Statistics>Stat Tests>
$df_{\rm err} = 16$	$df_{\rm err} = 16$	$df_{\rm err} = 16$	ANOVA I wo-way (D)

$$p_{A} = Fcdf(F_{A}, UB, df_{A}, df_{err})$$

$$p_{A} = Fcdf(1.04, 999999, 1, 16)$$

$$p_{A} = 0.324$$

$$\begin{split} p_{_B} &= Fcdf(F_{_{B'}} \; UB, \; df_{_{B'}} \; df_{_{err}}) \\ p_{_B} &= Fcdf(0.17, \; 9999999, \; 1, \; 16) \\ p_{_B} &= \; 0.689 \end{split}$$

$$\begin{split} p_{AB} &= Fcdf(F_{AB}, \, UB, \, df_{AB}, \, df_{err}) \\ p_{AB} &= Fcdf(0.373, \, 9999999, \, 1, \, 16) \\ p_{AB} &= \, 0.55 \end{split}$$

According to the Two-WAY ANOVA Test, for all tests, including variation of blood pressure reduction among drug type, variation of this metric among gender, as well as the interaction between these two factors, p = 0.324, 0.689, and 0.55, respectively. Thus, is for all three cases, the null hypothesis is not rejected. There is no statistically significant evidence to suggest that blood pressure reduction varies among different levels for either the gender or drug type factor. Nor is there statistically significant evidence to suggest that there is any significant interaction between these factors.

Senior STEM Project Statistical Tests

From Monday, 27 November 2023 1:45 PM to Thursday, 30 November 2023 10:00 PM, a form for information on statistical procedures used for Seniors' STEM Project was made available. 19 of 47 Seniors responded.

Type of Data	Count	Proportion
Categorical	5	0.2174
Quantitative	17	0.7391
Other	1	0.0434
Total	23	1

Distribution of Type of Data Collected

Source of Data	Count	Proportion
Human Subjects	6	0.2308
Experimental Design	7	0.2692
Random Sampling	0	0
Observational Data	6	0.2308
Generated Data (AI, ML Model, etc.)	7	0.2692
Total	26	1

Distribution of Source of Data Collected

Conclusions:

- Counts > People Sampled (indicative of checkboxes)
- Lack of Sampling Methodologies
 - Makes Sense: More Experimental Vulnerabilities/Overall Task of STEM Project
- Emphasis on Quantitative Data
- Similar amount of observational data and experimental data (Multifarious methodologies)

What statistical test(s) did you perform as part of your STEM Project? 19 responses



Conclusions:

- More quantitative data \rightarrow higher proportion of Senior used T-tests
 - As a corollary, use of ANOVA, LinReg T-Test, etc.

Anomalies:

- Post-Hoc Tukey Test
- Computational Modelling
 - Image Overlay
 - Confusion Matrix and Accuracy
- Wilcoxon Signed Rank Test

References/Extra Resources

Linear Regression T-Test: *Stats: Modeling the World* - David E. Bock, Paul F. Velleman, Richard D. De Veaux (4th Edition)

One-Way ANOVA: <u>https://milnepublishing.geneseo.edu/natural-resources-biometrics/chapter/chapter-5-one-way-analysis-of-variance/</u> <u>https://www.statslectures.com/</u> <u>https://www.youtube.com/watch?v=hJ4Bvh3YhPY</u>

Two-Way ANOVA: https://www.youtube.com/watch?v=7OY0DdFyJBg https://www.youtube.com/watch?v=9zIUbbWwYV8

Image Credit for F-Distribution: <u>https://www.yumpu.com/en/document/view/31022748/table-a-f-distribution-probability-table-table-a6-critical-values-</u>