# MA590 Topology – Syllabus

Instructor: William J. Martin

Office: SH 305A

Phone: 831-6764

MA 590 – Spring 2017

Special Topics: Topology

Text: "Topology", 2nd ed.

e-mail: martin@wpi.edu by James R. Munkres

**Office hours:** MW 4:15 to 5:15

or by appointment Meetings: 3:00-4:15pm Mon & Wed, SH309

We all hear that the donut and the coffee cup are indistinguishable to a topologist. Intuitively, topology is the study of smooth deformations. If one views mathematics as primarily the study of functions, then topology is the study of continuous functions. If one views mathematics as the study of precise structures, then topology is the study of topological spaces (ordered pairs  $(X, \mathcal{T})$  where  $\mathcal{T}$  is a topology on set X). Putting these together, we can view topology as the study of the *category* of topological spaces, where the objects of study are topological spaces and the morphisms are continuous functions between them.

But why study topology? Without going into a discussion of the history of the subject, I think it is safe to say that this "qualitative geometry" arose out of a desire to understand examples of spaces that "looked like" (or behaved like) our most obvious spaces — n-dimensional Euclidean space, spheres, balls — but were different at some level. What makes a torus different from a sphere or a Klein bottle? What are the possible shapes of the universe? Must a mathematical object which looks "locally" like n-space be the same as n-space at the global level? And, if not, what tools can we develop to distinguish the two? In analysis, we study convergence of sequences and convergence of functions — are these really different? How is it that a sequence can converge in one metric but not in another?

For me, though, topology presents the mathematician with a fundamental set of tools for seeing a problem differently. Algebraic topology, in particular, provides us with effectively computable invariants that capture both the local and global "shape" of a space. Since we are also interested in applications of topology, we will see at least one in this course. Topological data analysis is a young subject which applies the tools of algebraic topology to the study of large data sets. This gives us one indication of the power of the subject and the innovative ways to adapt applied problems so that they are amenable to topological analysis.

## TERM SCHEDULE

Here is a rough outline of what we will cover in the 28 meetings we have together:

| Chapter 1    | Review of Set Theory               | 1 week  |
|--------------|------------------------------------|---------|
| Chapter 2    | Topological Spaces and             | 2 weeks |
|              | Continuous Functions               |         |
| Chapter 3    | Connectedness and Compactness      | 3 weeks |
| Chapter 4    | Countability and Separation Axioms | 3 weeks |
| Chapter 7    | Complete Metric Spaces             | 3 weeks |
|              | and Function Spaces                |         |
| Applications | Topological Data Analysis          | 2 weeks |
|              | and more (as time permits)         |         |

#### WORKLOAD

Each student is expected to attend and participate in every scheduled class meeting. Students will regularly be called upon to prepare lecture material and present it at the board.

Each week, a number of exercises will be assigned for students to complete on their own or in groups. Some of the problems will not be collected for credit. The for-credit problems will be further broken down into two parts: assignments will consist of up to ten short-answer questions together with roughly three problems which require rigorous proofs. All problems submitted for credit are to be completed individually, without the aid of the internet or other persons.

We will have an in-class midterm exam on Wednesday, March 1st and an in-class final exam on Monday, May 1st.

Your grade will be apportioned as follows: %25 on your presentations to the class; %25 homework; %25 mid-term; %25 final exam.

## INFORMATION ON THE WEB

The course web page is

http://www.wpi.edu/~martin/TEACHING/current.html

#### ACADEMIC HONESTY

Each student is expected to familiarize him/herself with WPI's Academic Honesty policies which can be found at

http://wpiacademicintegrity.weebly.com

All acts of fabrication, plagiarism, cheating, and facilitation will be prosecuted according to the university's policy. If you are ever unsure as to whether your intended actions are considered academically honest or not, please see me.

## STUDENTS WITH DISABILITIES

Students with disabilities who believe that they may need accommodations in this class are encouraged to contact the Disability Services Office (DSO) as soon as possible to ensure that such accommodations are implemented in a timely fashion. The DSO is located at 157 West Street, (508) 831-4908.

If you are eligible for course adaptations or accommodations because of a disability (whether or not you choose to use these accommodations), or if you have medical information that I should know about please make an appointment with me immediately.