Coding Theory
MA533/CS525D, Spring 2015
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Coding Theory Assignment 2

DUE DATE: Monday, February 16, in class.

Please make sure your solutions are clearly legible. Please use only **one side** of the paper to improve readability.

1. Let $\mathcal{C}$ be an $[n, k, d]_q$-code with parity check matrix H. Prove:
   (i) $d(\mathcal{C}) \geq 2$ iff H has no all-zero column
   (ii) $d(\mathcal{C}) \geq 3$ iff no column of H is a scalar multiple of any other column of H
   (iii) $d(\mathcal{C}) \geq 4$ iff $d(\mathcal{C}) \geq 3$ and, in the projective geometry $PG(n-k-1, q)$, the set of columns of H forms a configuration with no three points collinear[1]

2. Let $\mathcal{C}$ be a binary linear $[n, k, d]$-code. The *even subcode* of $\mathcal{C}$ is the set of all codewords in $\mathcal{C}$ whose Hamming weight is even. Prove that either every codeword in $\mathcal{C}$ has even weight, or the even subcode of $\mathcal{C}$ is a (linear) $[n, k-1, d']$-code for $d' = 2\lfloor (d+1)/2 \rfloor$.

   In the latter case, give an efficient description of the dual code.

3. Prove that the dual of an MDS code is also an MDS code.

4. Problem 3.7.1 in Van Lint (p40 in 2nd ed.)

5. Problem 3.7.3 in Van Lint (p40 in 2nd ed.)

6. Problem 3.7.9 in Van Lint (p40 in 2nd ed.)

[1] In linear-algebraic terms, this means no three columns of H are collinear.