MA4891 Game Theory – Test 2

YOUR NAME: Time: 50 minutes.

Important Instructions: This test consists of 4 questions. Please answer all questions right on these pages, beginning each answer just below the statement of the question. If you need extra space (or space for rough work), use the back of the page, clearly indicating to which question the work pertains.

Dr. John Nash

Dr. Peter Morris

    "Life is not a game.
Still, in this life, we choose the games we live to play."
- J.R. Rim

1.) [*10 points*] (a) Apply the simplex method to find an optimal strategy for Column Player for the 2×2 (zero sum) matrix game given by

$$M = \begin{bmatrix} 1 & 8 \\ 3 & -4 \end{bmatrix}.$$

(b) When you are done, without further optimization, show how to read off an optimal strategy for Row Player.

2.) [*10 points*] Find all Nash equilibria for the two-person bi-matrix game (A, B) given by

$$A = \begin{bmatrix} 1 & 8 \\ 3 & -4 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 & 4 \\ 3 & -2 \end{bmatrix}$$

Finish with a table that lists all equilibrium pairs (p^*, q^*) together with the corresponding payoffs $\pi_1(p^*, q^*)$ and $\pi_2(p^*, q^*)$ for each.

3.) [*10 points*] (a) What is a *polyhedron*?

(b) Is it true that every polyhedron is a compact set? Explain.

(c) Prove that every polyhedron is a convex set.

4.) [*10 points*] Let M be an $m \times n$ matrix game.

(a) Define the *row value* $v_r(M)$.

(b) Define the *column value* $v_c(M)$.

(c) Prove that $v_r(M) \leq v_c(M)$.