MA535 ALGEBRA
W. J. Martin
September 16, 2015

Assignment 2

DUE DATE: Tuesday, September 22 at the beginning of class.

Please carefully review Dr. Martin's assignment presentation rules on the back of this sheet.

Provide neat and careful solutions to the following five problems:

1. Let p be a prime and let R_p denote the set of rational numbers whose denominators are relatively prime to p. Prove that R_p forms an abelian group under ordinary addition of rational numbers. *[HINT: First, be sure to rigorously define this set.]*

2. (a) If G is a group and for all $a \in G$, $a^2 = 1_G$, then G is abelian;

   (b) If G is a group and for all $a, b, c \in G$, $ab = ca$ implies $b = c$, then G is abelian.

3. (a) Exercise 21 on p60

   (b) Exercise 22 on p61

4. Exercise 17 on p65

5. (a) Prove that $\{+1, -1\}$ is a subgroup of $\mathbb{R}^\times$ and $\mathbb{R}^\times/\{+1, -1\} \cong \mathbb{R}^+$, the group of positive real numbers under multiplication.

   (b) Let G and H be groups, $K \trianglelefteq G$ and $L \trianglelefteq H$. Prove that $K \times L$ is a normal subgroup of $G \times H$ and that

   $$(G \times H) \Big/ (K \times L) \cong (G/K) \times (H/L) \ .$$

   (c) Prove that the special linear group[1] $SL(n, \mathbb{R})$ is a normal subgroup of the general linear group $GL(n, \mathbb{R})$ and that

   $$GL(n, \mathbb{R}) \Big/ SL(n, \mathbb{R}) \cong \mathbb{R}^\times \ ,$$

   the multiplicative group of non-zero real numbers.

[1] These are matrices of determinant one.

BASIC RULES FOR ALGEBRA ASSIGNMENTS

I) Each student must compose his/her assignments independently. However, rough work may be done in groups;

II) Write legibly and use only one side of each sheet of paper;

III) Show your work. Explain your answers using FULL SENTENCES;

IV) Late assignments will not, in general, be accepted for credit.